

## Signal Processing with Continuous Kernel Hough Transform

Ján Turán, Zoran Bojković, Peter Filo, Andreja Samčović,  
and L'uboš Ovsenik

**Abstract:** The paper deals with new modification of Hough transform - Continuous Kernel Hough transform. Definition of Continuous Kernel Hough transform, image processing, system identification and basics of parameter estimation are presented.

**Keywords:** Continuous kernel Hough transform, image processing, system identification, parameter estimation.

### 1 Introduction

People since the beginning have tried to find more effective and simpler way of doing every activity. During the 20-th century automatization of many processes becomes moving force of industry and economy powerful progress. The development of artificial intelligence has made possible to using computers to process images and to model reality using computer algorithms. The area of object recognition is widely branch-up in last decade. Much attention has been paid to object recognition independent of position, rotation and scale of the object. Generally the problem of modeling reality using algorithms is to determine performance of some system as a response of input signals. Opposite process is to find the internal mechanism of investigated system when the input signals and system response - the output signals are known. In this case it is necessary to find out information about the structure of investigated system, so there is a possibility to describe it by some model. The process of parameter estimation is useful in approximation of this model's parameters [1].

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J. Turán, P. Filo, and L. Ovsenik are with Department of Electronics and Multimedia Communications, University of Technology Kosice, Slovakia. Z. Bojković and A. Samčović are with Faculty of Transport and Traffic Engineering, University of Belgrade, Vojvode Stepe 305, 11000 Beograd, Serbia and Montenegro (e-mail: zoranb@sf.bg.ac.yu).

There are several well-known techniques commonly used for parameter estimation. The most common technique is parameter estimation based on classical least square method [1, 2], despite the problems encountered when it is applied to noisy or occluded data sets. The prime application for which the Hough transform (HT) [3] has been originally developed is the problem of curve detection in images, but HT can be used also to process a non-image data. The Hough transform significant features like robustness to impulsive noise and insensitivity of partial occlusion of patterns are suitable to use in non-image applications, e.g. parameter estimation [3, 4, 5].

The problem of system identification and CKHT is briefly presented and the parameter estimation tool is described in this paper. Finally the comparison between Least Square and CKHT parameter estimation method based on experimental results is presented.

## 2 Classical Hough Transform

The Hough transform is an image processing algorithm which has attracted some interest in literature. It possesses a number of desirable properties typically robustness to noise and data occlusion.

The Hough transform is a feature extraction technique used in digital image processing. The classical transform identifies lines in the image. Hough transform was first described by Duda and Hart (1972). Definition of Hough transform based on normal parameterization of a line

$$H(r, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(r - x \cos \theta - y \sin \theta) dx dy \quad (1)$$

where  $f(x, y)$  is binary image and  $\delta(\cdot)$  is Dirac delta function.

## 3 Continuous Kernel Hough Transform

The Hough transform (HT) is generally used in image processing to extract geometric primitives from digital images. The Continuous Kernel Hough transform (CKHT) [6] is a new modification of HT and improves its several features. Definition of Hough transform of a line with the continuous kernel

$$H_T(r, \theta) = \sum_{i=1}^L \sum_{j=1}^L f(x_i, y_j) \frac{T}{T + (x_i \cos \theta + y_j \sin \theta - r)^2} \quad (2)$$

where  $f(x_i, y_j)$  is an  $L \times L$  binary digital image,  $H_T(r, \theta)$  is corresponding parameter space and  $T$  is a constant determining a sensitivity of CKHT [7]. There is a

possibility to set together "wideness" of kernel selectivity and determine smoothness of approximation of the HT parameter space.

#### 4 Invariant Image Processing Using CKHT

In traditional Hough transform applications information is stored only in accumulator cells. In Invariant Recognition entire accumulator field represents unambiguous characterization of recognized image. The image recognition is commonly represented by two key components - feature extraction and pattern classification. The success of such system depends not only on the effectiveness of their execution. The feature extraction process has two major objectives: determination of certain attributes of the image classes which are invariant to as many kind of distortions as possible, and reduction of the dimensionality of feature vector by selecting the most discriminatory characteristics of these images. The efficient image recognition system must satisfy these requirements and must have fast execution time.

The proposed invariant recognition system [8] has two main sub-systems:

- Invariant Feature Extraction.
- Classification.

