

Optimal Design of Circular Inductors

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Abstract: The scope of this work is to introduce a software tool for optimization of circular spiral inductors. It is based on compact model, where the physical behavior is described through analytical expressions in geometric programming (GP) form. This paper describes three significant innovations: (a) optimization of circular inductor via GP, (b) new expressions for inductance and Q-factor in GP form, (c) globally optimal trade-off curves between maximum self-resonant frequency and inductance values or minimum inductor area and inductance values. The proposed optimization algorithm is flexible because the designers of RF integrated circuits can easily optimize the circular inductor for a desired performance.

Keywords: Circular inductor, geometric programming, quality factor.

1 Introduction

The integration of radio-frequency (RF) systems in silicon (Si) requires that the fabrication process provides high-quality passive components. The performances of RF systems are strongly influenced by the performance of inductors. Inductors are important elements that require high quality factor (Q-factor), high self-resonance frequency, and small size for desired inductance. Many researchers have been studying inductors for Si RF integrated circuits [1]-[4]. Most of them have used measurements to construct models. While this technique is most practical, it does not permit optimization. Otherwise, researchers have used commercial 3D electromagnetic simulators to design and analyze inductors. Although this approach is accurate, it can be computationally very expensive and time-consuming. A computer-aided optimization technique using GP has been recently used [5] to

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find the optimum design for square spiral inductors. The technique in which a patterned ground shield (PGS) is inserted between an on-chip square spiral inductor and a silicon substrate has been reported [6].

In this paper, we present a new method for the optimal design of circular inductors via GP. Based on our algorithm, a compact computer program INOPT (Inductor OPTimization) along with its graphical user interface has been developed as CAD tool for fast optimization of spiral inductors. We have previously reported the optimization of square inductor [7] or octagonal inductor [8] using INOPT. In this paper, the goal is to find optimal values of parameters (the number of turns and layout dimensions) of the circular inductor. Both square and circular spiral inductors are being used in RF and microwave integrated circuits. It has been reported that the circular geometry has about 10% - 20% higher Q-factor and self-resonance frequency values than the square configuration. Despite the relatively extensive literature on the implementation and modeling [9]-[11] of circular spiral inductors not much is known about the optimization of circular inductors. In paper [11] it was demonstrated increased Q-factor by up to 93% using thick polyimide layer in circular inductor. However, author has pronounced, by optimizing the inductor's layout, the Q-factor of planar circular inductors may further be increased.

The paper is organized as follows: in Section 2 we discuss the geometric programming, in Section 3 we show how the parameters of the inductor model can be formulated as expressions in GP form. Section 4 gives constraints and proposed improvement of GP formulation. Sections 5, 6, and 7 present examples of the software use, in optimization of quality factor, inductor area, and self-resonant frequency, respectively. Conclusions are given in Section 8.

2 Geometric Programming

Geometric programming (GP) is a recent development in optimization theory. It is based on the mathematical properties of inequalities and is capable of solving certain problems involving nonlinear terms in both the objective function and constraints. It is sometimes possible to locate the optimum solution by simple inspection of the exponents in the objective function. GP is a particularly powerful method for engineering design and optimization problems.

A generalized GP problem with N variables and M constraints has the form: minimize

$$y_o(x) = \sum_{t=1}^{T_0} \sigma_{ot} \cdot c_{ot} \prod_{n=1}^N x_n^{\alpha_{otn}}, \quad (1)$$

subject to constraints

$$\sum_{t=1}^{T_m} \sigma_{mt} \cdot c_{mt} \prod_{n=1}^N x_n^{a_{mnt}} \leq \sigma_m, \text{ for } m = 1, 2, \dots, M \quad (2)$$

where σ_{ot} and $\sigma_{mt} = \pm 1$ are the sign of each term in the objective function and m^{th} constraint, respectively, c_{ot} and $c_{mt} > 0$ are the coefficients of each term in the objective function and m^{th} constraint, respectively. $x_n > 0$ is the independent variables, $\sigma_m = \pm 1$ is the constant bound of the m^{th} constraint, a_{otn} and a_{mnt} are the exponents of the n^{th} independent variable of the t^{th} term of objective function and m^{th} constraint, respectively, M is the number of constraints, N is the number of variables, T_o is the number of terms in the objective function, T_1, T_2, \dots, T_M are the number of terms in each constraint, 1 to M , respectively, $\sigma = \pm 1$ assumed sign of the objective function.

