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Reduction Factor of Aerial Bundled Cables

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Abstract: This paper presents mathematical model for determination of reduction factor of aerial bundled cables. Knowledge of reduction factor of aerial bundled cables, that supplies a substation, is important for substation grounding design as well as for right estimation of safety conditions. Reduction factors of frequently used aerial bundled cables are determined by presented mathematical model. Influence of metal screen temperature as well as influence of supporting wire characteristics on reduction factor value are shown.

Keywords: Reduction factor, aerial bundled cable, single line to ground fault, impedance.

1 Introduction

Reduction factor of feeding line is one of the necessary parameters for calculation of the part of single-line-to-ground fault current returning through the ground, i.e. trough the grounding system of faulted terminal substation. For a cable feeding line, reduction factor is defined as ratio of the current running through the ground relative to the fault current, when metal screens (sheath) of cable are grounded at the both ends with connection of negligible resistance. Reduction factor comprehends influence of inductive coupling between phase conductors and metal screens (sheath) on decreasing of the current running trough the grounding system.

From the aspect of safety during single-line-to-ground fault at terminal substation, significant voltage appears on the grounding system. Since this voltage is proportional to the current running through the grounding system, knowledge of the reduction factor is important for right estimation of safety conditions. If design of the grounding system is calculated with reduction factor greater than real value, as

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the result we will have the grounding system with low resistance and great unnecessary expenditures. If we use reduction factor lower than real, safety conditions may not be satisfied. Previous example points out importance of right calculation of reduction factor of feeding line.

In city areas safety conditions are usually fullfled because of great number of cables in use (with paper insulation IPO, IPZO, NPO, NPZO, or with synthetic insulation XHP, XHE). Local grounding systems of MV/LV substations are connected via cable metal screens (sheets). Therefore, knowledge of exact value of the reduction factor is not a primary objective. In rural and suburban areas situation is different. Feeding lines are usually overhead lines, and there are no additional grounding installations. Therefore, we must pay greater attention in design and construction of grounding system.

So far not enough attention has been paid to determination of reduction factor of aerial bundled cables (ABC). References [1]-[4] discuss reduction factor of underground cables. Considering further development of distributive networks, greater usage of medium voltage ABCs is expected, especially in suburbs and small towns. Therefore this paper presents a mathematical model for calculation of reduction factor of ABC. The model is based on solving the system of equation for inductive coupled circuits with return path through the ground. Reduction factors for a few frequently used ABCs are calculated by presented model, as well influence of metal screen temperature and supporting wire characteristics are analyzed.

2 Mathematical Model

Medium voltage ABC consists of three single-core cables bundled with steel supporting wire. We will assume that metal screens of cables are connected with supporting wire and grounded at the both ends of the cable. Following self and mutual impedances will be considered in deriving the equation for reduction factor of ABC

- Self impedance of metal screen Z_s,
- Mutual impedances of metal screen and phase conductor of single-core cable Z_{cs},
- Mutual impedance of two metal screens Z_{ss},
- Mutual impedance of metal screen and a phase conductor of different cable Z_{sc},
- Mutual impedance of metal screen and supporting wire Z_{sw},
- Mutual impedance of phase conductor and supporting wire Z_{cw},
- Self-impedance of supporting wire Z_w.

Self impedance is impedance of the loop, that consists of mentioned element of the cable and earth return path [5]-[8]. Mutual impedance is impedance between loops of mentioned parts of the cable with earth return paths. Considering construction of ABC it is obvious that $Z_{sc} = Z_{ss}$ and $Z_{cw} = Z_{sw}$. If single-line-to-ground fault occurs from the open circuit state, and metal screens are connected to the ground at both ends, we can write following matrix equation

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{cs} & Z_s & Z_{ss} & Z_{ss} & Z_{sw} \\ Z_{sc} & Z_{ss} & Z_s & Z_{ss} & Z_{sw} \\ Z_{sc} & Z_{ss} & Z_{ss} & Z_s & Z_{sw} \\ Z_{cw} & Z_{sw} & Z_{sw} & Z_{sw} & Z_w \end{bmatrix} \begin{bmatrix} I_f \\ I_s \\ I_{s2} \\ I_{s2} \\ I_w \end{bmatrix}, \tag{1}$$

where I_f is fault current, I_s is current flowing through the metal screen of the faulted single-core cable, I_{s2} is current flowing through metal screens of non-faulted single-core cables, and I_w is supporting wire current.