Invariant Feature Extraction is a three steps system, which comprise subsystem using Continuous Kernel Hough transform for extracting shift invariant features. CT transforms subsystem [9] output is invariant to rotation and shift (shift invariance is result from previous subsystem) and normalization subsystem output is invariant to translation, shift and rotation.

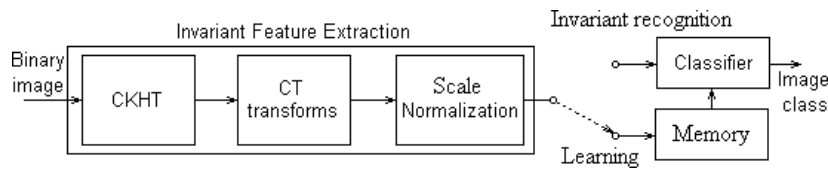


Fig. 1. Block scheme for invariant image recognition using CKHT.

Subsystem used for extraction shift invariant features is based on Continuous Kernel Hough transform.

Information concerning recognized object is distributed not evenly in the accumulator field. Transposition absolute zero point to recognized image center of gravity, most information will be stored in accumulator line  $r = 0$ . Doing this image feature becomes shift invariant. Rotation and scale of processed image express in cyclic shift or change of accumulator field values (line  $r = 0$ ).

In learning process the extracted invariant features are stored in memory and in recognition process the extracted feature is compared with every single invariant features stored in memory. Classification method is based on Euclidean classifier, which compares Euclidean distance between obtained invariant feature and invariant features stored in memory.

Sometimes there is a request for finding a difference of rotation, coordinates or scale change in image. Following this request there is a possibility to change proposed image recognition system (Fig.1) that way, the new system will be available to detect requested image parameters - sensitive image recognition system (Fig.2).

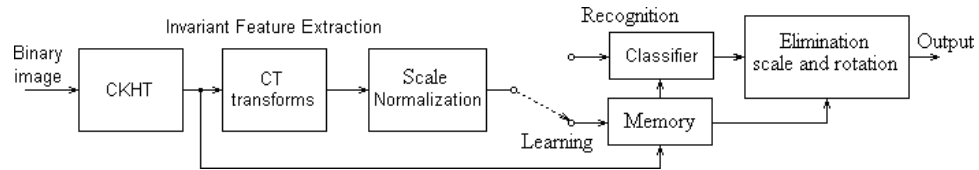


Fig. 2. Block scheme for sensitive image recognition using CKHT.

The significant Hough transform features make possible to use CKHT also in non-image data and estimation problem [3, 4, 5], i.e. when the data pixels are replaced with information concerning the system under investigation (input- output data).

System identification [10] allows build mathematical models of a dynamic system based on data obtained by measuring, where parameter estimation represents practical realization of this process. That means it is necessary to adjust parameters within a given model until its output coincides as well as possible with the measured output.

System identification problem contains:

- the data set
- the model structure
- the criterion of fit between data and models
- the routines to validate resulting model

The data set have to represent all aspects of investigated system, thus the input signal had not to be very simple. Sufficient input signal is often random signal (e.g. Gaussian white noise) because it contains all frequencies.

The model structure has to be chosen properly. There is needed to choose between linear and nonlinear model, how many parameters the model will contain, etc. The criterion of fit between data and models represents process of adjusting parameters within a given model until its output coincides as well as possible with

the measured output. The validation of resultant model is usually done by comparing the model's output with the validation data [2]. This could be done using one's eye or numerical measurement.

Every estimated model is referred to certain circumstances by what was input and output data obtained. By changing these circumstances the resultant model may not be available to suit a quality criterion.

Generally control systems are modelled by difference equation

$$y(k) = -a_1y(k-1) - \dots - a_ny(k-n) + b_1u(k-1) + \dots + b_nu(k-n) \quad (3)$$

where  $a_1 \dots a_n$  and  $b_1 \dots b_n$  are real constants with  $a_n, b_n \neq 0$ , then input and output at the sample interval  $k$  is  $u(k)$  and  $y(k)$ . Equation 3 represents also a linear model of investigated system.

