FACTA UNIVERSITATIS (NIŠ)
SER.: ELEC. ENERG. vol. 17, April 2004, 41-52

On the Impact of Antenna Diversity in IEEE 802.11b DCF with Capture

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Abstract: In this paper, we examined the influence of capture effect with L-fold antenna diversity at the Access Point over IEEE 802.11b DCF. We obtained an exact closed-form solution for the conditional capture probability in case of ideal selection diversity, and an approximate closed-form solution for the conditional capture probability in case of maximum selection diversity in a Rayleigh-faded channel. Obtained analytical expressions have general significance and can be applied for any other multiple access wireless network. We also analytically evaluated saturation throughput increase of the IEEE 802.11b DCF protocol exposed to capture.

Keywords: Antenna diversity, capture effect, IEEE 802.11b DCF.

1 Introduction

In classical analysis of random access protocols, it is assumed that all frames involved in collision are destroyed. Although such collisions are imminent in wired systems, where all frames are received with nearly equal powers, collisions are uncertain in wireless radio environments. In a typical wireless network employing multiple access protocol towards some Access Point (AP), the received signals are subject to deterministic path attenuation, shadowing and fast multipath fading. In this case, the AP can still successfully decode a single frame from the strongest received signal, thus increasing the network throughput. This occurrence is known as the capture effect.

The capture effect has been studied considerably, primarily over its influence on simple random access schemes, such as the Slotted Aloha [1, 2]. Capture effect

Manuscript received January 23, 2004. An earlier version of this paper was presented at the 6th Int. Conf. on Telecommunications in Modern Satellite, Cable and Broadcasting Services, TELSIKS 2003, October 1-3, 2003, 18 000 Niš, Serbia.

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is also feasible in IEEE 802.11 WLANs [3] that consists of low-power transmitting stations in an indoor environment, as studied in [4, 5]. The impact of the capture effect over the capacity improvement of IEEE 802.11b Wireless LANs in multipath-faded channels has been predicted accurately enough by the analytical approach in [6].

In this paper, we extend the analysis of capture effect by introducing L-fold antenna diversity at the Access Point in an IEEE 802.11b WLAN. We derived and compared expressions for the capture probabilities with and without antenna diversity. We also obtained the throughput increase of the IEEE 802.11b WLAN due to the capture effect with antenna diversity. Additionally, variation of signal strengths with distance between transmitter and central receiver yields to unequal probabilities of frame capture for stations at different distances, giving rise to the spatial unfairness into the network.

2 Classical Capture Model

During simultaneous transmission from multiple stations, a receiver captures a frame if the power of the detected frame P_0 sufficiently exceeds the joint power of the n interfering contenders (with signal powers P_i) by the capture ratio factor z_0 during a certain period (over which instantaneous power is assumed to remain approximately constant). The probability of this occurrence given n interfering frames is the *conditional capture probability*

$$p_c(z_0, n) = \Pr\left[\gamma = \frac{P_0}{\sum_{i=1}^{n} P_i} > z_0\right]. \tag{1}$$

In the case of IEEE 802.11b WLAN, typical values of z_0 appearing in classic modulation schemes ($z_0 > 10$ dB) are reduced due to the processing gain in correlation receiver during demodulation of frame's preamble and header. Namely, they are always transmitted by DSSS/DBPSK modulation by using fixed 11-chip Barker spread sequence, while rest of the frame can be transmitted at 1, 2, 5.5 or 11 Mbit/s (using DPSK or CCK modulation). We assume that a receiver determines weather a possible successful capture has occurred during transmission of the preamble/header part of the frame.

2.1 Propagation model and antenna diversity

Our propagation model takes the deterministic power loss and multipath fast fading of signals into account. The path-loss exponent for indoor channels in picocells is typically taken equal to 4. For the purpose of analytical tractability, we used somewhat arbitrary assumption for absence of direct path between the transmitter i and the receiver (at mutual distance r_i) within the Basic Service Set (BSS), which means the envelope of transmitted signal is Rayleigh-faded. Therefore, signal's instantaneous power is exponentially distributed according to the following pdf

$$f_{p_i}(x) = \frac{1}{P_{0i}} e^{-\frac{x}{P_{0i}}}, \quad x > 0 \tag{2}$$

where P_{0i} represents the local-mean power of the transmitted frame at the receiver. The local-mean power itself is determined by $P_{0i} = Ar_i^{-4} P_T$, where Ar_i^{-4} is the deterministic path-loss law, and P_T is the transmitted signal power. Constants A and P_T are assumed to be identical for all transmitted frames.

