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# Bit Error Probability of PSK Systems in the Presence of Impulse Noise

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**Abstract:** In this paper the phase error probability density function in the digital systems for the digital phase modulated signals transmission is derived. The circuit for the referent carrier extraction consists of phase locked loop. The referent carrier extraction is not ideal. The pulse interferences, narrow-band Gaussian noise and different interferences are present with useful signal at the input of the receiver. The error probability of the digital system for the phase modulated signal demodulation is also calculated in the paper.

**Keywords:** PSK coherent detection, imperfect carrier extraction, nonlinear first order PLL model, phase error, Gaussian noise, interference, fading.

## 1 Introduction

The receiver for coherent demodulation of the digital phase modulated signal is considered in this paper. The transmission of the signal is binary. The circuit for the reference carrier extraction from the arrival signal extracts the reference carrier. The arrival signal consists of the useful signal and Gaussian noise. The extraction of the reference carrier is not ideal.

The phase error is the difference between the arrival signal phase and reference carrier phase. This phase is the random process because of the Gaussian noise and the other interference influences at the input of the receiver. The phase error has the Tihonov probability density function. It has the influence on the signal amplitude at the correlator output, i.e. it has the influence on the error probability of the digital system for the phase modulated signal transmission. When the amplitude of the

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useful signal is variable or the some other interference in addition to Gaussian noise exists at the input of the coherent receiver, the probability density function of the phase error has conditional Tihonov distribution. The probability density function of the phase error in this case can be obtained by averaging of the conditional probability density function of the phase error by random parameters [1, 2, 3].

In this paper the useful signal is pulse signal. The equivalent amplitude of this signal consists of sum of useful signal amplitude and the pulse component. Pulse component is determined by intensity of the pulse process. When the intensity of the pulse process is constant this component has the Poison distribution. We will determine in this paper the phase error probability density function when the intensity of the pulse process is constant and when it is variable. The intensity of the pulse process is equals to square of the sum of the constant and the Gaussian signal. For that useful signal in the paper we will calculate the digital system error probability.

We will consider also the case when the interference is in addition to pulse signal and Gaussian noise at the input of the receiver. The interference is modelled by sinusoid wave with constant amplitude and uniformly distributed phase. In this case we will calculate the equivalent amplitude first and then determine the phase error probability density function and the digital system error probability. In mobile telephony the signal amplitude at the input of the receiver is variable. Because of that in this paper we will determine the phase error probability density function and the system error probability when the useful signal amplitude is variable.

An expression for the bit error probability was calculated when the signal and additive Gaussian noise are applied to the input of the phase-coherent communication receiver with the phase locked loop [2]. The lower bounds for digital communications with multiple interferences were determined in the reference [4].

## 2 System Model

The model of the receiver for our case is at the Figure 1. In the circuit for the carrier extraction the phase loop exists. This circuit can be realize as the circuit with square, as Costas loop, the circuit with remodulation and it can contain phase loop only. In this case the power of the coded signal is significant near the referent frequency. All these circuits are made because of increasing the signal power near the carrier frequency to make referent carrier extractors job easier. The extractor extracts referent carrier from the arrival signal corrupted by Gaussian noise.

The loop consists of phase discriminator, low pass filter and voltage controlled oscillator. At one discriminator input the useful signal and Gaussian noise come, and at the other the signal from the voltage controlled oscillator. The signal from

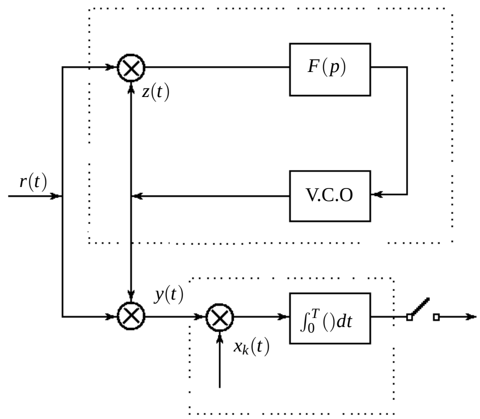


Fig. 1. The system model.

the output of the phase discriminator comes at the input of the low pass filter. The low pass filter output is connected with voltage controlled oscillator input.

