Introduction to Fuzzy Logic

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Abstract: This paper gives basics and reviews some classical as well as new applications of fuzzy logic. The main emphasis of the paper is on fuzzy decision making under a linguistic view of fuzzy sets.

Keywords: Fuzzy sets, fuzzy logic, linguistic variables, fuzzy decision making, fuzzy control.

1 Introduction

We are celebrating the 40th anniversary of the publication of the seminal paper of Lotfi A. Zadeh in 1965, introducing the concept of **fuzzy sets**, that opened a totally new view of systems, logic and models of reasoning [1]. In the meantime, thousands of papers in this area, covering both theory and applications, have been published all over the world. Textbooks on fuzzy sets are no longer a curiosity (see *e.g.* [2], [3], [4], [5], [6] or [7]) and many Universities presently offer regular courses on theory or applications of fuzzy sets. (See an ongoing list related to European universities at the following web page: `http://www.eusflat.org/research/teaching.htm`).

Consider the interval [0, 10] of the real line to be chosen as universe of discourse and consider the statement "x is between 3 and 5". This may be represented by a function $\mu_{3-5} : [0,10] \rightarrow [0,1]$, with $\mu_{3-5}(x) = 1$ if $3 \leq x \leq 5$ and $\mu_{3-5}(x) = 0$ otherwise, as shown on Fig. 1 (left part). It becomes apparent that this is the characterising function of the (classical) set [2], [8]. Consider now the statement "x is near 4". If ε is a very small positive real number, it seems quite reasonable to accept that $(4 - \varepsilon)$ is near 4. If the subtraction of ε is continued, new values will

be obtained, which will have a decreasing "*degree of nearness*" to 4, until a value will be reached, which depending on the context, will have a 0 degree of nearness to 4. Values smaller than this limit value will no longer be considered to be "near" 4. Furthermore, if the experiment is repeated with $4 + \varepsilon$, and continue adding ε, a symmetric behaviour will be obtained. If a function $\mu_{near-4} : [0,10] \rightarrow [0,1]$ is looked for to represent this statement, it cannot be of the same kind as μ_{3-5} (that lead to a classical set). Assuming that 3 and 5 are acceptable limit points for "near 4", then $\mu_{near-4}(x) = 0$ if $x \leq 3$ or $x \geq 5$; $\mu_{near-4}(4) = 1$, $\mu_{near-4}(x)$ will be continuous and increasing for $3 \leq x \leq 4$ and $\mu_{near-4}(x)$ will be continuous and decreasing for $4 \leq x \leq 5$. Without further information, linear transitions will be chosen as shown in Fig. 1 (right part). μ_{near-4} represents a **fuzzy set**. A distinguishing feature of fuzzy sets is that their elements have a degree of membership, expressed as a numerical value in [0,1]. For this reason the function μ will often be referred to as the "membership function" of the corresponding fuzzy set. It becomes apparent that classical sets are particular cases of fuzzy sets (where [0,1] is restricted to {0,1}). Statements like "x is between 3 and 5" are called *rigid* meanwhile those like "x is near 4" are called *flexible*.
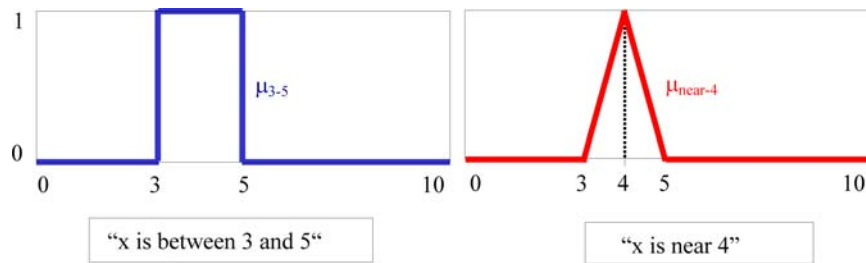

Fig. 1. Representation of a rigid (left) and a flexible (right) statement.

