FACTA UNIVERSITATIS (NIŠ)
SER.: ELEC. ENERG. vol. 17, August 2004, 185-197

On the Boundaries of the MIMO-Channel-Capacity with a Focus on Line-of-Sight-Connections

This paper is dedicated to Professor Karlheinz Tröndle on the ocassion of his 65th birthday

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Abstract: In this paper we show that geometric optimization is the most efficient way to reach maximum channel capacity when the channel is characterized by a powerful Line-Of-Sight signal component but it will result in larger antenna spacings than they are found customary. To have a quantitative criterion for comparison we firstly derive boundaries for the channel capacity when considering a pure Line-Of-Sight signal component. For finding the upper and lower bound we take into account signal pre-processing strategies in conjunction with systematic power allocation schemes. Having detected these bounds we calculate the capacity in the case of a linear antenna array spaced half-wavelength at both the transmitter as well as the receiver as it is commonly chosen in mobile communications. The capacity turns out adopting values close to the lower bound even if it is optimized best by means of signal processing and power allocation. On the contrary we present an examplary geometrically optimized set of antenna elements, which is dedicated to reach the upper bound of the channel capacity.

Keywords: Multiple Input - Multiple Output, MIMO, wireless communications, capacity, Line-Of-Sight, correlated channels, keyhole channels, parallel channels, non-frequency selective channels.

1 Introduction

Multiple Input - Multiple Output (MIMO) wireless communication systems provide a highly applicative possibility to increase the channel capacity and therefore

Manuscript received March 20, 2004

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the data rate that can be achieved for a particular connection. When using an identical number N of antennas at both the transmitter as well as the receiver with a matrix transfer function of independent complex Gaussian random variables the connection is characterized by a capacity, which grows linearly along with the number of antenna elements N for a fixed signal to noise ratio σ_x^2/σ_η^2 and bandwidth [1, 2]. This kind of channel matrix corresponds to a Rayleigh fading channel where the scattered signal parts arrive from a great number of different directions at the receiver so that a negligible amount of correlation between the various signal components can be assumed. To vindicate this assumption an antenna-spacing of at least about half the wavelength must be chosen and to get small antenna arrangements the spacing is commonly minimized [3, 4, 5]. However in many practical cases (e.g. in WLAN-applications) there is a strong deterministic Line-Of-Sight-signal component (LOS). As for a small antenna-spacing all the signal components arrive roughly with identical amplitude and phase-angle at each receiver array element the rank of the channel transfer matrix degenerates approximately to rank-one and the capacity not longer grows linearly but only logarithmically with the number of antenna elements. Due to this a strong LOS-component is often treated harmful to the channel capacity [4, 5].

Hence, only few approaches were made in the past to make use of the LOS-signal component. In [6] three fixed example geometries that maximize the capacity with respect to the LOS-component were presented. Another special geometry that is suitable to maximize the capacity has been described in [7] for a fixed LOS-Link in Point-to-Point Radio Transmission. After all in [8] several simulations were run to investigate the influence of a strong LOS-component on the capacity in mobile Rician channels using comparatively small-spaced linear antenna arrays.

Our work focuses on the capacity formula with special regard to the LOS-component. At a first glance we do not consider particular antenna-geometries and -spacings, we only discuss the capacity in dependence of the channel characteristics represented by the channel transfer matrix. Several formulas are derived for a communication system consisting of N transmit and M receive antennas ($M \times N$ system) to describe the capacity's maximum and minimum in the case of a solely occurring LOS component in a non-frequency selective fading channel (e.g. microwave radio). In our discussion we tell apart the case of complete channel knowledge at the transmitter with systematic power allocation and the case of equal power distribution due to missing channel knowledge.

Hence, the rest of this paper will be structured as follows: In the 2^{nd} section we briefly present our underlying system model by deriving the distinctive channel transfer function. Section 3 is divided into three subsections. The first one addresses the channel, which is unknown to the receiver and its capacity limit values and the second subsection attends the analogous topic for the known channel. In the third part a simple example for a geometrically optimized antenna arrangement that is suitable to reach the upper capacity bound is presented. The last section of our paper summarizes all decisive equations in a table and stresses our results with some figures we nourished from simulations. Finally we briefly discuss future prospects.