The signal at the output of the phase discriminator increases when the phase difference between the signals at the inputs of phase discriminator increases. This signal from low pass filter comes at the voltage controlled oscillator input. The voltage controlled oscillator is designed so that: as the voltage at its input increases, the phase difference between the signals at the inputs of phase discriminator reduces. The phase loop works in the same manner.

## 3 Phase Error Probability Density Function in the Presence of Noise

The general form of the pulse signal is

$$s(t) = (A + c\,n)\cos[\omega t + (\cos^{-1} m)x_k(t)], \quad k = 1, 2, \tag{1}$$

where $c\,n$ is the pulse random process, $A$ is the transmitted signal amplitude, $m$ is the coefficient which divides total power between the carrier and the lateral bands and $x_k(t)$, $0 \le t \le T$, is the binary signal which brings information. The case $x_k(t) = \pm 1$ is of the greatest practical interest.

The pulse number $n$ depends on pulse process intensity. When the pulse process intensity is constant, it has Poisons distribution. The probability of pulse number $n$ in some time interval is [1]

$$P(n) = \frac{\lambda^n}{n!} e^{-\lambda} \tag{2}$$

where $\lambda$ is the pulse process intensity. The pulse process intensity can be constant or variable. The amplitude $A$ can also be constant or variable.

The referent carrier, obtained at the output of the extraction circuit, is

$$z(t) = \sqrt{2}\cos[\omega_0 t + \Theta(t)] \tag{3}$$

where $\Theta(t)$ is the evaluated value of the phase movement formed in the cannel.

The signal at the coherent receiver input is in the form

$$r(t) = A_e \cos[\omega t + (\cos^{-1} m)x_k(t) + \theta(t)] + n(t), \tag{4}$$

where $A_e = A + cn$ is useful signal equivalent amplitude, $\theta(t)$ is the random phase movement produced in the channel and $n(t)$ is the Gaussian noise.

The product of the signals r(t) and $z(t)$, when the double frequencies terms are neglected, is

$$y(t) = \sqrt{S}x_k(t)\cos\varphi(t) + n'(t), \tag{5}$$

where $S = (1-m^2)P$, $P$ is the total transmitted power, $\varphi(t)$ is phase error process and $n'(t)$ is the Gaussian noise with single-sided power density spectrum $N_0$.

The decision is based on $q$. If $q \geq 0$ it can be taken that the transmitter sent the signal $x_1(t)$, if $q \leq 0$ the sent signal is $x_2(t)$. In the time interval of one digit, $T$, the conditional probability density function of $q$, for given $\varphi$ is normal. The mean value and variance of this distribution are

$$\begin{aligned} M_k &= (-1)^k 2\sqrt{S}\int_0^T \cos\varphi(t)\mathrm{d}t \\ \sigma_k^2 &= 2TN_0 \end{aligned} \tag{6}$$

where $k = 1, 2$. From(6) we can obtain the conditional probability density function of $q$ for both of hypothesis

$$\begin{aligned} p(q_1/\varphi) &= \frac{1}{\sqrt{2\pi}} e^{-\frac{(q_1 - \frac{\sqrt{2}M_1}{\sigma_1})^2}{2}} \\ p(q_2/\varphi) &= \frac{1}{\sqrt{2\pi}} e^{-\frac{q_2^2}{2}} \end{aligned} \tag{7}$$

The term for the conditional error probability for given $\varphi$ is from (7)

$$P_{e/\varphi} = Q(\sqrt{2R}Y), \tag{8}$$

where

$$R = \frac{ST}{N_0} = \frac{E}{N_0}$$
$$Y = \frac{1}{T}\int_0^T \cos\varphi(t)\mathrm{d}t \tag{9}$$
$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{z^2}{2}}\mathrm{d}z.$$

We observe the receiver for coherent demodulation of phase modulated signal. The circuit for the referent carrier extraction consists of phase locked loop. For the first order phase locked loop the phase error probability density function is [1]

$$p_\varphi(\varphi) = \frac{e^{\beta\varphi+\alpha\varphi}}{4\pi^2|I_{j\beta}(\alpha)|^2}\int_\varphi^{\varphi+2\pi} e^{-\beta x-\alpha\cos x}\mathrm{d}x \tag{10}$$

For the first order loop the parameters $\alpha$ and $\beta$ are: $\alpha = \alpha_0 A_e$ and $\beta = \beta_0\Omega$. $\Omega$ is the carrier frequent movement (frequency offset) and $A_e$ is the signal amplitude at the phase locked loop input; $\alpha_0$ and $\beta_0$ are the constants [3]. $I_\nu(x)$ is the modified Bessel function of order $\nu$ and argument $x$.

