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Optimal Pricing Strategies based on Time Delay in Multi-service Networks with Priority

This paper is dedicated to Professor Milić Stojić on the occasion of his 65th birthday

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Abstract: The problem of pricing equilibrium of multi-service priority-based network is studied by using incentive strategy in Stackelberg game theory. First, some concepts in game theory were introduced. Then, the existing results on two-user two-level Nash problem was introduced briefly. A new one-leader two-user two-level incentive Stackblberg strategy is presented by employing the time delay in the strategy.

Keywords: Game theory, incentive pricing, Nash equilibrium, priority based networks, Stackelberg strategy.

1 Introduction

Modern telecommunications represent multi-service network systems that are hierarchically structured large-scale complex systems carrying a wide variety of traffic classes and serving many users, and besides are undergoing a permanent process of development [1, 2]. Functionally they appear in various operating topologies as a result of the executed networking. In addition, the network is shared by a set of non-cooperative users each sending its communication flow in a fashion optimizing the individual performance objective.

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The users self-optimizing behaviour tendency, on one hand, leads to network operating behaviour that is usually referred to as *non-cooperative networking* [3]; it has been observed in various practical networks. Such common example networks with non-cooperative operating dynamics are the B-ISDN (Broadband Integrated Service Digital Network) network and the Internet [4, 5], and the incentive pricing is the realistic means to arrive at optimum operation [1, 2]. On the other hand, while the network is developing, the network users need and require strict Quality of Service (QoS). Then immediately how to supply the corresponding QoS becomes an urgent issue. It is therefore that the idea of employing some pricing strategy to manage the users actions and using different prices for different QoS supplies has been introduced [5]. Recently, DaSilva, Petr and Akar [1] have studied the pricing problem in multi-service priority-based networks by using the concept of Nash equilibrium in Game Theory [6], [7], [8].

The pricing of network services not only determines the economic viability of commercial networks but also plays an important role in traffic management through its influence on user behaviour. The principles of game theory have been used for years as tool of economic analysis [6] and for management problems [9]. Its applications to networking problems of recent date [3] and include not only pricing but also congestion control and call-admission control as well [2], [4][5]. While at times users and providers may have conflicting objectives [9], the pricing can be used as a means to encourage users to exhibit behaviour that is beneficial to the network as a whole [1, 2].

In our recent research [2], we have derived an optimal multi-user Nash-equilibrium strategy by studying this problem from the viewpoint of pricing based strategies in multi-service networks. A network authority, e.g. network manager, was introduced to mediate actively when users do not cooperate. The manager imposes certain limits to users behaviour by employing an incentive Stackelberg strategy thus obtaining the optimum in running the network. A kind of linear Stackelberg incentive strategy was proposed in [2] under which the users act as if they were a team, and it is further improved in this article on the to account of operating time delay.

In the subsequent section, the relationship between the behaviours and benefits of users and the strategies of manager is revealed first. Then, an improved incentive Stackelberg strategy based on the time delay in networks is developed, which enables the network manager to operate effectively the whole network at an operating Nash equilibrium. For this purpose a couple of new theoretical results are proved. Conclusions and references follow thereafter.

2 System Model

Consider a single FIFO queue with M levels of priority. In [1], a relatively simple system ($M = 2$) was studied and a method was given for finding the equilibrium in a priority system with any price difference between services and user's utility functions.

Let N be the number of customers utilizing the queue at a given time. The user i can choose to tag a percentage s_{ij} of his/her traffic as priority of level j, paying a price p_j for the bandwidth utilized. Denote $s_i = (s_{i1}, s_{i2}, \cdots, s_{iM})^T$. All available strategies of user i make a set S_i. The joint strategy space, denoted by S, is the Cartesian product of the individual strategy sets, so

$$S = S_1 \times S_2 \times \cdots \times S_N = \{s = (s_1, s_2, \cdots, s_N) \mid s_i \in S_i\}. \tag{1}$$

Denote the traffic statistics for user i by t_i which may include information such as average transmission rate and statistics of message size.