2 System Model

We will consider in this paper a single user channel with multiple transmitting and receiving antennas. We assume a system consisting of N transmit and M receive antennas, which are connected by a channel matrix

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{\eta}, \tag{1}$$

where $\boldsymbol{x} \in \mathbb{C}^{N\times 1}$ denotes the transmit data vector, $\boldsymbol{y} \in \mathbb{C}^{M\times 1}$ the receive data vector, $\boldsymbol{H} \in \mathbb{C}^{M\times N}$ the channel matrix and $\boldsymbol{\eta} \in \mathbb{C}^{M\times 1}$ the additive Gaussian noise. We assume the noise $\boldsymbol{\eta}$ to be zero-mean complex Gaussian with covariance matrix $\boldsymbol{R}_{\boldsymbol{\eta}} = \mathrm{E}[\boldsymbol{\eta}\boldsymbol{\eta}^{\mathrm{H}}] = \sigma_{\boldsymbol{\eta}}^2 \mathbf{1}_M$, where $\mathbf{1}_M \in \mathbb{C}^{M\times M}$ denotes the identity matrix. The channel matrix $\boldsymbol{H}$ can be approximated as

$$\boldsymbol{H} = \begin{bmatrix} \mathrm{e}^{j\alpha_{1,1}} & \dots & \mathrm{e}^{j\alpha_{1,N}} \\ \vdots & \ddots & \vdots \\ \mathrm{e}^{j\alpha_{M,1}} & \dots & \mathrm{e}^{j\alpha_{M,N}} \end{bmatrix} \in \mathbb{C}^{M\times N}, \tag{2}$$

where the angle $\alpha_{i,j} = 2\pi l_{i,j}/\lambda, \quad i = 1,\dots,M,\ j = 1,\dots,N$ depends on the distance $l_{i,j}$ between the i^{th} receive antenna and the j^{th} transmit antenna and the wave length λ used by the system. Thus $\boldsymbol{H}$ describes a particular geometric assembly of the transmit and receive antennas. In [6] a similar type of channel matrix was suggested with the amendment that due to the propagation loss the absolute values of the channel matrix coefficients were chosen as a function of the distance between the transmitter and the receiver and as the distance between each pair of transmit and receive antenna varies due to the given geometric assembly every coefficient has a different absolute value. We consider large distances compared to the antenna array dimensions of the transmitter and the receiver and for that these differences concerning the absolute values can be neglected and after a normalization we get the channel transfer matrix that is given in equation (2). Otherwise the attenuation due to fading has to be regarded.

3 Channel Capacity

At first the channel capacity of a MIMO-system and a possibility to maximize it generally should be derived. In the sequel we will use the results in adopting it to the pure LOS-signal component.

The channel capacity of an $M \times N$ system is given by [1]:

$$C = \max_{\boldsymbol{R}_x} \log_2 \det(\mathbf{1}_M + \frac{1}{\sigma_{\boldsymbol{\eta}}^2} \boldsymbol{H}\boldsymbol{R}_x\boldsymbol{H}^{\mathrm{H}}), \tag{3}$$

where $\mathbf{1}_M \in \mathbb{C}^{M \times M}$ again denotes the identity matrix and $\boldsymbol{R}_x = E[\boldsymbol{x}\boldsymbol{x}^{\mathrm{H}}]$ specifies the covariance matrix of the transmit signal vector $\boldsymbol{x}$. Further the capacity is normalized by the bandwith and thus is expressed in bit/(sHz).