When the frequency offset is $\Omega = 0$, i.e., $\beta = 0$, the expression (10) for the phase error probability density function becomes [5]

$$p_\varphi(\varphi) = \frac{e^{\alpha\cos\varphi}}{2\pi I_0(\alpha)}. \tag{11}$$

Note, $p_\varphi(\varphi)$ is the probability density function of the phase error in the form of a Tihonov distribution [6].

After formulas before, the conditional phase error probability density function obtains the form

$$p_\varphi(\varphi/n) = \frac{e^{\alpha_0(A+cn)\cos\varphi}}{2\pi I_0[\alpha_0(A+cn)]} \tag{12}$$

where $n$ is the pulse number in the some time interval. The phase error probability density is

$$p_\varphi(\varphi) = \sum_{n=0}^{\infty} p_\varphi(\varphi/n)p(n) = \sum_{n=0}^{\infty}\frac{e^{\alpha_0(A+cn)\cos\varphi}}{2\pi I_0[\alpha_0(A+cn)]}\frac{\lambda^n}{n!}e^{-\lambda} \tag{13}$$

The pulse process intensity $\lambda$ depends of the interferences which appear in transmission medium. It can be written in the form

$$\lambda = c_1(B+x)^2 \tag{14}$$

where $c_1$ and $B$ are constants and $x$ is a noise and it has Gaussian probability density

$$p_x(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \tag{15}$$

In this case the conditional phase error probability density is equals

$$p_\varphi(\varphi/x) = \sum_{n=0}^{\infty} \frac{e^{\alpha_0(A+cn)\cos\varphi}}{2\pi I_0[\alpha_0(A+cn)]} \frac{c_1^n(B+x)^{2n}}{n!} e^{-c_1(B+x)^2} \tag{16}$$

When the light intensity is variable the phase error probability density function is:

$$p_\varphi(\varphi) = \sum_{n=0}^{\infty} \frac{e^{\alpha_0(A+cn)\cos\varphi}}{2\pi I_0[\alpha_0(A+cn)]} \int_{-\infty}^{\infty} \frac{c_1^n(B+x)^{2n}}{n!} e^{-c_1(B+x)^2} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \tag{17}$$

In wireless telecommunication systems the signal amplitude is variable. The signal amplitude is variable because of the multipath fading and the shadow effect. In such case the equivalent amplitude can be written as a product of two random variables $A_1$ and $A_2$

$$A = A_1 A_2 \tag{18}$$

First of them has Nakagami distribution and second has log-normal distribution

$$p_{A_1}(A_1) = \frac{2}{\Gamma(m)} (\frac{m}{\Omega})^2 A_1^{2m-1} e^{-\frac{m}{\Omega}A_1} \tag{19}$$

$$p_{A_2}(A_2) = \frac{1}{\sqrt{2\pi}\,\sigma A_2} e^{-\frac{(\ln A_2-\mu)^2}{2\sigma_2^2}} \tag{20}$$

The probability density function of the amplitude A in this case is

$$\begin{aligned} p_A(A) &= \int_0^\infty \frac{1}{|A_2|} p_{A_1}(\frac{A}{A_2}) p_{A_2}(A_2) \mathrm{d}A_2 \\ &= \int_0^\infty \frac{1}{|A_2|} \frac{2}{\Gamma(m)} (\frac{m}{\Omega})^2 (\frac{A}{A_2})^{2m-1} e^{-\frac{m}{\Omega}\frac{A}{A_2}} \frac{1}{\sqrt{2\pi}\,\sigma A_2} e^{-\frac{(\ln A_2-\mu)^2}{2\sigma_2^2}} \mathrm{d}A_2 \end{aligned} \tag{21}$$

