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Use of Non-Linear Programming in Designing the Self-Starting Permanent Magnet Synchronous Micromotors

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Abstract: In this paper the possibility of a flexible use of non-linear mathematical programming is shown both for the optimal design of a self-starting permanent magnet synchronous micromotor and for resolving the problem of identification of its parameters without any essential changes of the program.

Keywords: Non-linear programming, Optimal design, Identification, Self-starting synchronous micromotor.

1 Introduction

The successful solution of the optimal design problem is the main aim of analysis of the electromagnetic processes in self-starting permanent magnet synchronous micromotors [1]. Formally, formulating this problem from mathematical point of view is not complicated because it comes to the general problem of non-linear programming [2]

$$\min\{f(\boldsymbol{x})|\boldsymbol{x}\in\boldsymbol{R}^n\} \tag{1}$$

where $\boldsymbol{x}=\left[x_1x_2\ldots x_n\right]^T$ is a vector of the variable quantities in Euclidean space $E^n(\boldsymbol{x}\in E^n)$ and $\boldsymbol{R}$ is the region of space, where the constraints of the problem are satisfied:

$$\boldsymbol{R}=\{\boldsymbol{x}|h_j(\boldsymbol{x})=0,g_j(\boldsymbol{x})\geq 0\forall j\}. \tag{2}$$

In (2) $h_j(\boldsymbol{x}),(j=\overline{1,m})$ are equality-constraints, and $g_j(\boldsymbol{x})\geq 0,(j=\overline{m+1,p})$ are inequality-constraints.

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The use of non-linear programming for finding out an extreme of a function at the presence of constraints brings often to some difficulties as an arithmetic interruptions of the program owing to the discrepancy of the co-ordinates of the current point in investigation space, and the parameters of the micromotor. Because of this, the problem of optimal design of a self-starting permanent magnet synchronous micromotor is carried out using the approximating functions method.

The objective function and the constraints are replaced by second order full polynomials of the kind [3]

$$\widetilde{Y} = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} b_{ij} x_i x_j + \sum_{i=1}^{n} b_{ii} {x_i}^2, \tag{3}$$

which are obtained on the base of a rotatable or orthogonal compositional design in accordance with the theory of experimental design. The code values of the factors presenting independent variables in the problem are expressed as

$$x_i = \frac{X_i - X_{i0}}{I_i}, \tag{4}$$

where X_{i0} is the value of the i-th variable X_i in the center of the design, and I_i is one-sided interval of variation of this variable. Consequently, the approximating expressions obtained by (3) are valid at $|x_i| \leq 1$, $i = \overline{1,n}$.

2 Optimal Designing of the Micromotor

2.1 Mathematical Modelling of the Problem

The object of investigation in this paper is a self-starting permanent magnet synchronous micromotor with one group of unshaded poles and one group of shaded poles of the stator. At the boundary of both groups the adjacent poles have identical polarity. A micromotor of this type is offered in [4].

A scheme of an unfolded air of the disposition of stator poles is shown in Fig. 1. A characteristic feature of this design is that the last two unshaded poles following the sense of rotation are displaced from its symmetric position backwards by an angle of $2\varphi_0$ and φ_0 , respectively. The angle $\varphi_j = 2\varphi_0$ is determined in accordance with the condition that the displaced stator pole interacting with the permanent pole of the same polarity creates the maximum force in desired sense of rotation. The value of this angle can be determined by the formula [5]

$$\varphi_j = \frac{1}{2}\left(\varphi_i + \frac{\delta}{R}\sqrt{\frac{27+\sqrt{665}}{2}}\right), \tag{5}$$

where:

φ_i is the angle of displacement of shaded poles from its symmetric position;

δ is the air gap value;

$R = R_M + \delta$ (R_M is the outer radius of the permanent magnet).

