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Shallow-Buried Ring Grounding Electrodes

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Abstract: The problem of two shallow-buried ring grounding electrodes of low frequencies is analysed in this paper. In solving this problem, the ground is assumed as linear, homogeneous and isotropic medium. The equivalent electrodes method, which is very simple numerical method of adequate accuracy of obtained results, is used for calculations. Numerical results are presented both in tables and figures.

Keywords: Equivalent electrodes method, shallow-buried ring grounding electrodes, image theorem.

1 Introduction

The problem of grounding of quasistationary and stationary currents have been analyzed up to now by a lot of authors, but in this paper just a few references are mentioned as interesting [1–5]. An approximate numerical method for solving low-frequency grounding problems is presented, too. The method is very simple and exact and it can be applied without limitations on ground electrodes of arbitrary, but known shape. General fundamentals of electrical grounding techniques are presented in [6] and one way of resistance to ground calculation in [7].

The base of this method is replacing the existing electrodes by the system of auxiliary, i.e. equivalent electrodes [8]. These equivalent electrodes are placed on the electrode surface. Depending on the electrode system geometry, equivalent electrodes can be very large cylindrical conductors of circle cross-section, linear circular loops or little conductive spheres. The radius of these linear loops, equivalent spheres and cylindrical conductors is equal to equivalent shell radius, i.e. to the part of electrode surface that they present. Since equivalent electrodes have the

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same potential and the same currents that go into the ground as from the shells, the system of linear equations from the condition that electrodes are equipotential is formed easily and after solving it the currents that go from equivalent electrodes i.e. from the shells can be determined. After that, the potential and the electric field in the ground electrode surroundings can be determined as well as the other parameters necessary for grounding system. Electric field strength and potential distribution of two shallow-buried ring electrodes are done in this paper.

2 Outline of the Method

Shallow-buried grounding system that consists of two thin ring electrodes is observed (Fig.1a). The ring electrodes have the same shape, external radius *a*, internal radius *b*, negligible thickness, $\delta \to 0$, and they are at depth *h* in homogeneous, linear, isotropic conductive ground. Above the ground is air (Fig.1b).

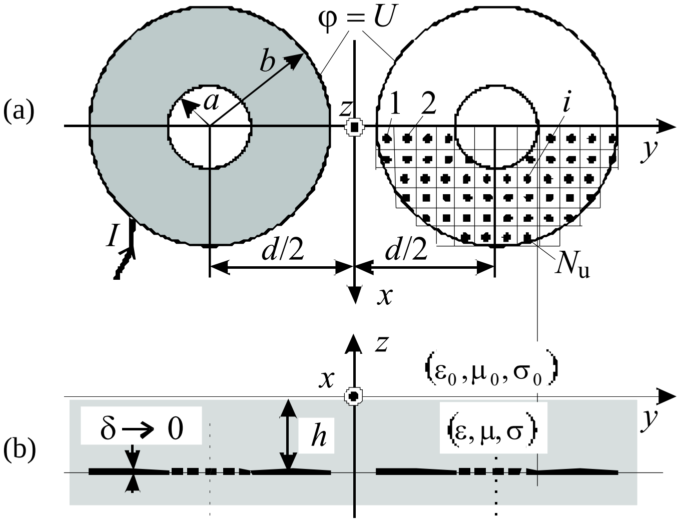

Fig. 1. Shallow-buried ring grounding electrodes.

The ground electrodes are equipotential and have the potential *U* regarding to the infinity. From the aboveground circuit, using an isolated conductor of negligible thickness, the current *I* is driven to the grounding electrodes.

In order to replace this ground electrodes by equivalent system of conductive spheres, according to Equivalent electrodes method, the square circumscribed around the ring electrode is, at first, divided into $2Nx2N$ square surfaces. Let one of square surfaces of side $\Delta = b/N$ be considered (Fig.1). Every square, whose cross of diagonals belongs to the ring electrode, is replaced by equivalent sphere electrode, whose center is in the cross of diagonals of the square and its radius is $a_e = 0.375\Delta$, [8]. The total number of equivalent electrodes is N_u.