For  $N$  input-output pairs  $\{[u(k), y(k)], k = 0, 1, 2, 3, \dots, N\}$  can be created equation system [11] (written in matrix formulation)

$$\mathbf{Y} = \Phi \boldsymbol{\beta} \quad (4)$$

where

$$\mathbf{Y} = \begin{bmatrix} y(n+1) \\ \vdots \\ y(N) \end{bmatrix}, \quad \boldsymbol{\beta} = [a_1 \dots a_n \quad b_1 \dots b_n]^T$$

and

$$\Phi = \begin{bmatrix} -y(n) & \dots & -y(1) & u(n) & \dots & u(1) \\ \vdots & & \vdots & \vdots & & \vdots \\ -y(N-1) & \dots & -y(N-n) & u(N-1) & \dots & u(N-n) \end{bmatrix}$$

The criterion of fit between data and models represents process of adjusting parameters within a given model until its output coincides as well as possible with the measured output.

Process of parameter estimation (Fig. 3) includes next steps [11]. After creating equation system (4) it is necessary to select  $P$  equations ( $P$  is number of estimated parameters). The solution of created system is determined using Gauss elimination method. Selecting other  $P$  equations other solution is determined. This procedure must be repeated until solutions of whole input-output pairs are obtained. The result is a set of  $[N - (P - 1)]$  approximations of vector of parameters  $\boldsymbol{\beta}$ , which represents co-ordinates of points in Hough parameter space. Each estimated linear model is described by difference equation, for which is necessary to create continuous kernel

of Hough transform (5).

$$H_T(\beta) = \sum_{i=1}^N \frac{T}{T + [y_i(k) + a_1 y_i(k-1) + \dots + a_n y_i(k_n) - b_0 u_i(k) - \dots - b_n u_i(k-n)]^2} \quad (5)$$

For each estimation of  $\beta$  using equation (5) can be obtained unambiguous value which subsistent to the given  $\beta$  value in corresponding Hough parameter space  $H_T(\beta)$ . Maximal value point represents starting point for searching Hough parameter space whereby of which is in neighbourhood of this point found real maximal value point, which indicate the estimated value of the parameters.

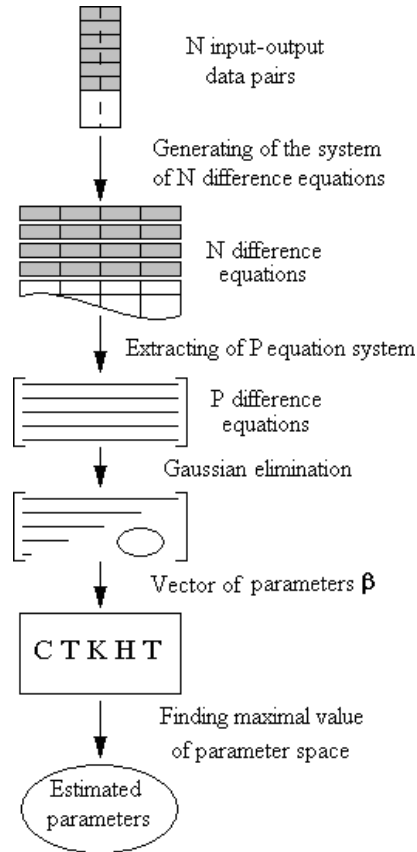


Fig. 3. Preprocessing steps for parameter estimation.

The validation of resultant model is usually done using one's eye and numerically. Quality evaluating is done using simulation with verification data and com-

puting square of the output deflection

$$\hat{\beta} = \left[ \sum_{t=1}^N \boldsymbol{\varphi}(t) \boldsymbol{\varphi}^T(t) \right]^{-1} \sum_{t=1}^N \boldsymbol{\varphi}(t) y(t) \quad (6)$$

The system identification tool has been designed for estimating parameters of linear and nonlinear models described by differential equation [11, 12, 13]. Obtaining the input and output data is done by simulation of chosen model. Here is necessary to choose number and in next step values of differential equation parameters. Also is required to choose input signal type (Heaviside function or Gaussian noise) and choose parameter values of given input signal if needed. Moreover is possible to simulate corrupting output data by noise (Gaussian white noise and impulse noise), which could represent errors arose by obtaining real data. After obtaining the input and output data succeed the estimation of parameters based on Least Square Method and Continuous Kernel Hough transform. The tool allows change some estimation settings, specifically the sensitivity of Hough transform kernel, number of iterations (number of sublimed searching of maximal value in accumulator field), the size of accumulator (distance of searched accumulator field in one direction) and number of searching points in one direction [12, 13].