To optimize the channel capacity the vector $\boldsymbol{x}$ is calculated from the transmitter data stream vector $\boldsymbol{s} \in \mathbb{C}^{N \times 1}$ by $\boldsymbol{x} = \boldsymbol{T}\tilde{\boldsymbol{P}}^{\frac{1}{2}}\boldsymbol{s}$. Thus $\boldsymbol{s}$ is assumed to be uncorrelated Gaussian with unity variance, i.e. $\mathrm{E}[\boldsymbol{s}\boldsymbol{s}^{\mathrm{H}}] = \mathbf{1}_N$. The matrix $\boldsymbol{T}$ is a unitary matrix that describes the distribution of the data stream elements to the transmit antennas and the diagonal matrix $\tilde{\boldsymbol{P}}$ contains the total signal power at the transmitter. Furthermore we assume the total transmit power increasing linearly with the number of the transmit antennas, so $\sum_{j=1}^{N} \tilde{p}_j = N\sigma_{\boldsymbol{x}}^2$. Here $\sigma_{\boldsymbol{x}}^2$ denotes the mean transmit power that is allocated to one transmit antenna. Accordingly the matrix power distribution $\tilde{\boldsymbol{P}}$ is given by:

$$\tilde{\boldsymbol{P}} = \sigma_{\boldsymbol{x}}^2 \boldsymbol{P} \in \mathbb{R}^{N \times N}, \tag{4}$$

with

$$\boldsymbol{P} = \begin{bmatrix} p_1 & & \\ & \ddots & \\ & & p_N \end{bmatrix} \quad \text{and} \quad \textstyle\sum_{j=1}^{N} p_j = N.$$

Recapitulating we get the signal covariance matrix $\boldsymbol{R}_{\boldsymbol{x}} = \boldsymbol{T}\tilde{\boldsymbol{P}}\boldsymbol{T}^{\mathrm{H}}$ and therefore for the channel capacity follows:

$$\begin{aligned} C = & \max_{\boldsymbol{T},\boldsymbol{P}} \{\log_2 \det(\mathbf{1}_M + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2} \boldsymbol{H}\boldsymbol{T}\boldsymbol{P}\boldsymbol{T}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}})\} \\ = & \max_{\boldsymbol{T},\boldsymbol{P}} \{\log_2 \det(\mathbf{1}_N + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2} \boldsymbol{T}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}\boldsymbol{T}\boldsymbol{P})\}. \end{aligned} \tag{5}$$

Taking into account the singular value decompostion of the matrix $\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H} = \boldsymbol{V}\boldsymbol{\Lambda}\boldsymbol{V}^{\mathbf{H}}$ we get:

$$C = \max_{\boldsymbol{T},\boldsymbol{P}} \{\log_2 \det(\mathbf{1}_N + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2} \boldsymbol{T}^{\mathrm{H}}\boldsymbol{V}\boldsymbol{\Lambda}\boldsymbol{V}^{\mathrm{H}}\boldsymbol{T}\boldsymbol{P})\}. \tag{6}$$

It can be shown now the capacity is maximized by assimilating the data stream to the channel characteristics what means a kind of signal pre-processing by setting

$\boldsymbol{T} = \boldsymbol{V}$ ([9, 10]):

$$\begin{aligned} C &= \max_{\boldsymbol{P}} \{\log_2 \det(\mathbf{1}_N + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2} \boldsymbol{\Lambda P})\} \\ &= \max_{p_1 \cdots p_N} \{\sum_{j=1}^{N} \log_2(1 + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2} \lambda_j p_j)\}. \end{aligned} \tag{7}$$

Obviously the maximization is done by allocating the whole transmit power to the subchannels represented by the eigenvalues of the matrix $\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}$ in such a way that the stronger a subchannel, the higher the allocated fraction of the total power. Therefore a systematic power allocation causes perfect channel knowledge to determine the eigenvalues.

In the following parts we will derive the limit values for the capacity when taking into account the LOS-signal component. We will differentiate between two cases, the case where no channel knowledge at the transmitter can be assumed and the case where the transmitter totally knows the channel characteristics and we can apply the denoted optimization strategy.