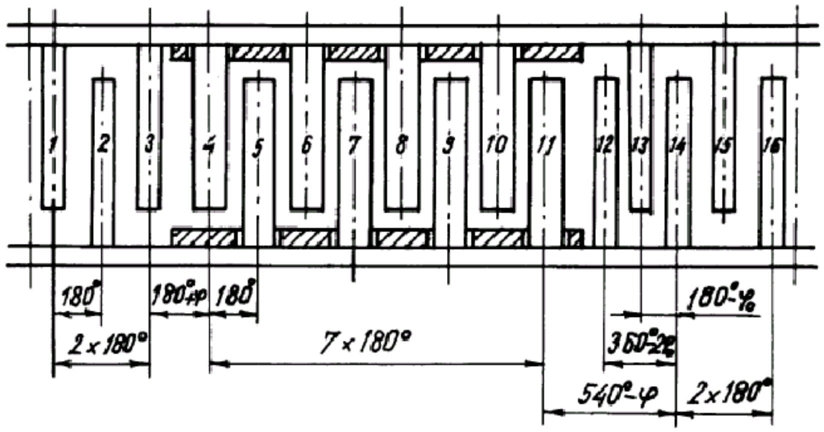

Fig. 1. Arrangement of the stator poles.

When starting designing, the internal impedance of the shade is determined approximately by the subroutine ECR9, when the influence of the frequency on the current distribution through the section of the shade is not taken into account. At the final stage of optimal design, a more precise value of the internal impedance of the shade [6] can be obtained by the finite element method [7].

In order to estimate more precisely the starting regime of the micromotor, the influence of the scalar magnetic potential, created by the permanent magnet within the stator poles, is taken into account.

Since some of the variable quantities in the problem have discrete values (for instance, the wire diameter of the exciting winding), when resolving the problem all variables as continuous values are considered, and finally their values are rounded-off.

At given outer diameter and axial length of the self-starting permanent magnet synchronous micromotor, its optimal design comes to solving the following problem of the non-linear mathematical programming:

Find the minimum of the function

$$f(\boldsymbol{x}) = -\frac{2}{\pi}\left(\widetilde{Y}_2 - \widetilde{Y}_1\right), \boldsymbol{x} \in E^4 \tag{6}$$

at the constraints:

$$
\begin{aligned}
& g_1(\boldsymbol{x}) = 1 - |x_1| \geq 0; \\
& g_2(\boldsymbol{x}) = 1 - |x_2| \geq 0; \\
& g_3(\boldsymbol{x}) = 1 - |x_3| \geq 0; \\
& g_4(\boldsymbol{x}) = 1 - |x_4| \geq 0; \\
& g_5(\boldsymbol{x}) = 1 - \frac{\widetilde{Y}_4}{\Theta} \geq 0; \\
& g_6(\boldsymbol{x}) = \frac{\tau + e_j - e_i - 0,5(a_{c1} + a_{c2})}{1.2 \times 10^{-3}} - 1 \geq 0; \\
& g_7(\boldsymbol{x}) = \frac{\tau - 0.5 e_j - a_{c1}}{0.3 \times 10^{-3}} - 1 \geq 0,
\end{aligned}
\tag{7}
$$

where:

τ is the pole pitch;

a_{c1}, a_{c2} is the width of the unshaded and shaded stator pole, respectively;

e_j is the displacement of the axis of the last unshaded pole backwards of rotation direction from its symmetric position;

e_i is the displacement of the axis of each shaded pole from its symmetric position;

$\widetilde{Y}_1, \widetilde{Y}_2$ is the approximating function for the electromagnetic torque of the forward and inverse rotating magnetic field, respectively;

$\widetilde{Y}_4$ is the approximating function for the overheat of the exciting winding;

Θ is the admissible overheat.

For a vector of variables is assumed

$$
\boldsymbol{x} = \left[V_{1m} \; \beta_1 \; \beta_2 \; \varphi_i \right]^T, \tag{8}
$$

where:

V_{1m} is the amplitude of the scalar magnetic potential of unshaded poles;

β_1, β_2 is the unshaded and shaded pole width in parts of the pole pitch, respectively;

φ_i is the phase angle between the scalar magnetic potentials of unshaded and shaded poles.