The consumers surplus $C_i(s)$ is defined as the difference between the utility obtained with given service choice and the price paid for the service.

$$C_i(s) = U_i(q_i) - \sum_{j=1}^{M} p_j s_{ij} \lambda_i \qquad i = 1, 2, \cdots, N \tag{2}$$

where $U_i(q_i)$ is the user's utility function and q_i represents the level of service the user i received from the network. λ_i is the average arrival rate.

Users will decide on a service request $s^* = (s_1^*, s_2^*, \cdots, s_N^*)$ that maximizes their surplus functions (2). A proper kind of equilibrium joint strategy is necessary to evaluated the effectiveness of a pricing policy. If the users act as team to maximize their surplus functions, the solution is easy to be obtained. In practice, however, the consumers are non-cooperative. A Nash equilibrium is a proper strategy combination where no user can unilaterally increase his/her utility by changing his/her strategy [3, 6]. In the two-user two-level case, [1] studied the Nash equilibrium and got some results. As pointed out in [1], the equilibrium is stable in some price range and not stable out of the range, *i.e.* at least one user would prefer another strategy combination to optimize his/her cost. To a proper strategy combination, a kind of so-called incentive Stackelberg strategy is best to be employed to force each user to obey this strategy combination. More precisely, the definitions of Nash equilibrium, incentive Stackelberg strategy and relative concept of Pareto optimality [6, 7, 8], respectively, are offered as follows

Definition 1 *(**Nash Equilibrium**) Strategy combination s^* is a Nash equilibrium if*

$$C_i(s^*) \geq C_i(s_i, s_{-i}^*) \qquad \forall s_i \in S_i, \quad i = 1, 2, \cdots, N \tag{3}$$

where $s_{-i} = (s_1, \cdots, s_{i-1}, s_{i+1}, \cdots, s_N)$.

Definition 2 *(**Pareto Optimality**) A strategy combination* $\tilde{s}$ *is Pareto optimal if there does not exist* $s' \in S$ *such that*

$$C_i(s') \geq C_i(\tilde{s}) \qquad \forall i = 1, 2, \cdots, N \tag{4}$$

and for at least one i,

$$C_i(s') > C_i(\tilde{s}) \tag{5}$$

Definition 3 *(**Incentive Stackelberg Strategy**) Assume* $\hat{s}$ *be the proper strategy combination of the entire network. As a leader, the network provider can make a leader strategy* s_0*, which is some kind of pricing decision effecting* $C_i(s)$ *in such way as* $C_i(s_0(s), s)$*, such that*

$$C_i(s_0(\hat{s}), \hat{s}) \geq C_i(s_0(s_i, \hat{s}_{-i}), s_i, \hat{s}_{-i}) \quad \forall s_i \in S_i, \quad i = 1, 2, \cdots, N \tag{6}$$

For the incentive Stackelberg strategy, there exists a necessary and sufficient condition by which we can determine an incentive Stackelberg strategy easily [10].

Lemma 1 *A strategy* s_0 *is an incentive Stackelberg strategy if and only if*

$$\arg\max_s C_i(s_0(s), s) = \hat{s}, \tag{7}$$

$$s_0(\hat{s}) = \hat{s}_0, \tag{8}$$

where, $\hat{s}_0$ *is the proper strategy of the leader, when the users act as a team.*

3 Two-user Two-level Nash Problem

In [1], a two-user two-level case was investigated by using Nash equilibrium concept with Pareto optimality. In that case, the surplus of user i was selected as

$$C_i(s) = A_i - B_i W_i^{d_i} - p_H s_{i1} \lambda_i - p_L s_{i2} \lambda_i, \quad i = 1, 2. \tag{9}$$

where, $A_i - B_i W_i^{d_i}$ is some nonlinear utility function of W_i if $d_i \neq 1$. For each user, A_i and B_i are constants. W_i is the average waiting time in the queue. p_H (and p_L) is the price of the high (and low) priority of the two levels of priority, respectively. Because s_{ij} represents the percentage of traffic, so we can rewrite (9) as

$$C_i(s) = A_i - B_i W_i^{d_i} - p_H s_i \lambda_i - p_L (1 - s_i) \lambda_i, \quad i = 1, 2. \tag{10}$$

where s_{i1} is replaced by s_i for simplicity. The joint strategy space for that case can be $S = [0,1] \times [0,1]$.