3.1 No channel knowledge at the transmitter

Considering an unknown channel the transmitter does not have any information on the channel matrix $\boldsymbol{H}$. Therefore no channel-accommodated power allocation can be applied, every transmit antenna is addressed with equal transmit power. Further no signal pre-processing can be done. For this all the eigenvalues and eigenvectors are assumed to be equal and therefore to the transmit antenna elements are allocated by an identical fraction of the total transmit power. We get

$$\boldsymbol{P} = \mathbf{1}_N, \tag{8}$$

$$\boldsymbol{T} = \mathbf{1}_N, \tag{9}$$

and the channel capacity follows from the above:

$$C = \log_2 \det(\mathbf{1}_M + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2} \boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}). \tag{10}$$

Taking this as a basis in the sequel we will derive the limit values for the channel capacity depending on the number N of transmit and the number M of receive antennas.

A. Parallel Channels

In this section we will show how the linear as well as the logarithmic increase of the capacity depends on the number of transmit and receive antennas. The maximum number of parallel channels marks the upper bound for the linear capacity increase in MIMO systems and its identical with the rank of the channel matrix $\boldsymbol{H}$. Here we will generally appoint it for an $M \times N$ system.

If in an $M \times N$ system the relation rank($\boldsymbol{H}\boldsymbol{H}^{\mathbf{H}}$) $= M \leq N$ holds the positive-definit matrix $\boldsymbol{H}\boldsymbol{H}^{\mathbf{H}}$ is shaped as:

$$\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}} = \begin{bmatrix} N & * & \dots \\ & \ddots & \\ \dots & * & N \end{bmatrix} \in \mathbb{C}^{M \times M}. \tag{11}$$

Here every element in the main diagonal is equal to the number of transmit antennas and the residual elements are arbitrary complex entries. Accordingly it is essential that:

$$\begin{aligned} \log_2 \det(\mathbf{1}_M + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2}\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}) &\leq \log_2 \prod_{j=1}^{M}(1 + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2}N) \\ &\leq \sum_{j=1}^{M} \log_2(1 + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2}N), \end{aligned} \tag{12}$$

and due to this the maximum capacity is ascertained:

$$C_{\max} = M \log_2(1 + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2}N). \tag{13}$$

Here it is obvious that for a fixed signal to noise ratio and provided that $M \leq N$ the upper capacity bound grows linearly with the number of receive antenna elements and logarithmically with the number of transmit antenna elements. This happens if the secondary diagonal of $\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}$ is completely zero-valued, i.e.

$$\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}} = N\mathbf{1}_M,$$

because every eigenvalue of $\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}$ reaches its maximum:

$$\lambda_1 = \dots = \lambda_M = N,$$

On the contrary, if the relation rank($\boldsymbol{H}\boldsymbol{H}^{\mathbf{H}}$) $= N \leq M$ holds by using the identity $\det(\mathbf{1}_m + \boldsymbol{A}\boldsymbol{B}) = \det(\mathbf{1}_n + \boldsymbol{B}\boldsymbol{A}), \boldsymbol{A} \in \mathbb{C}^{m \times n}, \boldsymbol{B} \in \mathbb{C}^{n \times m}$ we get

$$C = \log_2 \det(\mathbf{1}_M + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2}\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}) = \log_2 \det(\mathbf{1}_N + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}), \tag{14}$$

and analogous the maximum capacity is reached if

$$\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H} = M\mathbf{1}_N.$$

Here the upper limit for the capacity is denoted as

$$C_{\max} = N \log_2(1 + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2} M), \tag{15}$$

and it can be observed that it increases linearly with the number of transmit antennas and logarithmically with the elements at the receiver.

To put it in a nutshell it can be stated that for the case of no channel knowledge the number of subchannels constitutes the linear increase of the upper capacity limit and the larger one affects the logarithmic increase

$$C_{\max} = \min\{M,N\} \log_2(1 + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2} \max\{M,N\}). \tag{16}$$

B. Keyhole Channels

Similar to the maximum bound of the channel capacity a minimum case, regularly called keyhole channel, is found. The keyhole capacity therefore marks the lower bound.