Consider the statement "John is 1781.49231651850 mm high". This is a very accurate statement, but it becomes apparent that in this case this amount of accuracy is a burden rather than a semantic help. A group of high school pupils will probably agree that the meaning of the above statement is "John is *tall*", where "tall" is a flexible predicate representable as a fuzzy set, providing as much accuracy as needed to properly understand the statement, but at the same time, as little accuracy as possible, to improve the tractability of the statement. Notice however, that if John had been attending a meeting of the Serbian national selection of basketball players, *they* would have rather said that "John is *short*". The context of a statement and its use has to be known (and preserved) when moving from statements using numbers to statements using words. To formalize these aspects Zadeh introduced in a series of three articles, the concept of **Linguistic Variable** [9, 10].

A Linguistic Variable has a name, a definition domain, a set of values and an interpretation. The name of the variable can be freely chosen, but it is wise to name it after the real variable it will represent. The definition domain has to be consistent with the universe where it will be used. The definition domain contains a set of Linguistic Terms, which represent the values that may take the Linguistic Variable at different states of intensity. Linguistic Terms are labelled fuzzy sets, usually with a trapezoidal or bell-shaped structure. (Triangles are, in this context, trapezes with the upper side reduced to one point). The cores of these fuzzy sets –(sets of elements with membership degree 1)- are ordered, and the supports –(sets of elements with membership > 0)- overlap between neighbours.

Consider as example the Linguistic Variable *humidity* defined on a universe [0%, 100%] with Linguistic Terms labelled as {*low, below average, average, above average, high*}. See Fig. 2. The representation of the Linguistic Variable will certainly be different for an inhabitant of Serbia than for an inhabitant of north Finland. Assume that the one used here is adequate.
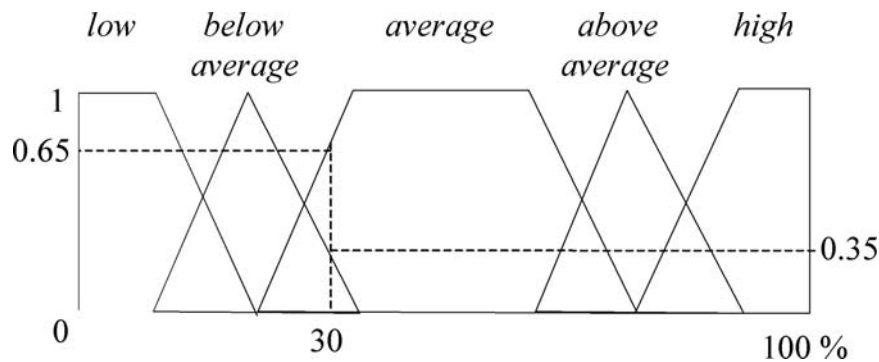

Fig. 2. Representation of the Linguistic Variable "humidity"

Notice that if the prevailing physical humidity were 30%, the corresponding linguistic representation would be given as a linear combination: 0.35 *below average* + 0.65 *average*. By defining proper operations upon linguistic variables [11], it is possible to have the basics for a fuzzy **logic** [7], which in its turn provides an adequate formalism for inferences, fuzzy modeling, fuzzy decision making and finally for fuzzy control, possibly the best known application of fuzzy systems.

2 Fuzzy Modeling, fuzzy decision making, fuzzy control

A model is a simplified representation of relevant aspects of the behaviour of a system, to help the user in obtaining a better understanding of the system, thus

being able to forecast and control its behaviour. If the model uses formalisms of fuzzy logic then it is called a fuzzy model. The simplest fuzzy model consists of a set of rules with an "if – then" structure:

$$If < \text{condition } 1 > and \ldots and < \text{condition } n > then < \text{conclusion} >$$

Where $<$ condition i $>$ is a statement of type "x_i is $L_{i,j}$". In this statement x_i represents the actual value of some i-th real world variable meanwhile $L_{i,j}$ is a flexible predicate naming the j-th linguistic term of the corresponding i-th Linguistic Variable. $L_{i,j}$ is given by a fuzzy set which represents the use of the flexible predicate on the domain of x_i. Statements of this kind are called "premises". The $<$conclusion$>$ is also a fuzzy set, which represents the linguistic term expressing a flexible predicate, which characterizes the output behaviour of the system if all conditions are satisfied.

Notice that "if – then" rules may be used both to model the state of a system (see Rule 1 below) and to take a decision to control the system (see Rule 2).