If $T_o = 1$, the objective function is called a monomial function. The sign of each term in the objective function or the constraint functions determines whether the polynomial is a posynomial (if σ_{ot} or $\sigma_{mt} = +1$) or a signomial (if σ_{ot} or $\sigma_{mt} = -1$).

3 Layout and Modeling of Circular Inductors

The computer-based algorithm for optimization square spiral inductors via GP is presented in [5]. No circular inductor optimization tools exist to help with the design of RF integrated circuits. It is well-known that ideal geometry of planar inductor is a circular spiral. The inductor parameters can be classified into two groups: technology and layout parameters. The technology parameters (thickness of substrate, thickness of silicon dioxide layer, ...) depend on fabrication process. The layout optimization parameters are the number of turns – n , the width of conductor – w , the spacing between adjacent conductors – s , the outer diameter – d_{out} , and the average diameter – d_{avg} , which can be expressed as $d_{avg} = 0.5(d_{out} + d_{in})$. These five variables are optimization variables. Some of these variables are shown in Fig. 1. Other geometry parameters of interest are the inductor length and the inductor area. The total length of the inductor can be expressed as $l = n \cdot d_{avg} \cdot N \cdot \tan(\pi/N)$ and the inductor area as $A = d_{out}^2$ [5]. For a square inductor $N = 4$, for octagonal $N = 8$, and for circular inductor, N is a big number. Simple mathematical operations lead to $l = \pi \cdot n \cdot d_{avg}$.

All the results in this paper are based on a relatively simple, reasonably accurate one-port inductor model shown in Fig. 2 (a), and simplified model is shown in Fig. 2 (b) [1], but INOPT solutions are not restricted to only one-port configurations. It is necessary to emphasize that for the inductor with PGS element Rp does not exist in Fig. 2 (b).

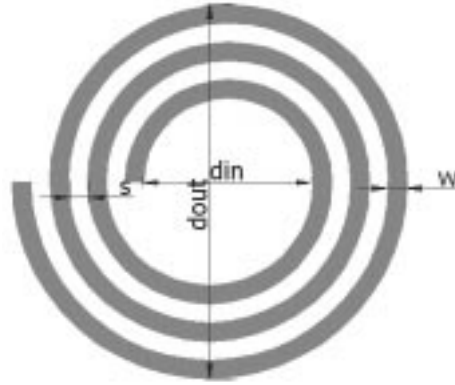


Fig. 1. Layout of planar circular inductor.

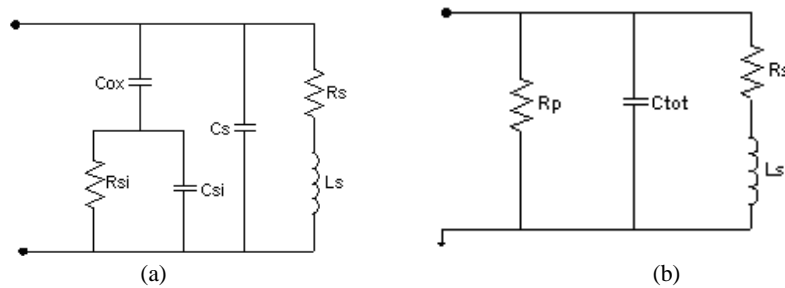


Fig. 2. (a) One-port model of a planar spiral inductor; (b) Simplified model of planar spiral inductor.

In this subsection, we give expressions for model parameters in the GP form.