2.2 Numerical results

For a micromotor with outer diameter $D = 42$ mm, axial length $l_0 = 18$ mm, and outer diameter of the permanent magnet $D_M = 20$ mm at voltage $U = 220$ V, frequency $f = 50$ Hz, and admissible overheat $\Theta = 50$ K, the following optimal solution is obtained:

Value of the objective function $f(\boldsymbol{x}^*) = -19.60964 \times 10^{-4}$ and optimal vector

$$\boldsymbol{x}^* = \begin{bmatrix} x_1^* \\ x_2^* \\ x_3^* \\ x_4^* \end{bmatrix} = \begin{bmatrix} 0.14431221 \times 10^{-1} \\ 0.9545939 \\ -0.9799074 \times 10^{-1} \\ 0.9961612 \end{bmatrix} \quad \text{or} \quad \boldsymbol{X}^* = \begin{bmatrix} V_{M1m}^* \\ \beta_1^* \\ \beta_2^* \\ \varphi_i^* \end{bmatrix} = \begin{bmatrix} 70.7 \\ 0.5940 \\ 0.8483 \\ 51.92 \end{bmatrix},$$

where:

$$x_1 = \frac{V_{M1m} - 70.6}{3.53};\ x_2 = \frac{\beta_1 - 0.5669}{0.02834};\ x_3 = \frac{\beta_2 - 0.8525}{0.04262};\ x_4 = \frac{\varphi_i - 49.46}{2.473}.$$

For obtained optimal solution the wire diameter is $d^* = 0.05192$ mm and the turns number of the winding is $w^* = 13214.7$. The wire diameter is rounded off to the nearest standard value $d = 0.05$ mm, and the turns number - to $w = 13000$.

Using a more precise value for the internal impedance of the shade, at unfavourable conditions of switching on the supply voltage, and with load torque $M_c = 5 \times 10^{-4}$ Nm, the value of the starting acceleration time $t_c = 0{,}0524$ s is obtained. If the influence of the permanent component of the scalar magnetic potential in the stator poles, due to the permanent magnet, is taken into account, this time is $t_c = 0.0156$ s.

Neglecting the insignificant coefficients of the global approximations in the area of the obtained optimal solution, for the criterion of optimality $M = f(\boldsymbol{x})$, the overheat Θ, and the dephasing angle between scalar magnetic potentials of unshaded and shaded poles, the following expressions are obtained:

$$\begin{aligned} M =& 21.412 \times 10^{-4} + 0.135841 \times 10^{-3} x_1 - 0.117859 \times 10^{-3} x_3 \\ & + 0.506381 \times 10^{-4} x_4 - 0.214423 \times 10^{-4} x_1^2 - 0.442241 \times 10^{-4} x_1 x_3 \\ & - 0.93782610^{-4} x_3^2 + 0.208855 \times 10^{-4} x_3 x_4; \end{aligned} \tag{9}$$

$$\begin{aligned} \Theta =& 50.4911 + 0.219192 x_1 - 0.282325 x_2 - 1.03107 x_3 + 0.131384 x_1^2 \\ & + 0.137932 x_1 x_3 + 0.0259308 x_2^2 + 0.0476745 x_2 x_3; \end{aligned} \tag{10}$$

$$\begin{aligned} \varphi =& 35.6185 - 0.546825 x_1 + 4.42347 x_3 - 0.157858 x_1^2 - 0.352779 x_1 x_3 \\ & - 0.0547119 x_2^2 + 0.626388 x_3^2, \end{aligned} \tag{11}$$

where:

$$x_1 = \frac{V_{M1m} - 69.6}{3.48}; \quad x_2 = \frac{\beta_1 - 0.5876}{0.02938}; \quad x_3 = \frac{\beta_2 - 0.857}{0.04285}; \quad x_4 = \frac{\varphi_i - 52}{2.6}.$$

When the insignificant coefficients are taken into account, the relative error of approximating expressions (9), (10), and (11) is 1.77 %, 0.13 %, and 0,40 %, respectively.

Often for self-starting permanent magnet synchronous micromotors an additional requirement for working both at frequency of 50 Hz and frequency of 60 Hz is presented. In Fig. 2 the dependence of the angular velocity, Ω , versus the time, t, is shown. The permanent component of the scalar magnetic potential in the stator poles, due to the permanent magnet, is taken into account. The solution is obtained using the program rkfixed [8].