Taking the already known capacity formula as a starting point for the case of equally allocated transmit power the capacity can be converted as follows:

$$\begin{aligned} C &= \log_2 \det(\mathbf{1}_N + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2} \boldsymbol{H}^{\mathrm{H}} \boldsymbol{H}) \\ &= \log_2 \det(\mathbf{1}_N + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2} \boldsymbol{V} \boldsymbol{\Lambda} \boldsymbol{V}^{\mathrm{H}}) \\ &= \log_2 \det(\mathbf{1}_N + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2} \boldsymbol{V}^{\mathrm{H}} \boldsymbol{V} \boldsymbol{\Lambda}). \end{aligned} \tag{17}$$

Taking the worst case szenario of the considered channel as a basis every eigenvalue equals to zero except of one eigenvalue. W.l.o.g. we assume the first eigenvalue to be non-zero and what follows is the lower bound for the capacity for the unknown channel case:

$$\begin{aligned} C &= \sum_{j=1}^{N} \log_2(1 + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2} \lambda_j) \\ &\geq \log_2(1 + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2} \lambda_1). \end{aligned} \tag{18}$$

For an $M \times N$ system the minimum case occurs if the matrix $\boldsymbol{H}^{\mathrm{H}} \boldsymbol{H}$ is rank-one and therefore has the form

$$\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H} = \begin{bmatrix} M & \dots & M \\ & \ddots & \\ M & \dots & M \end{bmatrix} \in \mathbb{C}^{N\times N}.$$

Taking into account our assumption that the first eigenvalue is non-zero from this it arises to $\lambda_1 = MN$. Hence, the keyhole capacity formula depending on the number of antenna elements at the transmitter and the receiver is developed to:

$$C_{\min} = \log_2(1 + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2} MN). \tag{19}$$

Here it turns out that having regard to the chosen power allocation strategy the minimum capacity grows logarithmically with the number of antenna elements at both the transmitter as well as the receiver.

3.2 Complete channel knowledge at the transmitter

Considering the case of complete channel knowledge the transmitter can use the channel information to execute signal pre-processing methods along with an efficient power allocation scheme as described in the introductory part of section 3. Here the instantaneous values of the channel transfer matrix $\boldsymbol{H}$ are assumed to be entirely known. Therefore we will use the expression of the capacity as stated in equation (7).

A. Parallel Channels

Analogous to the structure we used in part 3.1 we will differentiate between the two cases namely the case of the number of transmit antennas exceeding the number of receive antenna elements and vice versa. Considering the first case, $N \leq M$, the rank r of the channel matrix is constituted by $r = N \leq M$. Taking into account the fact of parallel channels in the next step we find all the eigenvalues having the identical value M, i.e. $\lambda_1 = \ldots = \lambda_N = M$. In using equation (7) and substituting the eigenvalues by M all the coefficients in the power allocation matrix turn out to be identical valued and due to the chosen total transmit power the value ensues to 1:

$$\begin{aligned} C &= \max_{p_1,\ldots,p_N} \sum_{j=1}^{N} \log_2(1 + \frac{\sigma_{\boldsymbol{x}}^2}{\sigma_{\boldsymbol{\eta}}^2} M p_j) \\ &\Rightarrow p_1 = \ldots = p_N = 1, \end{aligned} \tag{20}$$

and the capacity depending on M and N is assigned as:

$$C_{\max} = N \log_2(1 + \frac{\sigma_{\mathbf{x}}^2}{\sigma_{\boldsymbol{\eta}}^2} M). \tag{21}$$

In the second case with $r = M \leq N$ the first M eigenvalues are derived to $\lambda_1 = \ldots = \lambda_M = N$ and the residual ones ensue to $\lambda_{M+1} = \ldots = \lambda_N = 0$. Abutted to the above results in this case the total transmit power is equally allocated to the first M transmit antennas and the residual ones are ignored.

$$\begin{aligned} C &= \max_{p_1 \cdots p_M} \sum_{j=1}^{M} \log_2(1 + \frac{\sigma_{\mathbf{x}}^2}{\sigma_{\boldsymbol{\eta}}^2} N p_j) \\ \sum_{j=1}^{M} p_j &= N; \quad p_{M+1} = \ldots = p_N = 0. \end{aligned} \tag{22}$$