The phase error probability density with variable useful signal amplitude is

$$p_\varphi(\varphi) = \sum_{n=0}^{\infty} \int_0^\infty \frac{e^{\alpha_0(A+cn)\cos\varphi}}{2\pi I_0[\alpha_0(A+cn)]} \frac{\lambda^n}{n!} e^{-\lambda} p_A(A) \mathrm{d}A \tag{22}$$

## 4 Phase Error Probability Density Function in the Presence of Noise and Interference

The interferences appearing because of crosstalk are frequent in digital telecommunication systems. These interferences appear in wireless telecommunication systems, optical telecommunication systems, and wire and satellite digital telecommunication systems. In the optical telecommunication systems the interferences because of crosstalk can appear in optical transmitter, optical receiver and in the optical fiber. In the base band transmission the interferences because of the crosstalk are the base cause of the signal degradation [7].

The interferences appearing because of crosstalk can be described as one or more sinusoidal waves with constant amplitude and uniformly distributed phases. The interference with continual power spectral density can be represented approximately with more sinusoidal waves with different amplitudes and frequencies.

The changing of the signal amplitude appears in the wireless digital telecommunication systems because of fading influence and the shadow effect. The fading originates because of the signal transmission by more puts. In this case the signal amplitude can have Rayleigh, Rice, Nakagami, log-normal or some other distribution [8].

When the sinusoidal interference exists the total signal at the receiver input is

$$r(t) = (A + cn)\cos\omega t + A_1\cos(\omega t + \theta_1) + n(t) \tag{23}$$

where $A_1$ is the interference amplitude, $\theta_1$ is the interference phase and it is uniformly distributed

$$p_\theta(\theta_1) = \frac{1}{2\pi}, \qquad |\theta_1| \le \pi \tag{24}$$

The expression for the signal at the receiver input (23) can be given as

$$r(t) = R_e\cos(\omega_0 t + \Psi) + n(t) \tag{25}$$

Here:

$$\begin{aligned} R_e &= \sqrt{(A+cn)^2 + A_1^2 + 2A_1(A+cn)\cos\theta_1} \\ \Psi &= \arctan\frac{A_1\sin\theta_1}{A+cn+A_1\cos\theta_1} \end{aligned} \tag{26}$$

$R_e$ is the signal and interference equivalent amplitude and $\Psi$ is the signal and interference equivalent angle. The parameter $\alpha$ is

$$\alpha = \alpha_0 R_e = \alpha_0\sqrt{(A+cn)^2 + A_1^2 + 2A_1(A+cn)\cos\theta_1} \tag{27}$$

The frequency offset $\Omega$ is

$$\Omega = \frac{\mathrm{d}\Psi}{\mathrm{d}t} = \frac{\mathrm{d}\Psi}{\mathrm{d}\theta_1}\frac{\mathrm{d}\theta_1}{\mathrm{d}t} \tag{28}$$

Since $\mathrm{d}\theta_1/\mathrm{d}t = 0$ it follows $\Omega = 0$ and $\beta = 0$ and now the conditional phase error probability density function is, from (11), equals

$$p_\varphi(\varphi/\theta, n) = \frac{e^{\alpha_0 \cos\varphi \sqrt{(A+cn)^2 + A_1^2 + 2A_1(A+cn)\cos\theta_1}}}{2\pi I_0(\alpha_0\sqrt{(A+cn)^2 + A_1^2 + 2A_1(A+cn)\cos\theta_1})} \tag{29}$$

The phase error probability density in the presence of pulse signal and sinusoidal interference is obtained as:

$$p_\varphi(\varphi) = \int_{-\pi}^{\pi} \frac{\mathrm{d}\theta_1}{2\pi} \sum_{n=0}^{\infty} \frac{e^{\alpha_0 \cos\varphi \sqrt{(A+cn)^2 + A_1^2 + 2A_1(A+cn)\cos\theta_1}}}{2\pi I_0(\alpha_0\sqrt{(A+cn)^2 + A_1^2 + 2A_1(A+cn)\cos\theta_1})} \frac{\lambda^n}{n!} e^{-\lambda} \tag{30}$$