The estimation results with estimation error are shown after finishing the estimation process. Also the graphical results are shown for comparing these two methods.

The process of parameter estimation of non-linear systems is similar to parameter estimation process of linear systems. Data are substituted into the model, generating a list of pairs, similar to case for parameter estimation in linear systems.

Nonlinear system can be modelled by difference equation, e.g.

$$y(k) = a_1 y^2(k-1) + a_2 u(k-1) - a_3 u^2(k-2) \quad (7)$$

where  $a_1 \dots a_n$  are real constant with  $a_n \neq 0$ , then input and output at the sample interval  $k$  is  $u(k)$  and  $y(k)$ . Creating a substitution for observer input-output values  $y(k)$ ,  $y^2(k-1)$ ,  $u(k-1)$  and  $u^2(k-2)$ , equation (7) results in linear equation with constants  $C$ ,  $D$ ,  $E$  and  $F$  and three unknown model parameters  $a_1$ ,  $a_2$  and  $a_3$

$$C = a_1 D + a_2 E - a_3 F \quad (8)$$

For  $N$  input-output pairs  $\{[u(k), y(k)]; k = 0, 1, 2, 3, \dots, N\}$  can be created equation system [11] written in matrix formulation (4). The process of parameter estimation is then equal to process used above for the linear system.

## 5 Experiments

Performance of proposed invariant recognition system was tested in experiments with two image classes: Arabian ciphers and astrological symbols (Fig. 4).

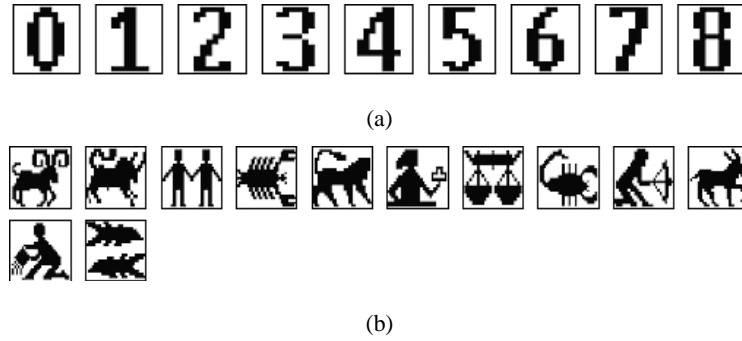


Fig. 4. Image classes: (a) Arabian ciphers, (b) Astrological symbols.

Size of reference images was  $64 \times 64$  pixels and the objects inside images were not rotated shifted and scaled. Tested images were created changing rotation, scaling and with shift of objects in images. The performance of the tested system depended on number of project lines (Fig. 6) and on used transform from the class CT. Results show that with 256 project lines (and more) and with using NT transform is image recognition 100% successful for both image classes. Number 256 for project lines sounds like optimal for explicit image recognition.

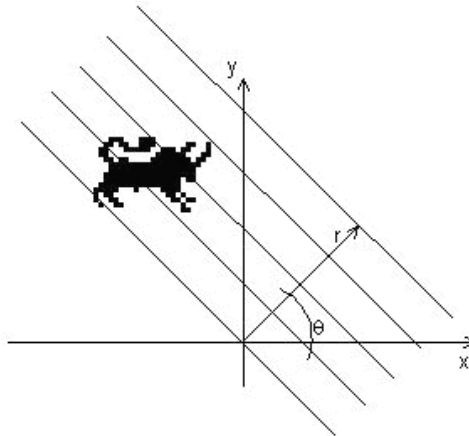


Fig. 5. Lines passing through object with constant.



Performance of proposed parameter estimation method was tested in experiments with linear models 4 and 6 parameters and in experiments on nonlinear models (4 parameters).