In the consequence every addressed transmit antenna gets the total transmit power scaled by the number of receive antennas, i.e. $p_1 = \ldots = p_M = N/M$ and thus the channel that marks the upper bound is determined to:

$$C_{\max} = M \log_2(1 + \frac{\sigma_{\mathbf{x}}^2}{\sigma_{\boldsymbol{\eta}}^2} \frac{N^2}{M}). \tag{23}$$

B. Keyhole Channels

In part 3.1 for the case of a keyhole channel for the eigenvalues we already found: $\lambda_1 = MN$ und $\lambda_2 = \ldots = \lambda_N = 0$. Further we will take into account equation (7) as we are looking at the keyhole channel when the channel matrix is completely known. Thus the capacity is calculated as:

$$C = \max_{p_1} \{\log_2(1 + \frac{\sigma_{\mathbf{x}}^2}{\sigma_{\boldsymbol{\eta}}^2} \lambda_1 p_1)\}. \tag{24}$$

In the consequence only one transmit antenna is allocated by the total transmit power,

$$p_1 = N$$

and in the sequel the capacity is ascertain to:

$$C_{\min} = \log_2(1 + \frac{\sigma_{\mathbf{x}}^2}{\sigma_{\boldsymbol{\eta}}^2} M N^2). \tag{25}$$

This capacity marks the lower bound in the case of a keyhole channel where the transmitter can make use of complete channel knowledge.

3.3 Capacity calculation for an exemplary geometrically optimized antenna arrangement

To show that its actually possible to find realistic geometric antenna arrangements that are suitable to reach the upper capacity bound a simple example is presented. Therefore a 2×3 system consisting of linear antenna arrays at the transmitter as well as the receiver is considered. This geometric setup is abutted to the commonly chosen one in mobile communications and MIMO capacity measurements [4, 5]. In our example in 1 we chose a symmetric arrangement with respect to the medium transmit antenna. To have a link to WLAN-szenarios a frequency of $f = 5.5$ GHz, which corresponds to the wavelength $\lambda = 5.4$ cm is assumed. Further we consider an indoor location with a distance among transmitter and receiver of $L = 10$ m and do not take into account any reflected signal component. The setup is illustrated in Figure 1. The task is now finding an antenna spacing D that is suitable to maximize the capacity. For this particular antenna setup it is found that a spacing

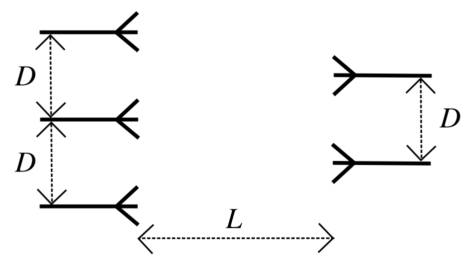

Fig. 1. Antenna arrangement for geometrical optimization

of approximately $D = 42.45$ cm is a perfect choice as for these parameters the channel transfer matrix is calculated

$$\boldsymbol{H} = \begin{bmatrix} 0.14 + j0.98 & 0.14 + j0.98 & -0.92 - j0.36 \\ -0.92 - j0.36 & 0.14 + j0.98 & 0.14 + j0.98 \end{bmatrix} \tag{26}$$

and with

$$\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}} = 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{27}$$

by using equation (3) we get a the capacity $C = 16.46$ bit/s/Hz for a fixed signal to noise ratio $10\log_{10}(\sigma_x^2/\sigma_\eta^2) = 20$ dB. In using equation (16) it can be stated that this geometric setup is suitable to reach the upper capacity bound. Admittedly normalized by the wavelength our antenna spacing is almost equivalent to $D = 7.86\,\lambda$, which broadly exceeds the common sense of $D = 0.5\,\lambda$. However this simple example indicates that it is possible to find realistic setup variants that facilitate reaching maximum capacity in the case of LOS.