Inductance L_s . A monomial expression, obtained by fitting technique, for calculation inductance of planar spiral inductor is given in [12]. Six fitting factors (β , α_1 , α_2 , α_3 , α_4 , and α_5) are given for square, hexagonal, and octagonal spiral inductor. No expression for inductance of circular spiral inductor in the GP form has been given.

Another simple and accurate expression for inductance of a planar spiral can be obtained by approximating the sides of the spiral by symmetrical current sheets of equivalent current densities [12]. The resulting expression is

$$L_s = \frac{\mu_o \cdot n^2 \cdot d_{avg} \cdot c_1}{2} \cdot \left(\ln \left(\frac{c_2}{\rho} \right) + c_3 \cdot \rho + c_4 \cdot \rho^2 \right) \quad (3)$$

where the coefficients c_1 , c_2 , c_3 , and c_4 are layout dependent. For a circular inductor, $c_1 = 1$, $c_2 = 2.46$, $c_3 = 0$, $c_4 = 0.2$, and ρ is the fill ratio, defined as $\rho = (d_{out} - d_{in}) / (d_{out} + d_{in})$. For simplicity, we introduce the following notation: $d_{out} = x_1$,

$w = x_2$, $d_{avg} = x_3$, $n = x_4$, and $s = x_5$. Equation 3 can be expressed as:

$$L_S = \frac{\mu_o}{2} \cdot x_4^2 \cdot x_3 \cdot \left[\ln(2.46) - \ln\left(\frac{x_1}{x_3} - 1\right) + 0.2 \cdot \left(\frac{x_1}{x_3} - 1\right)^2 \right] \quad (4)$$

Equation 4 is not in GP form because contains ln function. However, if $\ln(x_1/x_3 - 1)$ is expressed in posynomial or signomial form, the problem is solved. Used the fitting techniques, we obtained equality:

$$\ln(x - 1) = -46.257 + 102.091 \cdot x - 92.270 \cdot x^2 + 41.956 \cdot x^3 - 9.475 \cdot x^4 + 0.847 \cdot x^5, \quad (5)$$

where is $x = x_1/x_3$. Due to practical realization of planar inductor, this ratio is usually from 1.1 to 2. After the insertion of 5 in 4, new expression for inductance of circular inductor in GP form is:

$$L_S = 12.128 \cdot \mu_o \cdot x_3 \cdot x_4^2 - 19.575 \cdot \mu_o \cdot x_1 \cdot x_4^2 + 12.589 \cdot \mu_o \cdot x_1^2 \cdot x_3^{-1} \cdot x_4^2 - 3.640 \cdot \mu_o \cdot x_1^3 \cdot x_3^{-2} \cdot x_4^2 + 0.396 \cdot \mu_o \cdot x_1^4 \cdot x_3^{-1} \cdot x_4^2 \quad (6)$$

with x_1, x_3, x_4 in μm and inductance L_S in nH.

The signomial expression 6 is useful because, it can be used for optimal design of circular inductors using GP.

Series resistance R_S . Inductor's series resistance R_S is calculated in the conventional way [1]. After a simple mathematical calculation, it can be given as monomial expression

$$R_S = K_1 \cdot x_2^{-1} \cdot x_3 \cdot x_4 \quad (7)$$

Coefficient K_1 depends on technology and frequency. As well as, the all another coefficients $K_i, i = 2, 3 \dots 8$ in expressions which follow.