Using antenna diversity at the AP can effectively combat the presence of the multiple fast fading. APs typically use two, or more antennas separated by at least half wavelength so that multiple received signals can be affected by independent fading. Several strategies of combining the signal from the multiple antennas are possible, also including some potentially complicated signal processing. We assume two separate types of single antenna selection strategies are employed: *maximum selection* and *ideal selection* strategies among L antennas. Maximum selection insures the signal from antenna with the largest received power is selected, while ideal selection - the antenna with the best performance. It is well studied how antenna diversity assures improvement of frame error probability, while in this paper we analyze its impact over the frame capture probability.

2.2 Capture model without diversity (CM 1)

When the stations are not power-controlled, both the local-mean powers and Rayleigh fading envelopes of received frames differ and represent independent random variables. Assuming local-mean powers of the single captured and the n interfering frames are $P_{0u}, P_{01}, \ldots, P_{0n}$, respectively, the conditional capture probability at the AP with a single receive antenna can be expressed as in [2]

$$\begin{aligned} p_c^I(z_0, n) &= \int_0^\infty \mathrm{d}p_1 f_{P_1}(p_1) \cdots \int_0^\infty \mathrm{d}p_n f_{P_n}(p_n) \int_{z_0(p_1+\cdots+p_n)}^\infty \mathrm{d}p_u f_{P_u}(p_u) \\ &= \int_0^\infty \mathrm{d}p_1 f_{P_1}(p_1) \cdots \int_0^\infty \mathrm{d}p_n f_{P_n}(p_n) e^{-z_0 \sum_{i=1}^n \frac{p_i}{P_{0u}}} \\ &= \prod_{i=1}^n \frac{1}{1 + z_0 \frac{P_{0i}}{P_{0u}}} = \prod_{i=1}^n \frac{1}{1 + z_0 \left(\frac{r_i}{r_0}\right)^{-4}} \end{aligned} \tag{3}$$

where f_{P_u}, $f_{P_1}, \dots, f_{P_n}$ are the power pdfs of the useful signal and each of the n interferers, respectively, under assumption of their mutual statistical independence. Variables ri are the random distances from the transmitting stations to the AP receiver, which are distributed according to identical pdf $h(r)$.

Since all factors in the product of (3) are statistically equal, we can average over their distance distributions to obtain the *averaged conditional capture probability* given n interferers, as the following

$$\overline{p_c^l} = \int [I(r_0, z_0]^n h(r_0) \mathrm{d}r_0, \tag{4}$$

where

$$I(r_0, z_0) = \int \frac{h(r_i)\mathrm{d}r_i}{1 + z_0 \left(\frac{r_i}{r_0}\right)^{-4}}. \tag{5}$$

We assumed a single circular BSS with radius normalized to unity with the AP located in its center and uniformly spatially distributed stations, so that $h(r_i) = 2r_i$, $0 < r_i \le 1$. Thus, (5) can be solved as follows

$$I(r_0, z_0) = 1 - r_0^2 \sqrt{z_0} \arctan \frac{1}{r_0^2 \sqrt{z_0}} \tag{6}$$

By substitution of (6) into (4), averaged conditional capture probability $\overline{p_c^l}(z_0, n)$ can only be solved by numerical integration.

3 Capture Models with Diversity

In case of L antennas at the AP, each antenna receives an independently faded signal from the transmitted frame from each of the $(n+1)$ transmitting stations. We denote the instantaneous power of the captured frame at antenna j by P_{0j}, and the instantaneous power of an interfering frame from station i at antenna j by P_{ij}. In presence of diversity, the conditional capture probability can be expressed as

$$p_c^L(z_0, n) = \Pr\left[\gamma_k = \frac{P_{0k}}{\sum_{i=1}^n P_{ik}} > z_0\right]. \tag{7}$$