4 Conclusion

In this paper we derived the universal boundaries of the channel capacity for an $M \times N$ MIMO system. When there is a perfectly known channel in using signal pre-processing combined with a systematic power allocation scheme the capacity generally can be increased. Our results are summarized in the table below.

Table 1. Capacity for MIMO-Channels

Capacity / Channel	Parallel Channnels	Keyhole Channel
unknown	$\min\{M,N\} \log_2(1+\frac{\sigma_x^2}{\sigma_\eta^2}\max\{M,N\})$	$\log_2(1+\frac{\sigma_x^2}{\sigma_\eta^2}MN)$
known	$\min\{M,N\} \log_2(1+\frac{\sigma_x^2}{\sigma_\eta^2}N\frac{\max\{M,N\}}{\min\{M,N\}})$	$\log_2(1+\frac{\sigma_x^2}{\sigma_\eta^2}MN^2)$

In addition Figure 2 illustrates the capacity in every discussed case in dependence of the signal to noise ratio. On the left hand side the case $N = 3, M = 2$ is illustrated and on the right hand side the opposite case. Again it is stated that the smaller number of antenna elements in reference to a given transmitter-receiver set constitutes the linear capacity increase whereas the major number affects the logarithmic one. Further for the case of complete channel knowledge compared to the unknown channel it is observed that assuming parallel channels the capacity can be enhanced only if the number of transmit antennas exceeds the number of receive antenna elements. Vice versa no capacity enhancement is possible and therefore the capacity curves are identical (Figure 2). Beyond it can be noticed that there seems to be a difference considering the keyhole situation if a known channel is looked at or if a unknown one is viewed. In the latter case in our model the capacity increases logarithmically with both the number of receive antennas as well as the number of transmit antenna elements. If a known channel is considered the number of receive antennas contributes quadratic whereas the number of receive antennas are linearly incorporated. This fact causes in our choice of the total transmit power as we assumed the total transmit power increasing linearly with the number of transmit antennas. If this assumption is substituted with the constant transmit power case of course in every equation the signal to noise ratio must be normalized by the number of transmit antennas and due to this normalization the mentioned difference disappears.

After deriving the boundaries for the capacity we compared a set of small-spaced antennas at the transmitter as well as the receiver to a set of perfectly arranged antennas when regarding the maximum MIMO capacity. We proved that even when using signal processing techniques and power allocation schemes for the small antenna spacing the capacity adopts values close to the keyhole case

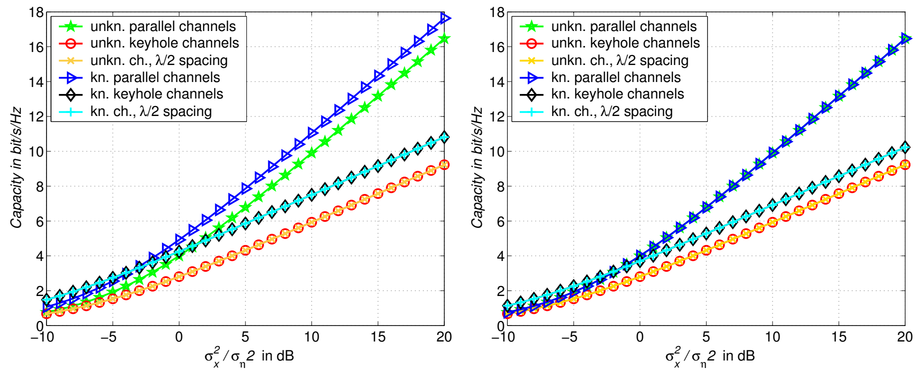

Fig. 2. left: N=3, M=2   right: N=2 , M=3

as illustrated in figure 2 where the capacity curve for an 0.5 λ spaced setup is identical with the keyhole capacity. In the contrary the connection that uses perfectly arranged sets of antenna elements approximately reaches the upper capacity bound. Therefore it becomes obvious that a geometric optimization marks the only efficient way to make use of the LOS signal component. Due to this fact finding optimal spaced geometric antenna arrangements is an important task when aiming on reaching maximum MIMO capacity and so further research on this topic is in progress.

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