Previous matrix equation can be written as

$$\begin{bmatrix} I_s \\ I_{s2} \\ I_{s2} \\ I_w \end{bmatrix} = -I_f \begin{bmatrix} Z_s & Z_{ss} & Z_{ss} & Z_{sw} \\ Z_{ss} & Z_s & Z_{ss} & Z_{sw} \\ Z_{ss} & Z_{ss} & Z_s & Z_{sw} \\ Z_{sw} & Z_{sw} & Z_{sw} & Z_w \end{bmatrix}^{-1} \begin{bmatrix} Z_{cs} \\ Z_{sc} \\ Z_{sc} \\ Z_{cw} \end{bmatrix}. \tag{2}$$

The current running through the ground is

$$I_g = I_f + I_s + I_{s2} + I_{s2} + I_w, \tag{3}$$

thus

$$I_g = I_f + \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_s \\ I_{s2} \\ I_{s2} \\ I_w \end{bmatrix}. \tag{4}$$

Considering that reduction factor is defined as ratio of the current running through the ground relative to the total fault current

$$r = \frac{I_f + I_s + I_{s2} + I_{s2} + I_w}{I_f}, \tag{5}$$

and substituting (2) and (4) into (5) yields

$$r = 1 - \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} Z_s & Z_{ss} & Z_{ss} & Z_{sw} \\ Z_{ss} & Z_s & Z_{ss} & Z_{sw} \\ Z_{ss} & Z_{ss} & Z_s & Z_{sw} \\ Z_{sw} & Z_{sw} & Z_{sw} & Z_w \end{bmatrix}^{-1} \begin{bmatrix} Z_{cs} \\ Z_{sc} \\ Z_{sc} \\ Z_{cw} \end{bmatrix}. \tag{6}$$

This is general equation for reduction factor calculation. Considering $Z_{sc} = Z_{ss}$ and $Z_{cw} = Z_{sw}$ simplified expression is obtained. Matrix equation (1) gives in this case following equations

$$\begin{aligned} Z_{cs}I_f + Z_sI_s + 2Z_{ss}I_{s2} + Z_{sw}I_w &= 0, \\ Z_{ss}I_f + Z_{ss}I_s + (Z_s + Z_{ss})I_{s2} + Z_{sw}I_w &= 0, \\ Z_{sw}I_f + Z_{sw}I_s + 2Z_{sw}I_{s2} + Z_wI_w &= 0. \end{aligned} \tag{7}$$

Solving these equations we obtain

$$\begin{aligned} I_s &= -\frac{2Z'^2_{ss} - Z'_{cs}Z'_s - Z'_{ss}Z'_{cs}}{2Z'^2_{ss} - Z'^2_s - Z'_sZ'_{ss}} I_f, \\ I_{s2} &= \frac{Z'_{ss}Z'_s - Z'_{ss}Z'_{cs}}{2Z'^2_{ss} - Z'^2_s - Z'_sZ'_{ss}} I_f, \\ I_w &= -\frac{Z_{sw}}{Z_w}\left(I_f + I_s + 2I_{s2}\right). \end{aligned} \tag{8}$$

where $Z'_s = Z_s - Z^2_{sw}/Z_w$, $Z'_{cs} = Z_{cs} - Z^2_{sw}/Z_w$ and $Z'_{ss} = Z_{ss} - Z^2_{sw}/Z_w$.