The potential in the ground electrodes surroundings, if image theorem into the flat mirror and existing symmetry are used, can be presented in the following form

$$\begin{aligned}\varphi_M(x,y,z) = \sum_{i=1}^{N_u} \frac{I_i}{4\pi\sigma} \Bigg[\frac{1}{r_{oi}} + \frac{1}{r_{ooi}} + \frac{1}{r'_{oi}} + \frac{1}{r'_{ooi}} + \frac{1}{r_{oli}} \\ + \frac{1}{r_{ooli}} + \frac{1}{r'_{oli}} + \frac{1}{r'_{ooli}} \Bigg],\end{aligned} \tag{1}$$

where I_i is the current that goes into the ground from the equivalent conductive sphere,

$$\begin{aligned}
r_{oi} &= \sqrt{(x-x_i)+(y-y_i)^2+(z+h)^2} & r_{oli} &= \sqrt{(x-x_i)+(y-y_i)^2+(z-h)^2}\\
r_{ooi} &= \sqrt{(x+x_i)+(y-y_i)^2+(z+h)^2} & r_{ooli} &= \sqrt{(x+x_i)+(y-y_i)^2+(z-h)^2}\\
r'_{oi} &= \sqrt{(x-x_i)+(y+y_i)^2+(z+h)^2} & r'_{oli} &= \sqrt{(x-x_i)+(y+y_i)^2+(z-h)^2}\\
r'_{ooi} &= \sqrt{(x+x_i)+(y+y_i)^2+(z+h)^2} & r'_{oi} &= \sqrt{(x+x_i)+(y+y_i)^2+(z-h)^2}.
\end{aligned}$$

Since the equivalent electrodes are equipotential, for unknown values I_i determination, the following system of linear equations is formed:

$$\begin{aligned}U = \sum_{i=1}^{N_u} \frac{I_i}{4\pi\sigma} \Bigg[\frac{1}{r_{oi,j}} + \frac{1}{r_{ooi,j}} + \frac{1}{r'_{oi,j}} + \frac{1}{r'_{ooi,j}} + \frac{1}{r_{oli,j}} \\ + \frac{1}{r_{ooli,j}} + \frac{1}{r'_{oli,j}} + \frac{1}{r'_{ooli,j}} \Bigg], \quad j = 1,2,\ldots,N_u,\end{aligned} \tag{2}$$

where:

$$\begin{aligned}
r_{oi,j} &= \sqrt{(x_j-x_i)^2+(y_j-y_i)^2+\delta_{i,j}a_e^2}\\
r_{ooi,j} &= \sqrt{(x_j+x_i)^2+(y_j-y_i)^2}\\
r'_{oi,j} &= \sqrt{(x_j-x_i)^2+(y_j+y_i)^2}\\
r'_{ooi,j} &= \sqrt{(x_j+x_i)^2+(y_j+y_i)^2}
\end{aligned}$$

$$r_{oli,j} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + 4h^2}$$
$$r_{ooli,j} = \sqrt{(x_j + x_i)^2 + (y_j - y_i)^2 + 4h^2}$$
$$r'_{oli,j} = \sqrt{(x_j - x_i)^2 + (y_j + y_i)^2 + 4h^2}$$
$$r'_{ooli,j} = \sqrt{(x_j + x_i)^2 + (y_j + y_i)^2 + 4h^2},$$

where $\delta_{i,j}$ is Kronecker symbol.

After solving this system of linear equations, the total current that this ground electrode gives into the ground is

$$I = 4\sum_{i=1}^{N_u} I_i, \tag{3}$$

so the resistance of grounding is

$$R = \frac{U}{I}. \tag{4}$$

One analitical expesion for resistance of grounding system is given in [9]

3 Numerical Results

In order to illustrate the procedure given above for shallow-buried ring grounding solving, in the Table 1 are given numerical results for ratios of resistance of grounding of two ring grounding electrodes and the resistance of deeply interred ring grounding electrodes for b/a, distance between the centers of the ring electrodes $d/a = 5$, $d/a = 7$ and $d/a = 11$ and different values of ratio h/a.