Oxide capacitance C_{ox} . The total oxide capacitance between the spiral and silicon can be calculated by using expression from [1] and also can be given by the following monomial expression

$$C_{ox} = K_2 \cdot x_2 \cdot x_3 \cdot x_4 \quad (8)$$

Series capacitance C_S . In the similar way, the series capacitance is calculated as the capacitance of the cross-overlap between the spiral and the underpass. It can be expressed as the monomial

$$C_S = K_3 \cdot x_2^2 \cdot x_4 \quad (9)$$

Substrate capacitance C_{Si} . The substrate capacitance is given by the monomial expression as

$$C_{Si} = K_4 \cdot x_2 \cdot x_3 \cdot x_4 \quad (10)$$

Substrate resistance R_{Si} . The substrate resistance can be expressed as monomial

$$R_{Si} = K_5 \cdot x_2^{-1} \cdot x_3^{-1} \cdot x_4^{-1} \quad (11)$$

Parallel resistance R_p . This resistance is given by the following monomial

$$R_p = K_6 \cdot x_2^{-1} \cdot x_3^{-1} \cdot x_4^{-1} \quad (12)$$

Parallel capacitance C_{tot} . The total parallel capacitance is given by the posynomial expression

$$C_{tot} = K_3 \cdot x_2^2 \cdot x_4 + K_7 \cdot x_2 \cdot x_3 \cdot x_4 + K_8 \cdot x_2^2 \cdot x_3^2 \cdot x_4^2 \quad (13)$$

In all above expressions x_1, x_2, x_3, x_4 are in μm , resistances are in Ω and capacitances are in fF.

4 Constraints and Improvement for Optimal Design of Circular Inductors

A geometric program requires each of the constraint functions (or objective function) to be a monomial, posynomial or a signomial for a difference to [5] which is restricted to be either monomial or posynomial. Each constraints function can be less than or equal to 1, or equal to 1.

Constraint for L_S . New expression for inductance in GP form is given by 6 and our software tool requires the inductance to equal some specific value

$$L_S = L_{req} \quad (14)$$

Geometry constraints. Since, width of conductor (w or x_2) and spacing between adjacent conductors (s or x_5) belong to optimization variables, very simple, it can be constrained. For example,

$$x_2 \geq w_{\min} \text{ or } x_5 \geq s_{\min} \quad (15)$$

Quality factor. The degree at which the inductor deviates from an ideal device is described by the quality factor. Q-factor of an inductor, via elements of the one-port model shown in the Fig. 2 (b), can be defined as follows

$$Q_L = \frac{\omega \cdot L_S}{R_S} \cdot \frac{R_P \left(1 - \frac{R_S^2 \cdot C_{tot}}{L_S} - \omega^2 \cdot L_S \cdot C_{tot} \right)}{R_P + \left[\left(\frac{\omega \cdot L_S}{R_S} \right)^2 + 1 \right] \cdot R_S} \quad (16)$$

However, again, the problem is that equation 16 is not written in GP form. In the open literature [5], this problem is solved by introducing one new variable Q_{Lmin} or x_6 and in the objective function, this variable is maximized. Then, an additive constraint must be written as $Q_L \geq Q_{Lmin}$. This approach is correct, but introduced one variable more and one new constraint more.

It is well-known that patterned ground shield – PGS (beneath of silicon dioxide layer) improves performance of inductors [6]. Using INOPT, spiral inductors with or without PGS can be optimized successfully. For inductors with PGS, we found the following expression for the Q-factor in the GP form

$$Q_L = \frac{\omega \cdot L_S}{R_S} - \omega \cdot R_S \cdot C_{tot} - \frac{\omega^3 \cdot L_S^2 \cdot C_{tot}}{R_S} \quad (17)$$

This approach is enabled type of the proposed and implemented algorithm in which objective function and constraints functions can be written in monomial, posynomial or signomial form (for difference to program in [5]). This difference is crucial in the context to reduce the CPU time for optimization.