The current running through the ground is

$$\begin{aligned} I_g &= I_f + I_s + 2I_{s2} + I_w \\ &= (I_f + I_s + 2I_{s2})\left(1 - \frac{Z_{sw}}{Z_w}\right), \end{aligned} \tag{9}$$

and substituting (8) yields

$$I_g = \frac{Z'_s - Z'_{cs}}{2Z'_{ss} + Z'_s}\left(1 - \frac{Z_{sw}}{Z_w}\right) I_f. \tag{10}$$

The reduction factor of ABC is therefore

$$r = \frac{Z'_s - Z'_{cs}}{2Z'_{ss} + Z'_s}\left(1 - \frac{Z_{sw}}{Z_w}\right). \tag{11}$$

Substituting (9) into (12) gives

$$r = \frac{Z_s - Z_{cs}}{2Z_{ss} + Z_s - 3\dfrac{Z^2_{sw}}{Z_w}}\left(1 - \frac{Z_{sw}}{Z_w}\right), \tag{12}$$

i.e. using equations for self and mutual impedances given on the appendix

$$r = \frac{R_s}{2Z_{ss} + Z_s - 3\dfrac{Z^2_{sw}}{Z_w}}\left(1 - \frac{Z_{sw}}{Z_w}\right), \tag{13}$$

where R_s is resistance of the metal screen.

3 Example

Using presented mathematical model, reduction factor of aerial bundled cable XHE 48/O-A 3×(1×70)+50 6/10 kV is calculated. It is a bundle of thee single-core cables with 70 mm^2 aluminum conductors and XLPE insulation. Cross sectional area of copper screen is 16 mm^2. Single-core cables are bundled with 50 mm^2 steel supporting wire.

Lack of knowledge of the electrical and magnetic characteristics of supporting wire causes certain difficulties in reduction factor calculations. Therefore we use the data for 50 mm^2 stranded steel wire given in [1](resistance per unit length on 50 Hz is 4.13 Ω/km and magnetic relative permeability is $\mu_r = 30$).

Electrical resistance of metal screen depends on temperature. Consequently, reduction factor also depends on temperature. Table 1 comprehends reduction factors of the cable under consideration for different temperatures of metal screen, and specific soil resistivity of 50 Ωm. Table 1 shows the influence of temperature on reduction factor, for example reduction factor modulus at the temperature of 70$^{\mathrm{o}}$C (0.49) is 13% larger than modulus at 20$^{\mathrm{o}}$C (0.433). With the temperature rise reduction factor argument is decreasing, but these changes are negligible. For the temperature of 20$^{\mathrm{o}}$C reduction factor argument is -59.53^{o}, while for 70$^{\mathrm{o}}$C is -55.81^{o}.

Table 1. Reduction factors of aerial bundled cable XHE 48/O-A 3×(1×70)+50 6/10 kV for different temperatures of metal screen

$\theta_s(^{\mathrm{o}}\mathrm{C})$	r	$\|r\|$
20	$0.2168 - j0.375$	0.433
30	$0.228 - j0.3825$	0.445
40	$0.239 - j0.389$	0.457
50	$0.25 - j0.396$	0.468
60	$0.261 - j0.402$	0.479
70	$0.272 - j0.408$	0.49

Beside the cable with 70 mm^2 phase conductors , reduction factor were determined for cross sectional areas of 35, 50, and 95 mm^2. Cross sectional area of copper screens of these cables is also 16 mm^2. Table 2 shows reduction factor for copper screen temperature of 50$^{\mathrm{o}}$C. With the rise of cross sectional area the reduction factor slightly increases. Therefore it is considered that cross sectional area of phase conductor almost does not influence on reduction factor value. Definition of reduction factor clearly points out that cross sectional area of phase conductor has no influence on its value, but changes in geometrical dimensions of the cable have. The argument of reduction factor is also changing with cross sectional area variations. For $35\,\mathrm{mm}^2$ conductor argument is -57.66^{o}, and for $95\,\mathrm{mm}^2$ is -57.1^{o}.

Table 2. Reduction factor for different cross sectional areas of phase conductors, and copper screen temperature of 50^{o}C

Conductor Area $\left(\mathrm{mm}^2\right)$	r	$\lvert r\rvert$
35	0.245 - j0.394	0.464
50	0.2475 - j0.395	0.466
70	0.25 - j0.396	0.468
95	0.252 - j0.397	0.47

Because of lack of the supporting wire magnetic characteristics, calculation has been made for magnetic relative constant $\mu_r = 1$, and for infinite self-impedance of supporting wire. In this way we define upper and lower border for the reduction factor.