Due to the scarcity of closed-form results for delay in $G/G/1$ priority queueing systems, we assume Poisson arrivals to the queue, with $t_i = (\lambda_i, \overline{x_i}, \overline{x_i^2})$, where $\overline{x_i}$ and $\overline{x_i^2}$ are the first two moments of message length for user i, $i = 1,2$. Utilizing the well-known queuing theory results [11], and assuming $\overline{x_i} = \overline{x}$, $\overline{x_i^2} = \overline{x^2}$, $\forall i$, the average waiting time W_i in the G/G/1 queue will be

$$W_i = K \frac{1 - \overline{x}\lambda_T s_i}{1 - \overline{x}(s_1\lambda_1 + s_2\lambda_2)}, \quad i = 1,2 \tag{11}$$

where $\lambda_T = \lambda_1 + \lambda_2$, $K = \dfrac{\overline{x^2}\lambda_T}{2(1 - \overline{x}\lambda_T)}$ which is unrelated to s_i.

So we can calculate $\partial W_i^{d_i} / \partial s_i$, $i = 1,2$, as follows

$$\begin{aligned} \frac{\partial W_i^{d_i}}{\partial s_i} =& K^{d_i} d_i \left(\frac{1 - \overline{x}\lambda_i s_i}{1 - \overline{x}(s_1\lambda_1 + s_2\lambda_2)} \right)^{d_i - 1} \left(\frac{-\overline{x}\lambda_T}{1 - \overline{x}(s_1\lambda_1 + s_2\lambda_2)} \right. \\ &+ \left. \frac{(1 - \overline{x}\lambda_T s_i)\overline{x}\lambda_i}{(1 - \overline{x}(s_1\lambda_1 + s_2\lambda_2))^2} \right) \\ =& K^{d_i} d_i \overline{x}\lambda_j \frac{(1 - \overline{x}\lambda_T s_i)^{d_i - 1}}{(1 - \overline{x}(s_1\lambda_1 + s_2\lambda_2))^{d_i + 1}} (-1 + \overline{x}\lambda_T s_j). \end{aligned} \tag{12}$$

Let

$$\frac{\partial C_1(s)}{\partial s_1} = B_1 K^{d_1} d_1 \overline{x}\lambda_2 \frac{(1 - \overline{x}\lambda_T s_1)^{d_1 - 1}(1 - \overline{x}\lambda_T s_2)}{(1 - \overline{x}(s_1\lambda_1 + s_2\lambda_2))^{d_1 + 1}} - \triangle p \lambda_1 = 0, \tag{13}$$

and

$$\frac{\partial C_2(s)}{\partial s_2} = B_2 K^{d_2} d_2 \overline{x}\lambda_1 \frac{(1 - \overline{x}\lambda_T s_2)^{d_2 - 1}(1 - \overline{x}\lambda_T s_1)}{(1 - \overline{x}(s_2\lambda_2 + s_1\lambda_1))^{d_2 + 1}} - \triangle p \lambda_2 = 0, \tag{14}$$

where, $\triangle p = p_H - p_L$.

If there exists a solution for (13) and (14), it is just the pure strategy Nash equilibrium. A closed-form result, under the assumption of exponentially-distributed message lengths of mean μ, as well as numerical results, were given in [1].

Lemma 2 *A two-user system achieves a unique Nash equilibrium that is Pareto optimal and maximizes revenue if and only if*

$$\left(\min_{i=1,2} B_i \right) \frac{2\lambda}{\mu(\mu - 2\lambda)(\mu - \lambda)} < \triangle p < \left(\max_{i=1,2} B_i \right) \frac{2\lambda}{\mu(\mu - 2\lambda)(\mu - \lambda)} \tag{15}$$

The Lemma 2 gave a range of $\triangle p$ for the existence of an optimal equilibrium. In that range, there exists the Nash equilibrium with Pareto optimality. If $\triangle p$ is out of the range given by (15), there maybe exist a Nash equilibrium which has, however, the probability of deviation by at least one user who would prefer other strategy combination to maximize his/her surplus. We refer to such case as instability of the network.