Table 1. The ratio of resistance of grounding of two ring grounding electrodes and the resistance of deeply interred ring electrodes, R/R_{us}, for $b/a = 2$ and different values of ratios d/a and h/a.

d/a h/a	5	7	11
0.1	1.873488	1.866919	1.859175
0.5	1.656004	1.643308	1.624586
1	1.512902	1.501127	1.478552
5	1.173613	1.177928	1.173385
10	1.087346	1.091687	1.093620

Potential distribution dependence along y-axis for different values of distance between the ring electrodes and for $b/a = 2$ and $h/a = 2$ is shown in Fig.2. The ratio of potential and the potential of the center of the system φ_c decreases along y-axis. As the ratio d/a increases, the pick of the ratio φ/φ_c increases, too.

Fig.3 shows electric field strength dependence on the distance between the ring grounding electrodes for $b/a = 2$ and $h/a = 2$. As the distance between the centers of the rings decreases, the local maximum of the electric field strength in the near vicinity of the center of the system decreases, too.

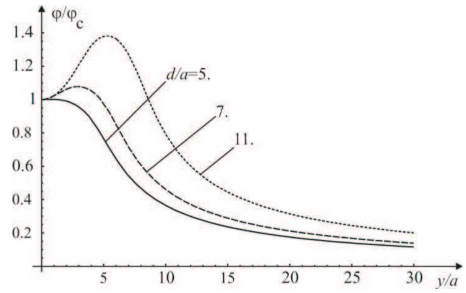

Fig. 2. Potential distribution.

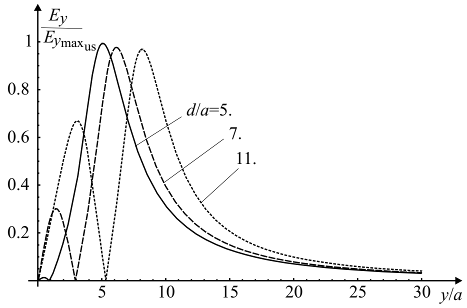

Fig. 3. Electric field strength along y axis, for $b/a = 2$, and $h/2 = 2$ different values of ratio d/a.

The largest value of electric field strength in the grounding surroundings and on the ground surface is marked as E_{max}. If $E\Delta h$ is step voltage, where Δh is the length of the step, then these curves present at the same time the step voltage distribution dependences in the radial direction on the ground surface.

Equipotential curves for $b/a = 2$, $d/a = 5$ and $h/a = 2$ are shown in Fig.4 Singular equipotential curves can be noticed. The distribution shown bellow is very complex.

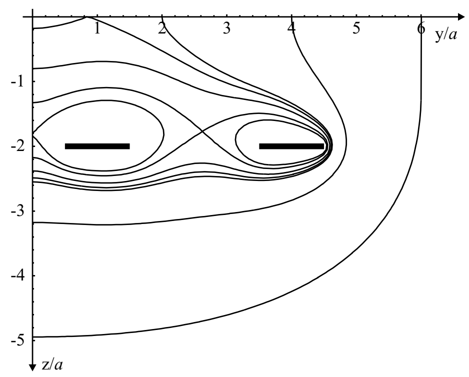

Fig. 4. Equipotential curves for $b/a = 2$, $d/a = 5$ and $h/a = 2$.

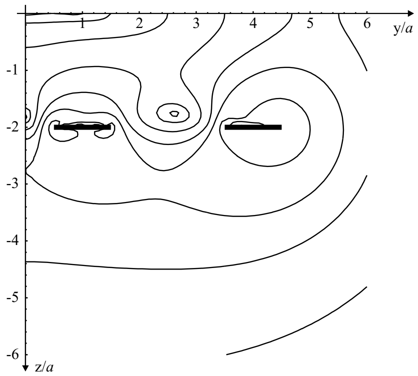

Fig. 5. Equienergetic curves for $b/a = 2$, $d/a = 5$ and $h/a = 2$.