Differences between reduction factor values given in Table 2 and the ones obtained with $\mu_r = 1$ are negligible. If we ignore influence of supporting wire (supposing its self impedance is infinite) reduction factor modules of 0.5 were obtained for copper screen temperature of 50^{o}C and any cross-sectional area of phase conductors. Underground cable line consisting of three single-core cables with $16\,\mathrm{mm}^2$ copper screen also has reduction factor modulus close to 0.5. This result is expected, because substituting $Z_s \to \infty$ in equation (20) leads to equation for reduction factor of three single-core cables buried in the ground.

Based on the calculation results we can conclude that the value of reduction factor modulus is not greater than 0.5. That should be the value for fault current distribution analysis and safety condition estimations.

4 Conclusion

Mathematical model for determination of reduction factor of aerial bundled cables is presented in this paper. Reduction factors of a few frequently used cables are calculated by presented model. The results show that copper screen temperature certainly influences the reduction factor. For the aerial bundled cables with different cross sectional areas of phase conductor, but with the same cross sectional area of copper screen, almost equal result (lower the 0.5) is obtained. Disregarding influence of the supporting wire we obtain modulus of reduction factor close to 0.5. Results of calculations show that influence of supporting wire presence on reduction factor value is negligible.

Appendix

Calculation of self impedance Z_{ii} and mutual impedance Z_{ij}

Self-impedance per unit length of the conductor-earth loop is

$$Z_{ii} = R_c + R_g + j\omega L_{ii}, \tag{A.1}$$

where R_c is electrical resistance per unit length of the conductor (phase conductor or metal screen), R_g is electrical resistance per unit length of the ground, ω is angular frequency ($\omega = 2\pi f$), and L_{ii} is self inductance per unit length of the conductor-earth loop.

Mutual impedance per unit length of conductors i and j with earth return path is

$$Z_{ij} = R_g + j\omega L_{ij}, \tag{A.2}$$

where L_{ij} is mutual inductance per unit length of mentioned loops.

Ground resistance per unit length depends on frequency f

$$R_g = \frac{\omega\mu_0}{8} = \pi^2 f 10^{-7}. \tag{A.3}$$

In this relation R_g is expressed in Ω/m, and f in Hz.

Self inductance per unit length of the conductor-earth loop, when conductor is made of non-magnetic material is

$$L_{ii} = \frac{\mu_0}{2\pi} \ln \frac{D_{eq}}{r_{ceq}}, \tag{A.4}$$

where D_{eq} is equivalent depth of return path, and r_{ceq} is equivalent radius of conductor.

Equivalent radius of conductor usually can be calculate as $r_{ceq} = 0.779 r_c$. For metal screen, because of small thickness, we can use its mean radius for r_{ceq}.

Equivalent depth of earth return path depends on frequency and soil resistivity and can be calculated as

$$D_{eq} = 658\sqrt{\frac{\rho}{f}}. \tag{A.5}$$

In this relation ρ is expressed in Ωm, f in Hz, and D_{eq} in m.

Self-impedance per unit length of the conductor-earth loop, when conductor is made of magnetic material

$$L_{ii} = \frac{\mu_0\mu_r}{8\pi} + \frac{\mu_0}{2\pi} \ln \frac{D_{eq}}{r_c}, \tag{A.6}$$

where μ_r is relative magnetic permeability of conductor.

Mutual inductance per unit length L_{ij} of the loops i and j with earth return path is

$$L_{ij}=\frac{\mu_0}{2\pi}\ln\frac{D_{eq}}{a_{ij}}, \tag{A.7}$$

where a_{ij} is center to center distance between conductors i and j. If axes of conductors i and j are collinear (phase conductor and metal screen of single-core cable), for a_{ij} we should use larger radius (electrical screening radius).

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