As illustration, after simple mathematical operation, expression for Q-factor in the GP form is as follows (for specific value for inductance 10nH and for operating frequency 2.5GHz)

$$\begin{aligned} Q_L = & 1171.67 \cdot x_2 \cdot x_3^{-1} \cdot x_4^{-1} - 2.19 \cdot 10^{-8} \cdot x_3^2 \cdot x_4^2 - 1.79 \cdot 10^{-17} \cdot x_2 \cdot x_3^3 \cdot x_4^3 \\ & - 7.62 \cdot 10^{-8} \cdot x_2 \cdot x_3 \cdot x_4^2 - 3.21 \cdot 10^{-2} \cdot x_2^2 - 2.45 \cdot 10^{-11} \cdot x_2^3 \cdot x_3 \cdot x_4 \\ & - 9.98 \cdot 10^{-14} \cdot x_2^3 \cdot x_3 \end{aligned} \quad (18)$$

where values $x_1 = d_{out}$, $x_2 = w$, $x_3 = d_{avg}$, $x_5 = s$ are in μm . In optimization process this objective function 18 is maximized in accordance with specifications for constraints. Thus, another specific value for inductance gives different coefficients of each term, but function form and the exponents of the independent variables stay the same.

Constraint for minimum self-resonance frequency. The self-resonance frequency ω_{sr} is the frequency at which the quality factor, expression 16, is 0. We want that the self-resonance frequency is greater than or equal to exactly specific value $\omega_{sr} \geq \omega_{sr,min}$. This condition can be written as follows

$$\omega_{sr,min}^2 \cdot L_S \cdot C_{tot} + \frac{R_S^2 \cdot C_{tot}}{L_S} \leq 1 \quad (19)$$

5 Results for Optimal Design of Circular Inductors

One often used figure of merit for the inductor characterization is the Q-factor, which we have already introduced in the previous section. The Q-factor of an inductor is a figure of merit for the quality of the component. With practical aspect, we want to maximize Q-factor for a specific value inductance. In order to demonstrate our optimization program we represented GP problem as:

$$\begin{aligned} & \text{maximize} \\ & \quad Q_L \end{aligned} \quad (20)$$

subject to

$$L_s = L_{req}, s \geq s_{min}, \omega_{sr} \geq \omega_{sr,min} \quad (21)$$

In the Fig. 3 is shown maximum Q-factor at 2.5GHz versus inductance value for the circular inductor with PGS, obtained using INOPT. Constraints are $s \geq 2 \mu\text{m}$ for a curve 1, while for curve 2 is added a constraint for minimum self-resonance frequency as $\omega_{sr} \geq 7 \text{ GHz}$.

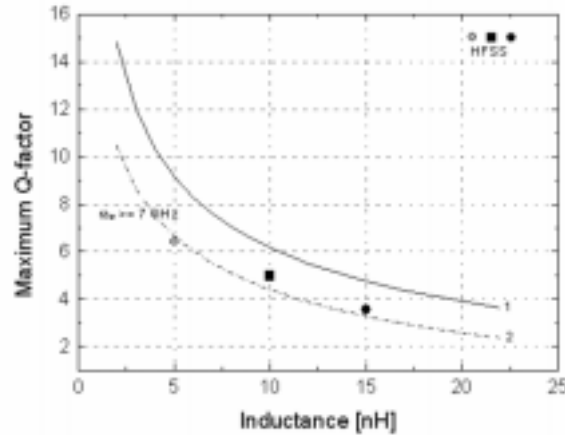


Fig. 3. Maximum Q-factor of circular inductor versus inductance.

The proposed program gives graphical view for trade-off curves in a approximately one second. The corresponding geometrical dimensions are all in a feasible technical range, as it can be seen in Table 1. It means that an optimal design of the circular inductors is possible in the considered cases.

Again, it is necessary to emphasize, that coefficients $K_i, i = 1, 2 \dots 8$ in expressions for elements of equivalent model inductors are dependent on technological

Table 1. Maximum Q-factor and optimal value of geometry parameters for some inductance values.

Inductance [nH]	Maximum Q-factor
	d_{out} [μm]; w [μm]; d_{avg} [μm]; n ; s [μm]
without constrained ω_{sr}	
5	9.14 394.1; 25.0; 301.5; 3.5; 2
10	6.15 303.9; 10.7; 236.1; 5.5; 2
15	4.76 263.8; 6.4; 204.6; 7.25; 2
with constrained ω_{sr} ($\omega_{sr} \geq 7$ GHz)	
5	6.67 241.0; 10.3; 190.5; 4.25; 2
10	4.39 193.3; 4.4; 152.2; 6.75; 2
15	3.27 175.3; 2.6; 137.4; 8.75 ; 2

parameters and frequency.