When $\lambda$ is given by equation (8) we obtain

$$\begin{aligned} p_\varphi(\varphi) = & \int_{-\pi}^{\pi} \frac{\mathrm{d}\theta_1}{2\pi} \sum_{n=0}^{\infty} \frac{e^{\alpha_0 \cos\varphi \sqrt{(A+cn)^2 + A_1^2 + 2A_1(A+cn)\cos\theta_1}}}{2\pi I_0(\alpha_0\sqrt{(A+cn)^2 + A_1^2 + 2A_1(A+cn)\cos\theta_1})} \\ & \times \int_{-\infty}^{\infty} \mathrm{d}x \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \frac{c_1^n (B+x)^{2n}}{n!} e^{-c_1(B+x)^2} \end{aligned} \tag{31}$$

## 5 The Bit Error Probability of PSK System

The conditional error probability for coherent receiver for the PSK signal demodulation is [4]

$$P_{e/\varphi} = Q(\sqrt{2R_b}\cos\varphi) \tag{32}$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{z^2}{2}} \mathrm{d}z \tag{33}$$

The constant $R_b$ can be written as

$$R_b = 2R_1 A_e^2 \tag{34}$$

where constant $R_1$ corresponds to the case when there is no interference.

We consider first the case when the input signal is given by equation (4)

In this case the conditional error probability is:

$$P_{e/n} = Q(\sqrt{2R_1}(A+cn)\cos\varphi) \tag{35}$$

The error probability is

$$P_e = \sum_{n=0}^{\infty} \int_{-\pi}^{\pi} Q(\sqrt{2R_1}(A+cn)\cos\varphi) \frac{e^{\alpha_0 (A+cn)\cos\varphi}}{2\pi I_0[\alpha_0(A+cn)]} \frac{\lambda^n}{n!} e^{-\lambda} \mathrm{d}\varphi \tag{36}$$

If the intensity of the pulse noise has the form as is given by equation (14) the error probability is

$$\begin{aligned} P_e = \sum_{n=0}^{\infty} \int_{-\pi}^{\pi} & Q(\sqrt{2R_1}(A+cn)\cos\varphi) \frac{e^{\alpha_0 (A+cn)\cos\varphi}}{2\pi I_0[\alpha_0(A+cn)]} \\ & \times \int_{-\infty}^{\infty} \mathrm{d}x \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \frac{c_1^n (B+x)^{2n}}{n!} e^{-c_1(B+x)^2} \end{aligned} \tag{37}$$

When the amplitude A has Rayleigh probability density

$$p_A(A) = \frac{A}{\sigma_F^2} e^{-\frac{a^2}{2\sigma_F^2}}, \quad A \geq 0 \tag{38}$$

it follows that

$$\begin{aligned} P_e = \sum_{n=0}^{\infty} \int_{-\pi}^{\pi} \mathrm{d}\varphi \int_{0}^{\infty} & \mathrm{d}A Q(\sqrt{2R_1}(A+cn)\cos\varphi) \frac{e^{\alpha_0 (A+cn)\cos\varphi}}{2\pi I_0[\alpha_0(A+cn)]} \\ & \times \frac{\lambda^n}{n!} e^{-\lambda} \frac{A}{\sigma_F^2} e^{-\frac{A^2}{2\sigma_F^2}} \end{aligned} \tag{39}$$

But if $\lambda$ is given by equation (8) and

$$p_A(A) = \frac{A}{\sigma_F^2} e^{-\frac{A^2+D^2}{2\sigma_F^2}} I_0(\frac{AD}{\sigma_F^2}) \tag{40}$$

then

$$\begin{aligned} P_e = \sum_{n=0}^{\infty} \int_{-\pi}^{\pi} \mathrm{d}\varphi \int_{0}^{\infty} & \mathrm{d}A Q(\sqrt{2R_1}(A+cn)\cos\varphi) \frac{e^{\alpha_0 (A+cn)\cos\varphi}}{2\pi I_0[\alpha_0(A+cn)]} \\ & \times \int_{-\infty}^{\infty} \mathrm{d}x \frac{c_1^n (B+x)^{2n}}{n!} e^{-c_1(B+x)^2} \frac{A}{\sigma_F^2} e^{-\frac{A^2+D^2}{2\sigma_F^2}} I_0(\frac{AD}{\sigma_f^2}) \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \end{aligned} \tag{41}$$

The total signal at the receiver input if the sinusoidal interference is present is given by (23).