Spaces of the same electrostatic energy density, which are very important for corona phenomena, are shown in Fig.5. Those spaces have very complex shape.

Potential distribution at ground surface for $d/a = 5$, $d/a = 7$, and $d/a = 11$ are shown in Fig.6, Fig.7 and Fig.8, respectively.

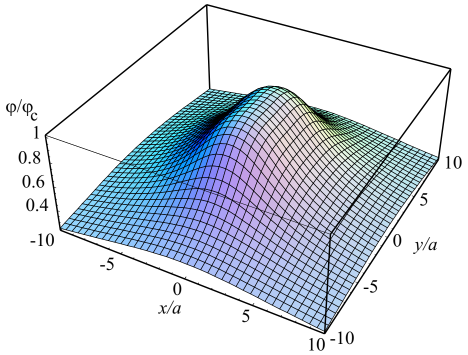

Fig. 6. Potential distribution at ground surface for $b/a = 2$, $d/a = 5$ and $h/a = 2$.

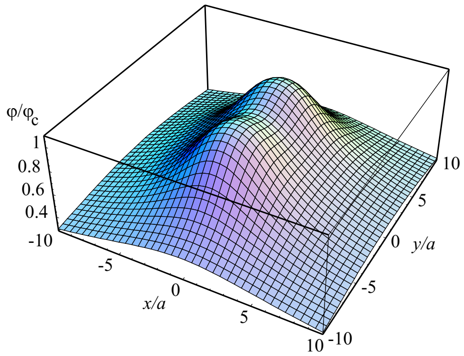

Fig. 7. Potential distribution at ground surface for $b/a = 2, d/a = 7$, and $h/a = 2$.

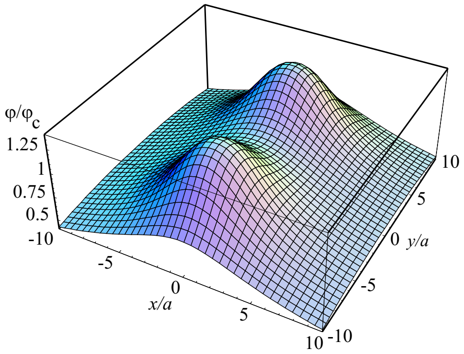

Fig. 8. Potential distribution at ground surface for $b/a = 2$, $d/a = 11$ and $h/a = 2$.

Under fault conditions, voltage rises in the vicinity of electrical installations and that situation can be very dangerous. Accurate knowledge of the impedance of grounding and potential distribution at ground surface in the vicinity of grounding system are very important for their safe operation.

The system of rings is used for step voltage and touch voltage modelling, which are very important parameters of grounding system. The potential distribution at ground surface is shown for near vicinity of grounding electrodes. Shown potential decreases very fast with the distance from the place where grounding rings are buried.

The idea of burring two grounding electrodes of ring shape one above the other was presented by M. J. Nahman [10]. In that way step voltage and touch voltage have been decreased and grounding system has been made more safely. In this paper, two ring electrodes are shallowly buried one close to the other. In that way, step voltage and touch voltage are made more lower than it was done in [10]. So, this is one successful attempt of step and touch voltage distribution modelling.

4 Conclusion

The problem of shallow-buried ring grounding electrodes using equivalent electrodes method is solved in this paper. After successful solving a large number of electromagnetic problems, this method presents one approximate numerical method with adequate accuracy, very useful and simple for applying. Program package in Fortran is made. Using this package, a lot of calculations are done and the results for equipotential curves, electric field strength distribution, potential distribution at ground surface and equienergetic curves are shown graphically. This paper presents one attempt of step voltage distribution modelling.

Acknowledgement

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