Rule 1: *If* outside is freezing *and* the window does not close properly *and* the heating is off *then* the room will become very cold

Rule 2: *If* outside is freezing *and* the window does not close properly *and* the heating is off *then* repare the window *and* switch on the heating

Fuzzy control is then closely related to fuzzy decision making. The earliest known proposal for fuzzy control may be found in [12] however the first successful realizations were reported in [13], [14]. The first application at industrial level was done to control the kiln of a cement fabric [15] and possibly the most impressive results of those years was the automatic fuzzy control of the subway train in Sendai, Japan [16]. An important collection of relevant results obtained in the following decade may be found in [17]. Many universities allover the world presently offer graduate courses in Fuzzy Control. An important advanced text-book on Fuzzy Control is due to [18].

Take as example the following set of "if – then" rules constituting a fuzzy control-model for an extreme simple irrigation system for a particular kind of cereal:

R1:   *if* the amount of rainfall last night was scarce
        then the watering should be gallonwise

R2:   *if* the amount of rainfall last night was regular
        then the watering should be literwise

R3:   *if* the amount of rainfall last night was large
        then the watering should be dropwise

To use the rules, the meaning of "scarce", "regular" and "large" in a universe with a liter/m^2 scale as well as that of "watering gallonwise", "watering literwise" and "watering dropwise" in a universe with a scale in volume of water is needed. Assume that the representation shown in the first two columns of Fig. 3 is adequate and corresponds to the <u>use</u> of these concepts by the farmers. Furthermore assume that the universes [0, K$_1$] and [0, K$_2$] have been defined by agricultural experts.

If the measured (or estimated) rainfall in a given night is "r" liter/m^2, then as may be seen on Fig. 3, "r" is not considered to be "scarce"; accordingly, R1 does not apply. On the other hand, "r" is close to "regular". Its degreee of membership is given by μ_{reg}(r). Similarly, μ_{large}(r) gives the degree of membership of "r" to "large". It is clear that μ_{reg}(r) > μ_{large}(r). This means that R2 is more satisfied than R3. It is reasonable to expect that the behaviour of the system should be closer to "literwise" than to "dropwise".
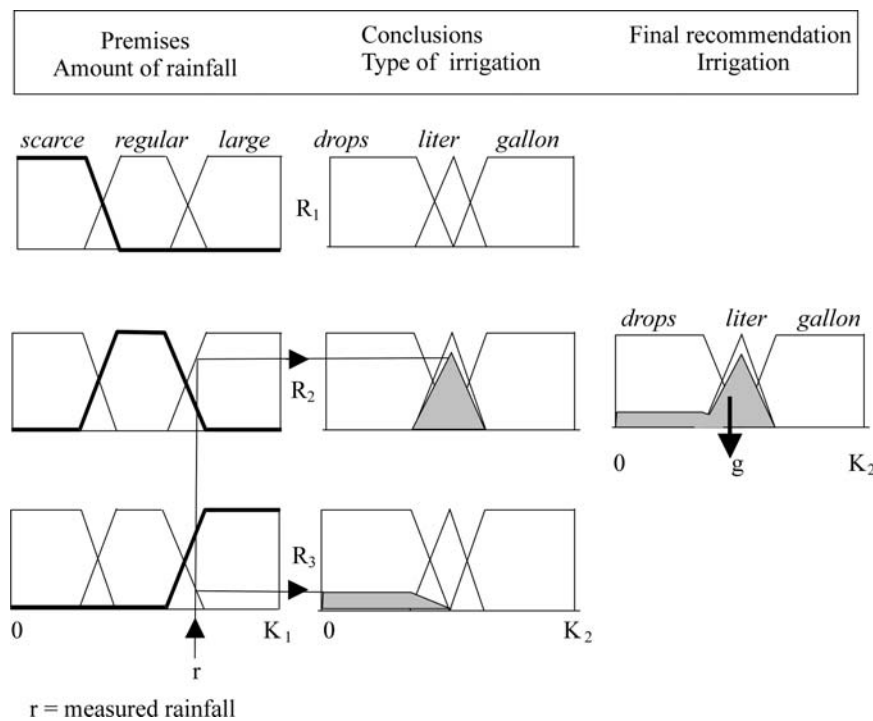