In order to verify our results, commercial FEM simulation software of HFSS [13] was conducted for some layout parameters depicted in Table 1. Fig. 4 shows dependence of the Q-factor versus frequency (in the range of interest for practical applications) for some fixed inductance values (for example 5nH, 10nH, and 15nH) simulated in HFSS. In Fig. 4, it can be seen characteristic values of Q-factor at 2.5 GHz operating frequency, which are depicted in Fig. 3, also.

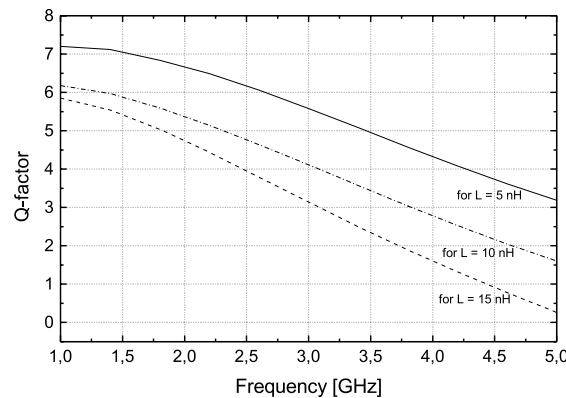


Fig. 4. Q-factor of circular planar inductor versus frequency simulated in HFSS.

The results of our program presented in Fig. 3 show very good agreement when it is compared with the simulated data in 3D commercial electromagnetic simulator – HFSS. To demonstrate simulated structure, the solid 3D model of the circular inductor (for $L=5\text{nH}$, with constrained ω_{sr}) is shown in Fig. 5.

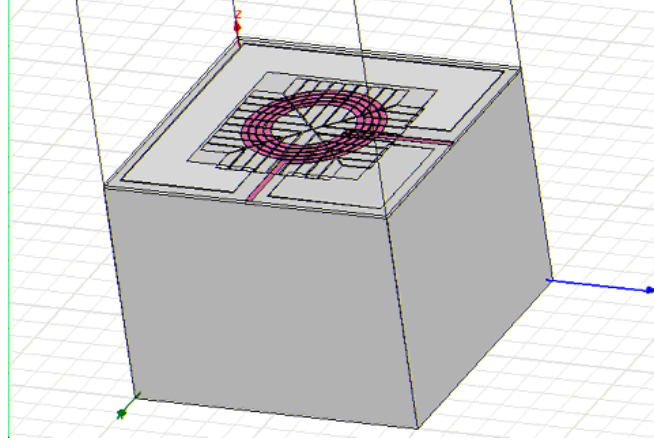


Fig. 5. Geometry of an circular inductor, with PGS and ground ring, in HFSS.

6 Optimizing Inductor Area via GP

Inductors are usually larger than any other element used in communication circuits. Due to intention towards miniaturization electronic components, essentially is, that inductor area be minimally, with exactly specific values for Q-factor and inductance.

Thus, it is necessary minimized inductor area A , in reference to outer diameter d_{out} (variable x_1). In accordance to, GP problem can be represented as:

minimize

$$x_1 \quad (22)$$

subject to (for our example):

$$Q = 4, L_s = L_{req}, s \geq 2 \mu\text{m}, \quad (23)$$

Simulation results are given in Fig. 6 and exactly number values are shown in Table 2.