When

$$\begin{aligned} \Delta p &= \left(\min_{i=1,2} B_i\right)\frac{2\lambda}{\mu(\mu-2\lambda)(\mu-\lambda)} \\ &\text{or} \\ \Delta p &= \left(\max_{i=1,2} B_i\right)\frac{2\lambda}{\mu(\mu-2\lambda)(\mu-\lambda)} \end{aligned} \tag{16}$$

there is no longer a unique equilibrium, since more than one strategy will be equivalent in the eye of one of the users.

Furthermore, when B_i for all users are the same, say B, $\triangle p$ has to be just a point exactly rather than in an interval to guarantee the sufficient and necessary condition hold. In such case, the network provider has to put the high-priority level at the point where is just $\triangle p$ higher than the low-priority level, or he cannot control the entire network.

4 One-leader Two-user Two-level Incentive Problem

To overcome the instability of Nash equilibrium solution, we employ the incentive Stackelberg strategy. At first, we should introduce a leader into the system. And then, a cost function for the leader should be selected. At the third step, a proper strategy combination for the entire network should be determined. Finally, we will construct an incentive Stackelberg strategy to achieve our goal.

The network provider is the best and natural choice to be the player with leadership in the game. The leader represents the interest of the entire system. We can consider the following linear combination of C_i as the leader's cost function.

$$C_0(s_0,s) = \alpha_1 C_1(s_0,s) + \alpha_2 C_2(s_0,s) \tag{17}$$

where α_1 and α_2 are the weights of user 1 and user 2, respectively, and $\alpha_1+\alpha_2 = 2$. s_0 is the strategy of the leader, which is as a function of the strategy combination s so that the leader has the ability to punish the users who want to deviate the proper strategy combination. For two-user system, s_0 may be a 2-dimensional vector, i.e. $s_0 = (s_{01}, s_{02})^T$. In practice, s_{0i} can be taken as the extra price charged to the use

i if he makes the deviation from the team solution. Therefore, s_{0i} will never be negative.

In (17), $C_i(s_0,s)$, $i=1,2$, are the extensions of $C_i(s)$, i.e.

$$C_i(s_0,s) = C_i(s) + P_i(s_0), \qquad i = 1,2 \tag{18}$$

where $P_i(s_0)$, $i=1,2$, play the punitive or incentive part and $P_i(0)=0$, $P_i(s_0)<0$ for $s_0 \neq 0$. Usually, we can take a linear function as $P_i(s_0)$. So, we have

$$C_i(s_0,s) = A_i - B_i W_i^{d_i} - p_H s_i \lambda_i - p_L(1-s_i)\lambda_i + \xi_i s_{0i}, \quad i = 1,2. \tag{19}$$

where $\xi_i < 0$, $i=1,2$, are the parameters of the linear functions. Usually, $\xi_i = -1$, for the convenience of calculations.

When $\alpha_1 = \alpha_2$ and $s_0 = 0$, $C_0(s_0,s)$ is just the sum of the surplus $C_i(s), i = 1,2$.

The proper strategy combination can be obtained by taking derivative of (17) with respect to $s_i,\ i=0,1,2$. Denote the proper strategy combination as $(\tilde{s}_0,\tilde{s})$. We can calculate $\partial C_0(s_0,s)/\partial s_i$, $i=1,2$, as follows.

$$\frac{\partial C_0(s_0,s)}{\partial s_0} = (\alpha_1\xi_1, \alpha_2\xi_2)^T, \tag{20}$$

$$\begin{aligned}\frac{\partial C_0(s_0,s)}{\partial s_1} &= \alpha_1\frac{\partial C_1(s_0,s)}{\partial s_1} + \alpha_2\frac{\partial C_2(s_0,s)}{\partial s_1} \\ &= \alpha_1\frac{\partial C_1(s)}{\partial s_1} + \alpha_2\frac{\partial C_2(s)}{\partial s_1}\end{aligned} \tag{21}$$

$$\begin{aligned}\frac{\partial C_0(s_0,s)}{\partial s_2} &= \alpha_1\frac{\partial C_1(s_0,s)}{\partial s_2} + \alpha_2\frac{\partial C_2(s_0,s)}{\partial s_2} \\ &= \alpha_1\frac{\partial C_1(s)}{\partial s_2} + \alpha_2\frac{\partial C_2(s)}{\partial s_2}\end{aligned} \tag{22}$$