To demonstrate robustness of CKHT to noisy data, the output data was distorted by Gauss white noise with zero means

$$y_n(t) = y(t) + \varepsilon_\sigma(t) \quad (9)$$

and with bipolar impulse noise with variation  $\sigma$

$$\varepsilon_\sigma(t) \begin{cases} g(0, \sigma) \\ 150g(0, \sigma) \end{cases} \text{ if } |g(0, \sigma)| \geq 1.5\sigma \quad (10)$$

where  $g(0, \sigma)$  represents Gaussian white noise with zero mean and variance  $\sigma$ . The 4 parameter linear model used for tests was represented by difference equation

$$\begin{aligned} y(t) = & y(t-1) - 0.7y(t-2) \\ & + 0.81u(t-1) + 0.42u(t-2) \\ & + \varepsilon(k) - \varepsilon(k-1) + 0.2\varepsilon(k-2) \end{aligned} \quad (11)$$

In this experiment was compared parameter estimation based on CKHT with parameter estimation based on Least Square method (LMS) for value 0.2 noise variation and for 500 samples of input-output data (the result is presented in Table 1 and Fig. 6).

Table 1. Linear parameter estimation of 4 parameters.

Original values and estimation error		CKHT	LMS
a1	1.00	0.86091	-0.41191
a2	-0.70	-0.58036	-0.07951
b1	0.81	0.74707	-0.53814
b2	0.42	0.52145	4.64212
e		1.605E-01	2.612E+01

The 6 parameters linear model used for tests was represented by difference equation

$$\begin{aligned} y(t) = & 0.764y(t-1) - 0.3y(t-2) - 0.122y(t-3) \\ & + 0.471u(t-1) - 0.174u(t-2) - 0.04u(t-3) \\ & + \varepsilon(k) + 0.423\varepsilon(k-1) + 0.038\varepsilon(k-2) \end{aligned} \quad (12)$$

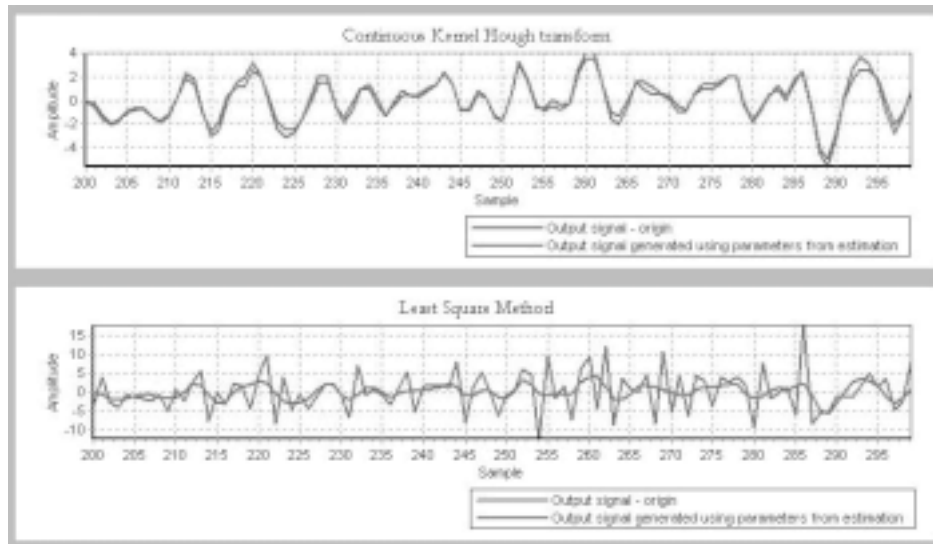


Fig. 6. Example of linear estimation process outputs using CKHT and Least Square Method (LMS) -4 parameters.

In this experiment was compared parameter estimation based on CKHT with parameter estimation based on Least Square method (LMS) for value 0,05 noise variation and for 500 samples of input-output data (the result is presented in Table 2 and Fig. 7).

Table 2. Linear parameter estimation of 6 parameters.