The conditional error probability in this case is

$$P_{e/\varphi} = Q(\sqrt{2R_1}\sqrt{(A+cn)^2 + A_1^2 + 2A_1(A+cn)\cos\theta_1}\cos\varphi) \tag{42}$$

By averaging

$$P_e = \int_{-\pi}^{\pi} \mathrm{d}\varphi \int_{-\pi}^{\pi} \frac{\mathrm{d}\theta_1}{2\pi} \sum_{n=0}^{\infty} Q(\sqrt{2R_1}\sqrt{(A+cn)^2 + A_1^2 + 2A_1(A+cn)\cos\theta_1}\cos\varphi)$$
$$\times \frac{e^{\alpha_0 \cos\phi \sqrt{(A+cn)^2 + A_1^2 + 2A_1(A+cn)\cos\theta_1}}}{2\pi I_0(\alpha_0\sqrt{(A+cn)^2 + A_1^2 + 2A_1(A+cn)\cos\theta_1})} \frac{\lambda^n}{n!} e^{-\lambda} \tag{43}$$

Because of $\lambda$ is as in equation (14) we have

$$P_e = \int_{-\pi}^{\pi} \mathrm{d}\varphi \int_{-\pi}^{\pi} \frac{\mathrm{d}\theta_1}{2\pi} \sum_{n=0}^{\infty} Q(\sqrt{2R_1}\sqrt{(A+cn)^2 + A_1^2 + 2A_1(A+cn)\cos\theta_1}\cos\varphi)$$
$$\times \frac{e^{\alpha_0 \cos\phi \sqrt{(A+cn)^2 + A_1^2 + 2A_1(A+cn)\cos\theta_1}}}{2\pi I_0(\alpha_0\sqrt{(A+cn)^2 + A_1^2 + 2A_1(A+cn)\cos\theta_1})} \int_{-\infty}^{\infty} \mathrm{d}x \frac{c_1^n (B+x)^{2n}}{n!} e^{-c_1 (B+x)^2} \tag{44}$$

When the useful signal amplitude has Nakagami distribution

$$p_A(A) = \frac{2}{\Gamma(m)} (\frac{m}{\Omega})^2 A^{2m-1} e^{-\frac{m}{\Omega}A} \tag{45}$$

the error probability is

$$P_e = \int_{0}^{\infty} \mathrm{d}A \int_{-\pi}^{\pi} \mathrm{d}\varphi \int_{-\pi}^{\pi} \frac{\mathrm{d}\theta_1}{2\pi} \sum_{n=0}^{\infty} Q(\sqrt{2R_1}\sqrt{(A+cn)^2 + A_1^2 + 2A_1(A+cn)\cos\theta_1}\cos\varphi)$$
$$\times \frac{e^{\alpha_0 \cos\phi \sqrt{(A+cn)^2 + A_1^2 + 2A_1(A+cn)\cos\theta_1}}}{2\pi I_0(\alpha_0\sqrt{(A+cn)^2 + A_1^2 + 2A_1(A+cn)\cos\theta_1})} \frac{\lambda^n}{n!} e^{-\lambda}$$
$$\times \frac{2}{\Gamma(m)} (\frac{m}{\Omega})^2 A^{2m-1} e^{-\frac{m}{\Omega}A} \tag{46}$$

When the interference consists of more sinusoidal waves, the error probability can be calculated in the similar way.

## 6 Conclusion

In this paper the coherent receiver for the binary phase modulated signal demodulation is considered. The phase loop exists in the circuit for the referent carrier extraction. The referent carrier is not ideal because of Gaussian noise and the interferences appeared in the circuit for the referent carrier extraction. The probability density function of the PLL phase error and the receiver error probability are calculated in the paper. The case of the interferences appearance with Gaussian noise in the circuit for the referent carrier extraction is also considered. Because of these circuits utilization in wireless telecommunication systems we considered the fading influence to the phase error probability density and to the system error probability.

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