The first part of the right hand in (21) is just (13) and the second part is

$$\begin{aligned}\frac{\partial C_2(s)}{\partial s_1} &= -B_2K^{d_2}d_2\left(\frac{1-\bar{x}\lambda_T s_2}{1-\bar{x}(s_1\lambda_1+s_2\lambda_2)}\right)^{d_2-1} \\ &\quad \times (1-\bar{x}\lambda_T s_2)\left(\frac{\bar{x}\lambda_1}{(1-\bar{x}(s_1\lambda_1+s_2\lambda_2))^2}\right) \\ &= -B_2K^{d_2}d_2\bar{x}\lambda_1\frac{(1-\bar{x}\lambda_T s_2)^{d_2}}{(1-\bar{x}(s_1\lambda_1+s_2\lambda_2))^{d_2+1}}\end{aligned} \tag{23}$$

The first part of the right hand in (22) is

$$\begin{aligned}\frac{\partial C_1(s)}{\partial s_2} =& -B_1K^{d_1}d_1\left(\frac{1-\bar{x}\lambda_T s_1}{1-\bar{x}(s_1\lambda_1+s_2\lambda_2)}\right)^{d_1-1}\\ &\times(1-\bar{x}\lambda_T s_1)\left(\frac{\bar{x}\lambda_2}{(1-\bar{x}(s_1\lambda_1+s_2\lambda_2))^2}\right)\\ =& -B_1K^{d_1}d_1\bar{x}\lambda_2\frac{(1-\bar{x}\lambda_T s_2)^{d_1}}{(1-\bar{x}(s_1\lambda_1+s_2\lambda_2))^{d_1+1}}\end{aligned} \tag{24}$$

and the second part is just (14). As $C_i(s_0,s)$ is a linear function of s_{0i} as in (19), $\partial C_0(s_0,s)/\partial s_0$ never be zero but less than zero. So, we can see $\tilde{s}=(\tilde{s}_0,\tilde{s}_1,\tilde{s}_2)^T=(0,\tilde{s}_1,\tilde{s}_2)^T$.

We have just established the following lemma when it is in the interior of the joint strategy set $[0,\infty)\times S$, i.e. in $(0,\infty)\times(0,1)\times(0,1)$.

Lemma 3 *If $\tilde{s}$ is a solution for the system of equations:*

$$\frac{\partial C_0(s_0,s)}{\partial s_i}=0, \quad i=1,2 \tag{25}$$

then $\tilde{s}$ is a proper strategy combination on which the users act as a team.

If $\partial C_0(s_0,s)/\partial s_i \neq 0$ for all $s_i \in (0,1), i=1,2$, $\tilde{s}$ will not be in $(0,1)\times(0,1)$. $\tilde{s}_i=1$, when $\partial C_0(s_0,s)/\partial s_i \geq 0$ over S. Conversely, $\tilde{s}_i=0$, when $\partial C_0(s_0,s)\partial s_i \leq 0$.

If the users are cooperative, then the system will be in the optimal situation by taking the team strategy combination $\tilde{s}$. The users of the network, however, are usually noncooperative. In such case, a Nesh strategy can be taken among the users who are in the same position. But, if a leader is introduced, the leader will be in the charge of management of network. Therefore, it is his/her duty to manage the entire system optimal. Stackelberg strategy is, then, the best to that purpose.