Original values and estimation error		CKHT	LMS
a1	0.746	0.42810	2.44804
a2	0.300	0.56994	0.51198
a3	-0.122	-0.09122	-3.05522
b1	0.471	0.49285	0,057529
b2	-0.174	-0.04022	0.79511
b3	-0.040	-0.12743	-0.55749
e		1.64E-03	5.83E+263

The non-linear model used for tests was Hammerstein model with first- order dynamics, third-degree non-linearity with additive Gaussian and impulse noise,

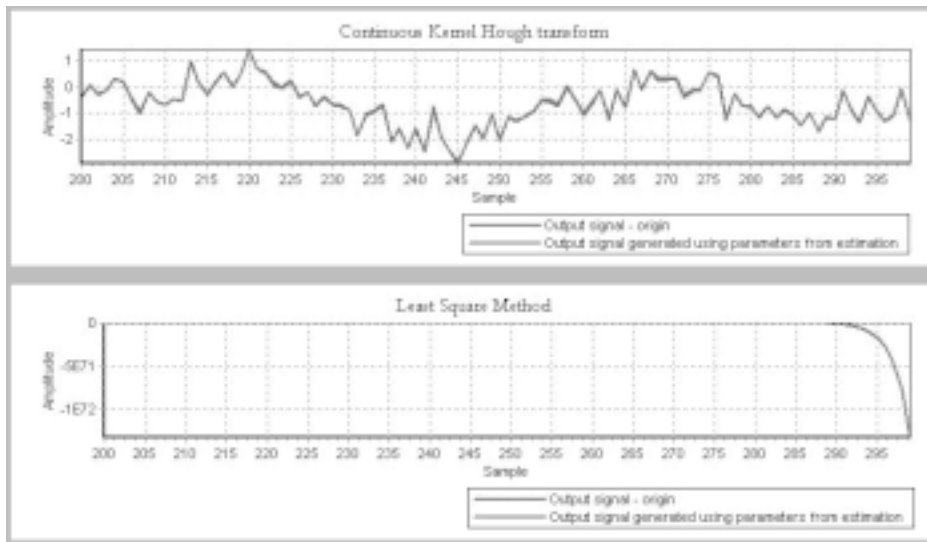


Fig. 7. Example of linear estimation process outputs using CKHT and Least Square Method (LMS) -6 parameters.

presented by non-linear difference equation

$$y(k) = 0.8y(k-1) + 0.4u(k-1) + 0.4u^2(k-1) + 0.4u^3(k-1) + \varepsilon_\sigma(k-1) \quad (13)$$

In experiments was compared parameter estimation based on CKHT with parameter estimation based on Least Square method (LMS) for several different values of noise variation and samples of input-output data. The results for 250 samples of input-output data and 0.1 noise variation, and 2000 samples of samples of input-output data and 0.3 noise variation are presented in Table 3, Table 4 and Fig. 8.

Table 3. Nonlinear parameter estimation - 250 i/o samples, 0.1 noise variation.

Original values and error of estimation		CKHT	LMS
a1	0.8	0.79948	0.84658
b1	0.4	0.37140	-0.00902
b2	0.4	0.40547	1.73297
b3	0.4	0.40427	-0.46212
e		5.641E-01	1,236E+01

Table 4. Nonlinear parameter estimation - 2000 i/o samples, 0.3 noise variation.

Original values and error of estimation		CKHT	LMS
a1	0.8	0.80218	0.91797
b1	0.4	0,37000	-0,00089
b2	0.4	0,39390	1.47328
b3	0.4	0,40524	-0.30014
e		4.016E-01	1.343E+01

Although the Continuous Kernel Hough transform provides more robust parameter estimation methods than least squares, transform methods have the disadvantage of greater complexity that means they need more time and memory space than Least Square method.