Fig. 3. Representation of the rule base of a fuzzy model. Use of the model for a given rainfall

This way of approximate reasoning may be formalized (among other alternatives), as follows: use the degree of membership of the actual input to the corresponding fuzzy sets representing the premises, to *scale* the fuzzy sets of the corresponding conclusions –(see the second column in Fig. 3)– and combine the scaled

conclusions by means of a *pointwise maximum*. This will lead to the shaded fuzzy set shown on the third column of Fig. 3. Its equation is:

$$\mu_{irrigation}(x) = \max [\ \mu_{reg}(r) \cdot g\mu_{liter}(x)\ ,\ \mu_{large}(r) \cdot g\mu_{drops}(x)\ ]\ \forall x \in [0, K_2\ ]$$

This resulting fuzzy set represents the flexible predicate assignable to the required watering. As mentioned earlier, this might quite well be "*closer to literwise than to dropwise*". However, in the scenario under consideration, farmers will not be interested in the *kind* of needed watering, but on the *amount* of water to be spread on the field. This implies the conversion of the obtained fuzzy set into a numerical value of the same universe $[0, K_2\ ]$. One way (but not the only one) of doing this, quite often used in fuzzy control, is to calculate the abscise "g" of the gravity center of the fuzzy set (as shown in Fig. 3) and use it as the numerical answer of the model.

Fuzzy control systems, usually named "fuzzy controllers" in the specialized literature, have been shown to be universal approximators [19], [20]. This means that *it is possible* to design a fuzzy controller to approximate with any desired accuracy a given target function. The known proofs are of the kind called "existence proofs" in mathematics, i.e., they only guarantee that at least one fuzzy controller does exist to approximate a given target function, but they do not give indications on how to design the controller. As seen in the simple example illustrated above, which in spite of its simplicity is representative for the design steps of many fuzzy control systems, decisions have to be taken on the number and content of the rules, on the number, shape and distribution of the fuzzy sets representing the Linguistic Terms of every Linguistic Variable, on the dimensions of the related universes, on the operations that will be used to combine degrees of satisfaction of premises – (when there are more than one per rule), on the operations to transfer the degree of satisfaction of the premises to the conclusions, on the operation to combine partially activated rules and on the operation to convert the resulting fuzzy set into a numerical value. There is not known method to automatically determine all these parameters. Prediction-correction iterative strategies are often used and lead to quite satisfactory results.

3   Other Applications

The fact that Linguistic Variables have a semantic level close to (possibly formalized) natural languages, supported by the fuzzy set representation of their Linguistic Terms and well defined operating rules, has opened a wide horizon for a variety of applications beyond fuzzy control. The World Wide Net, with its capability of making available and transferable huge amounts of information, states clear challenges for the development of intelligent search machines, able to cope

with uncertainty, vagueness and flexibility. This is an area with a continuously increasing number of contributions from the fuzzy logic community. See, for instance, the books [21] and [22]. Closely related to this subject is the extraction of knowledge from very large amounts of data, presently called "data mining" [23], [24]. Applications of fuzzy logic may also be found, among others, in the areas of Environmental Protection [25], [26], Economy [27], [28], Picture Processing [29], Power Systems [30] Social Sciences [31], Music [32], Hardware [33] and Telecommunications [34], [35]. Special mention deserve the many applications of fuzzy logic in Medicine [36] like *e.g.* in support of diagnosis [8], in Medical Image Processing [37], in medical data mining [38], and in medical modelling [39], [40]. This list is by all means not exhaustive; it only pretended to show the wide spectrum of applications that have been developed in the last decade. Parallel to these applications a similarly intensive development in theoretical aspects of fuzzy sets and fuzzy logic has taken place (as may be observed in the Proceedings of relevant international Conferences, like *e.g.* the annual World Conference organized by the International Fuzzy Sets Association IFSA). The present trend includes the search for synergy by combining fuzzy systems and evolutionary algorithms [41], [42], [43], fuzzy systems and neural networks [44], [45] as well as fuzzy systems and probabilities [46].

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