The Strakelbreg Strategy of [2] has simple formula, can not embody the relationship between profits of the user and the network provider. Here, we derive a kind of incentive strategy that is improved considerably on the account of the networking time delay. First we consider the effect of time delay.

$$W_i = K\frac{1-\bar{x}\lambda_T s_i}{1-\bar{x}(s_1\lambda_1+s_2\lambda_2)} \quad i=1,2; \tag{26}$$

$$\frac{\partial W_i}{\partial s_i} = K\bar{x}\lambda_j\frac{-1+\bar{x}\lambda_T s_j}{(1-\bar{x}(s_1\lambda_1+s_2\lambda_2))^2} < 0 \tag{27}$$

where W_i is the decrease function about s_i. When s_i increase, the value of W_i decrease.

$$\frac{\partial W_j}{\partial s_i} = K\bar{x}\lambda_i \frac{1-\bar{x}\lambda_T s_j}{(1-\bar{x}(s_1\lambda_1+s_2\lambda_2))^2} > 0 \tag{28}$$

where W_j is the increase function about s_i,when s_i increase, the value of W_j increase.

By analyzing above formula, we know that if the user i use the advanced priority bandwidth then it's time delay is decreased and the other's is increased. We can chose the following incentive Stackelberg strategy.

$$s_0 = \tilde{s}_0 + \begin{bmatrix} Q_1((-B_1(W_1^{d_1} - \widetilde{W}_1^{d_1}) + B_2(\widetilde{W}_{-2}^{d_2} - \widetilde{W}_2^{d_2})) \\ Q_2(B_1(\widetilde{W}_{-1}^{d_1} - \widetilde{W}_1^{d_1}) - B_2(W_2^{d_2} - \widetilde{W}_2^{d_2})) \end{bmatrix} \tag{29}$$

where

$$\widetilde{W}_i = K\frac{1-\bar{x}\lambda_T\tilde{s}_i}{1-\bar{x}(\tilde{s}_1\lambda_1+\tilde{s}_2\lambda_2)} \quad i=1,2 \tag{30}$$

$$W_i = K\frac{1-\bar{x}\lambda_T s_i}{1-\bar{x}(s_i\lambda_i+\tilde{s}_j\lambda_j)} \quad i=1,2;\ j=1,2;\ i\neq j \tag{31}$$

$$\widetilde{W}_{-i} = K\frac{1-\bar{x}\lambda_T\tilde{s}_i}{1-\bar{x}(\tilde{s}_i\lambda_i+s_j\lambda_j)} \quad i=1,2;\ j=1,2;\ i\neq j \tag{32}$$

where Q_1 and Q_2 are the punishment parameters. Substituting(29) into (19) yields

$$\begin{aligned} C_i(s_0,s) &= A_i - B_iW_i^{d_i} - P_Hs_i\lambda_i - P_L(1-s_i)\lambda_i \\ &+\xi_iQ_i(-B_i(W_i^{d_i}-\widetilde{W}_i^{d_i}) + B_j(\widetilde{W}_{-j}^{d_j}-\widetilde{W}_j^{d_j})) \quad i=1,2;\ j=1,2;\ i\neq j \end{aligned} \tag{33}$$

The derivative with respect to s_i are given by means of

$$\begin{aligned} \frac{\partial C_1(s_0,s)}{\partial s_1} =& B_1K^{d_1}d_1\bar{x}\lambda_2\frac{(1-\bar{x}\lambda_Ts_1)^{d_1-1}(1-\bar{x}\lambda_Ts_2)}{(1-\bar{x}(s_1\lambda_1+s_2\lambda_2))^{d_1+1}} - \Delta p\lambda_1 \\ &+\xi_1Q_1\left(-B_1K^{d_1}d_1\bar{x}\lambda_2\frac{(1-\bar{x}\lambda_Ts_1)^{d_1-1}(-1+\bar{x}\lambda_T\tilde{s}_2)}{(1-\bar{x}(s_1\lambda_1+\tilde{s}_2\lambda_2))^{d_1+1}}\right. \\ &\left.+B_2K^{d_2}d_2\bar{x}\lambda_1\frac{(1-\bar{x}\lambda_T\tilde{s}_2)^{d_2}}{(1-\bar{x}(s_1\lambda_1+\tilde{s}_2\lambda_2))^{d_2+1}}\right) \end{aligned} \tag{34}$$