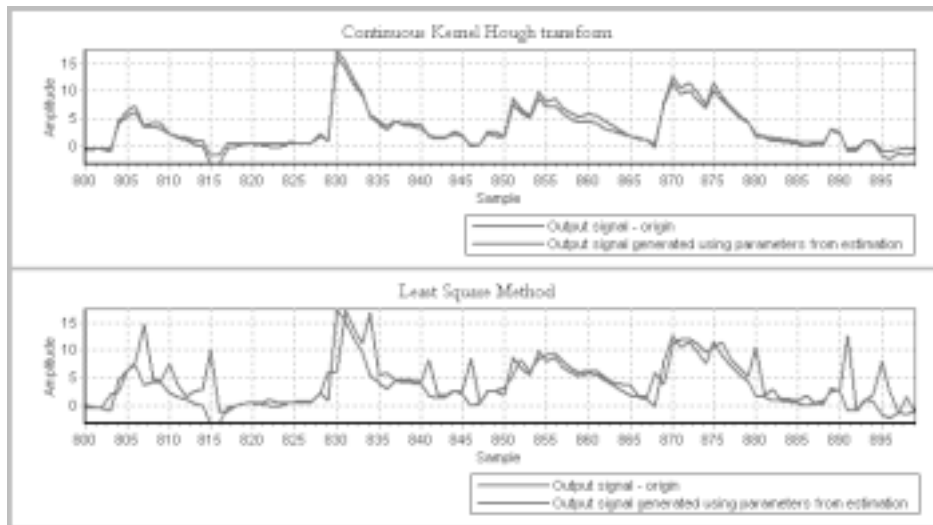


Fig. 8. Example of nonlinear estimation process outputs using CKHT and Least Square Method (LMS) - number of samples 2000, noise variation 0.3.

## 6 Conclusion

The paper presents a new approach to parameter estimation using Continuous Kernel Hough transform. This technique has been successfully applied to the problem of invariant image recognition and parameter estimation of linear and nonlinear systems. Performance of the CKHT based invariant recognition system was tested

in experiments with recognition of Arabian ciphers and astrological symbols. The system was 100 % successful with 256 project lines and using NT transform for both image classes. In parameter estimation comparison was made with the most popular method used for parameter estimation - Last Square method. The result is the method using CKHT is more efficient primarily in cases when the data set used for parameter estimation (the input and output data) is few. In those cases parameter estimation using Least Square method is not able to adjust parameters within a given model at such a small data set, especially when the data are occluded by some heavy impulsive noise, unlike to parameter estimation method using Continuous Kernel Hough transform. Preferability of parameter estimation based on CKHT is also visible primarily by using models with more parameters.

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### References

- [1] A. C. O. Mutambara, *Design and analysis of Control systems*. New York: CRC Press, 1999.
- [2] L. Ljung, *System Identification; Theory for the User*. Englewood Cliffs, N.J.: Prentice Hall, 1987.
- [3] T. Flint, M. Ingleby, and D. Morton, "A new generalisation of hough transform in trend analysis," in *Proc. IEEE Conference on Intelligent Control '92*, Glasgow, 1995, pp. 261–267.
- [4] V. S. Donchenko and V. M. Shpak, "Estimation of autoregression parameters on the basis for the hough transform," *Izv. VUZ. Radioelektronika*, vol. 37, no. 8, pp. 38–44, 1994.
- [5] M. Henderson, M. Ingleby, and D. Ford, "Robust system identification using the hough transform," in *Proc. the Institution of Mechanical Engineers, Part I (Journal of Systems and Control Engineering)*, vol. 211, no. I2, 1997, pp. 135–144.
- [6] J. Illingworth and J. Kittler, "A survey to the hough transform," *Comput. Vision, Graphics, Image Process.*, no. 1, pp. 87–116, 1988.
- [7] J. Turán and P. Farkaš, "Line fitting using hough-like procedure," *Radioengineering*, vol. 10, no. 1, pp. 25–30, 2001.
- [8] P. Farkaš, J. Turán, and K. Fazekas, "Invariant object recognition based on hybrid hough-rapid transform," in *Proc. 9th International Conference on Systems, Signals and Image Processing, IWSSIP 2002*, Nov. 7 – 8, 2002, pp. 144–149.

- [9] J. Turán, *Fast Translation Invariant Transforms and their Applications*. Kosice: Elfa, 1999.
- [10] P. Hough, "Method and means for recognizing complex patterns," US patent Patent 3 069 654, 1962.
- [11] J. Turán, Šišková, P. Farkaš, and P. Filo, "Parameter estimation using continuous kernel hough transform," in *Proc. Third COST # 276 Workshop on Information and Knowledge Management for Integrated Media Communication*, Budapest, Hungary, Oct. 11–12, 2002, pp. 121–126.
- [12] J. Turán, P. Filo, and D. Šišková, "Experiments with new systems identification tool based on continuous kernel hough transform," in *Proc IEEE region 8 EUROCON 2003*, Ljubljana, Slovenia, Sept. 22–24, 2003.
- [13] J. Turán and P. Filo, "Development parameter estimation using continuous kernel hough transform method," in *Proc. IEEE ICIT 2003*, Maribor, Slovenia, Dec. 10–12, 2003, pp. 288–292.