For the small value of inductance ($< 10 \text{ nH}$) is necessary small number of turn, but that d_{out} have to be great due to great the width of conductor – w , attained for specific value of Q-factor. For the great value of inductance $> 10 \text{ nH}$ number of

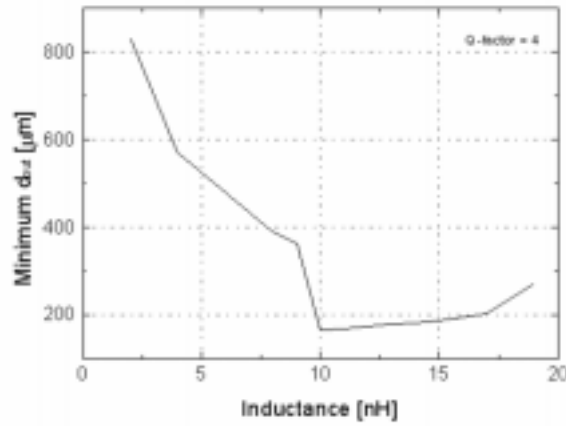


Fig. 6. Minimum the outer diameter of inductor versus inductance value.

Table 2. Minimum the outer diameter and minimum inductor area versus some inductance value.

Inductance (nH)	Minimum d_{out} (μm)	Minimum inductor area ($\mu\text{m} \times \mu\text{m}$)
2	829.65	688333.65
4	571.24	326897.01
6	478.82	229268.59
8	388.30	150781.79
10	163.75	26816.25
12	172.32	29702.58
14	181.70	33018.11
16	193.33	37376.48
18	236.47	55918.06
20	213.33	45941.04

turn is approximately equal (from 4 to 6) and the width is relatively small therefore minimum value for d_{out} are attained (inductor area, also).

7 Optimizing Self-resonance Frequency ω_{SR} via GP

In addition to maximized Q-factor, very important is maximized self-resonance frequency, in the context to practical application an inductor. In this section we represent results of our simulation for maximize self-resonance frequency via GP. Thus, a design problem can be posed as:

maximize

$$\omega_{srmin}, \quad (24)$$

subject to:

$$\omega_{sr} \geq \omega_{sr,min}, Q = 3, L_s = L_{req}, s \geq 5 \mu\text{m}, \quad (25)$$

We can maximize the self-resonance frequency by maximizing ω_{srmin} subject to constraint. The constraints given in 25 are just for our example and other constraints can be change or added very simple. Simulation results are depicted in the Fig. 7, for circular geometry of planar spiral inductors at operating frequency 1 GHz. We compared maximum self-resonance frequency predicted by program INOPT with values from previously published paper [14] (for circular inductor on silicon), and we found excellent agreement, as can be seen in the Fig. 7.

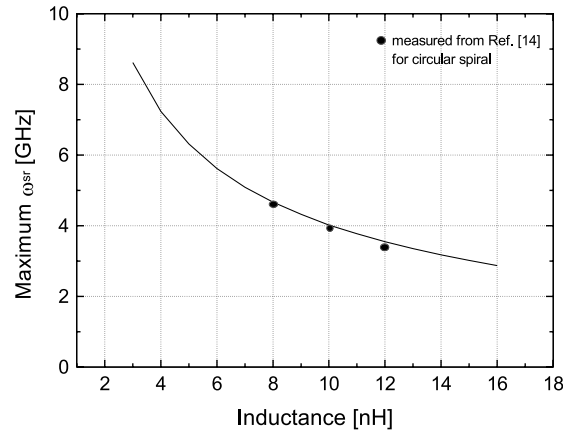


Fig. 7. Maximum the self-resonance frequency versus inductance value.

8 Conclusion

Results of an efficient optimization method via geometric programming have been demonstrated in this paper. We presented how elements of model circular inductor and objective function can be written in the signomial, monomial or posynomial form. Our developed software tool can radically reduce optimization times for optimal design circular inductor, while still keeping the desired accuracy. After a successful optimization it is possible to draw quickly the global trade-off curves (maximum Q-factor versus inductance, minimum inductor area versus inductance,

maximum ω_{sr} versus inductance, etc.). The numerous results of the proposed software tool indicate the usefulness of the maximum Q-factor of circular inductors for wireless telecommunication applications in terms of miniaturization and performance enhancement.

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