$$\begin{aligned} \frac{\partial C_2(s_0,s)}{\partial s_2} =& B_2K^{d_2}d_2\bar{x}\lambda_1\frac{(1-\bar{x}\lambda_Ts_2)^{d_2-1}(1-\bar{x}\lambda_Ts_1)}{(1-\bar{x}(s_1\lambda_1+s_2\lambda_2))^{d_2+1}} - \Delta p\lambda_2 \\ &+\xi_2Q_2\left(B_1K^{d_1}d_1\bar{x}\lambda_2\frac{(1-\bar{x}\lambda_T\tilde{s}_1)^{d_1}}{(1-\bar{x}(\tilde{s}_1\lambda_1+s_2\lambda_2))^{d_1+1}}\right. \\ &\left.-B_2K^{d_2}d_2\bar{x}\lambda_1\frac{(1-\bar{x}\lambda_Ts_2)^{d_2-1}(1-\bar{x}\lambda_T\tilde{s}_1)}{(1-\bar{x}(\tilde{s}_1\lambda_1+s_2\lambda_2))^{d_2+1}}\right) \end{aligned} \tag{35}$$

Let $s = \tilde{s}$ and

$$\begin{aligned} F_i(s) =& B_i K^{d_i} d_i \bar{x} \lambda_j \frac{(1-\bar{x}\lambda_T s_i)^{d_i-1}(1-\bar{x}\lambda_T s_j)}{(1-\bar{x}(s_1\lambda_1+s_2\lambda_2))^{d_i+1}} \quad i=1,2;\ j=1,2;\ i\neq j \\ H_i(s) =& B_i K^{d_i} d_i \bar{x} \lambda_i \frac{(1-\bar{x}\lambda_T s_i)^{d_i}}{(1-\bar{x}(s_1\lambda_1+s_2\lambda_2))^{d_i+1}} \qquad i=1,2 \end{aligned} \tag{36}$$

Let

$$\frac{\partial C_i(s_0,s)}{\partial s_i} \;=\; 0 \qquad i=1,2 \tag{37}$$

We have

$$Q_i \;=\; \frac{\Delta p \lambda_i - F_i(\tilde{s})}{\xi(H_i(\tilde{s}) - F_i(\tilde{s}))} \qquad i=1,2; \tag{38}$$

where Q_i is the punishment parameter of manager, so the user can get maximum surplus at $\tilde{s}$ point.

The leader's strategy (29) with Q_i given in (38) is just the incentive Stackelberg strategy which can make the users get their maximal surplus at the point $\tilde{s}$.

Theorem 1 *If Q_i, $i=1,2$, in (29) are chosen as (38), then the strategy s_0 given in (29) can be taken as an incentive Stackelberg strategy for the leader of the network.*

Proof : According to Lemma 1, we just need to show that s_0 presented in (29) with Q_i given in (38) satisfies the conditions (7) and (8). It is evident that (8) is held for this strategy. Now, let us consider the condition (7). Substituting (38) into (34) and (35), we can see that $\partial C_i(s_0,s)/\partial s_i = 0$ only when $s_i = \tilde{s}_i$. It indicates that only $s_i = \tilde{s}_i$ can be the optimum point of $C_i(s_0,s)$, i.e.

$$\arg\max_s C_i(s_0(s),s) = \tilde{s}. \tag{39}$$

Whatsoever $s_i = \tilde{s}_i, i = 1,2$ is or not the extreme point, it always make $\partial C_i(s_0,s)/\partial s_i = 0$ with Q_i chosen as in (38).

5 Conclusions

In this article, we improved the existing result on pricing strategies that reinforce Nash equilibrium in multi-service networks by means of the incentive Stackelberg strategy concept of game theory. The new theoretical results, presented in terms of Lemma 3 and Theorem 1, provide a new solution to the problem in two-user two-priority-level case in the game-theoretic setting of leader-follower concept.

The network provider was introduced into the game-theoretic representation model of the problem as the player who has the leadership role in the priority-based networking game. Then an appropriate linear incentive Stackelberg strategy of network-service pricing was derived. This incentive Stackelberg strategy, however, is a non-linear function of users time delays.

Network provider, who employs this strategy, shall be able to manage all the network users at Nash equilibrium by effectively forcing them to act as if they were a team. Thus, during the network operation, the network provider can inter-link users connections and manage users behaviour much better. Hence he will ensure the benefits for all the users and the network.

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