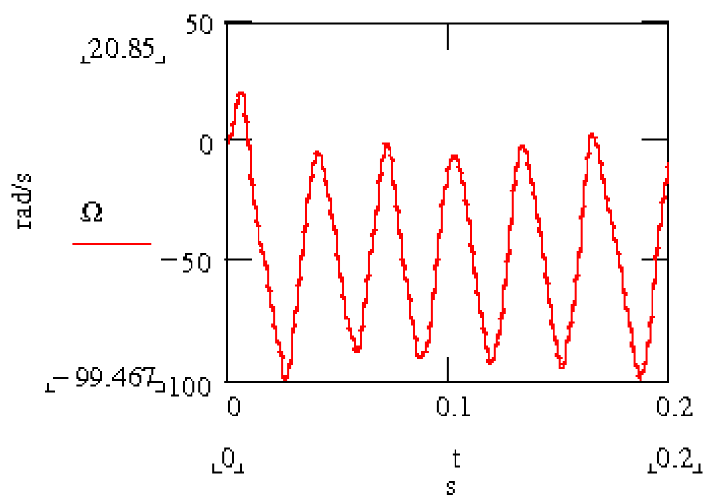

Fig. 2. Start regime of the micromotor at frequency of 60 Hz.

Consequently, the designed micromotor correspond to this additional requirement, too.

3 Identifying the Micromotor Parameters

The use of non-linear programming involves a great flexibility in the process of optimal design because this process can be continued at fixed values of some variables without any essential changes of the program. Practically, this presents a problem for identifying the micromotor parameters. For this purpose assuming small values of variation intervals I_i for this variables is quite sufficient.

The problem for identification of micromotor parameters at known dimensions differs from the problem of its optimal design by the form of constraints, and can be written as follows

$$f(\boldsymbol{x}) = -\frac{2}{\pi}\left(\widetilde{Y}_2 - \widetilde{Y}_1\right), \quad \boldsymbol{x} \in E^4 \tag{12}$$

at the constraints

$$\begin{aligned}
h_1(\boldsymbol{x}) &= \frac{\widetilde{Y}_8}{\widetilde{Y}_6} - 1 = 0;\\
g_2(\boldsymbol{x}) &= 1 - |x_1| \geq 0;\\
g_3(\boldsymbol{x}) &= 1 - |x_2| \geq 0;\\
g_4(\boldsymbol{x}) &= 1 - |x_3| \geq 0;\\
g_5(\boldsymbol{x}) &= 1 - |x_4| \geq 0;\\
g_6(\boldsymbol{x}) &= \frac{\tau + e_j - e_i - 0.5(a_{c1} + a_{c2})}{1.2 \times 10^{-3}} - 1 \geq 0;\\
g_7(\boldsymbol{x}) &= \frac{\tau - 0.5e_j - a_{c1}}{0.3 \times 10^{-3}} - 1 \geq 0,
\end{aligned} \tag{13}$$

In (12), $\widetilde{Y}_1$ and $\widetilde{Y}_2$ are approximating functions for the electromagnetic torque of forward and inverse rotating magnetic field, respectively. In (13), $\widetilde{Y}_6$ and $\widetilde{Y}_8$ are approximating functions for magneto-motive force, calculated by means of the magnetic circuit and the current, accordingly; $g_6(\boldsymbol{x})$ and $g_7(\boldsymbol{x})$are technological constraints.

The comparative data from computations and experimental investigations of the micromotor are presented in Table 1. In this case the interval of variation for x_1 is 0.1, and for other variables an interval of 0.1×10^{-6} is assumed. Thus the magnetic flux density varies only.

Table 1. Comparative data of some characteristics for designed micromotor with width of unshaded poles $a_{c1} = 2.4$ mm, width of shaded poles $a_{c2} = 3.1$ mm at voltage $U = 220$ V and frequency $f = 50$ Hz

Quantity	Computation	Experiment	Relative error
Rated current [A]	11.36×10^{-3}	11.75×10^{-3}	-3.32
Power consumption [W]	2.18	2.35	-7.23
Starting torque [Nm]	12.04×10^{-4}	10.4×10^{-4}	15.77
Pull-out torque [Nm]	27.7×10^{-4}	27.5×10^{-4}	0.73

Taking into consideration the small values of measured quantities, it can be accepted that the computed and measured characteristics are in a relatively good agreement from engineering point of view.

4 Conclusion

Using the non-linear mathematical programming as a base of the general approach to resolve the problem of optimal design of a self-starting permanent magnet synchronous micromotor, a possibility exists to identify its parameters, too. So, a real assessment of the used mathematical model can be carried out.

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