In (7), we assume antenna k is selected for decoding the received signal(s) according to the corresponding combining strategy. In case of maximum selection strategy, antenna k is selected if

$$\sum_{i=0}^n P_{ik} \ge \sum_{i=0}^n P_{ij}, \quad 1 \le j \le L, \tag{8}$$

while in case of ideal selection, the condition is

$$\frac{P_{0k}}{\sum_{i=1}^{n} P_{ik}} \geq \frac{P_{0j}}{\sum_{i=1}^{n} P_{ij}}, \quad 1 \leq j \leq L. \tag{9}$$

3.1 Capture model with maximum selection diversity (CM 2)

We now derive an approximate expression for conditional capture probability at AP's antennas, where the signal from an antenna with largest received instantaneous power is selected. Capture ratio is assumed to have relatively high value, e.g. $z_0 \geq 10$. We actually determine capture probability at selected antenna assuming signal-to-interference ratio γ_k exceeds z_0.

It is reasonable to assume that for each of the two sums in (8), the power of captured signal at any antenna significantly exceeds joint interference power, that is

$$P_{0k} \gg \sum_{i=1}^{n} P_{ik} \quad \text{and} \quad P_{0j} \gg \sum_{i=1}^{n} P_{ij}, \quad 1 \leq j \leq L. \tag{10}$$

Now, (8) can be relieved to become $P_{0k} \geq P_{0j}$, $1 \leq j \leq L$, i.e. $P_{0k} = \max\{P_{0j}\}$, $1 \leq j \leq L$. In this case, p.d.f. of signal at the selected P_{0k} is well known

$$f_{P_u}(x) \equiv f_{P_{0k}}(x) = L[1 - e^{-\frac{x}{P_{0u}}}]^{L-1} \frac{1}{P_{0u}} e^{-\frac{x}{P_{0u}}}, \quad x \geq 0. \tag{11}$$

After developing (11) in binomial series, we introduce it in (3), so (7) can be expressed as follows

$$\begin{aligned} p_c^L(z_0, n) &= \int_0^\infty \mathrm{d}p_1 f_{P_1}(p_1) \cdots \int_0^\infty \mathrm{d}p_n f_{P_n}(p_n) \times L \sum_{j=0}^{L-1} \binom{L-1}{j} \frac{(-1)^j}{1+j} e^{-(j+1) z_0 \sum\limits_{i=1}^{n} \frac{p_i}{P_{0u}}} \\ &= \sum_{j=0}^{L-1} \binom{L}{j+1} (-1)^j \prod_{i=1}^{n} \frac{1}{1 + (j+1) z_0 \frac{P_{0i}}{P_{0u}}} \\ &= \sum_{j=0}^{L-1} \binom{L}{j+1} (-1)^j \prod_{i=1}^{n} \frac{1}{1 + (j+1) z_0 (\frac{r_i}{r_0})^{-4}} \end{aligned} \tag{12}$$

Then, we average (12) over distance r.v.s r_0 and r_i to obtain the averaged conditional capture probability

$$\overline{p_c^L}(z_0, n) = \sum_{j=0}^{L-1} \binom{L}{j+1} (-1)^j \int [I(r_0, (j+1) z_0)]^n h(r_0) \mathrm{d}r_0, \tag{13}$$

where integral $I(\cdot)$ can be calculated according to (6), when z_0 is substituted with $(j+1)z_0$. Given $z_0 > 10$, there is a match between actual capture probabilities evaluated by Monte Carlo simulations and the approximate solution (13) calculated by numerical integration.

3.2 Capture model with ideal selection diversity (CM 3)

In case of ideal selection L-fold diversity, we were able to establish accurate expressions for the conditional capture probabilities. Namely, ideal selection means that antenna with highest signal-to-interference ratio γ_k is selected, possibly by involving some complicated signal processing. Therefore, the capture probability of this antenna needs to be estimated. If there is at least one antenna for which $\gamma_k > z_0$, it will certainly be selected. Thus, conditional capture probability given L-fold diversity can be expressed through capture probability of a no-diversity receiver, defined as in (3), as follows:

$$\begin{aligned} p_c^L(z_0,n) &= 1-[1-p_c^l(z_0,n)]^L \\ &= 1-\Big[1-\prod_{i=1}^{n}\frac{1}{1+z_0(\frac{r_i}{r_0})^{-4}}\Big]^L \\ &= 1-\sum_{j=0}^{L}\binom{L}{j}(-1)^j\prod_{i=1}^{n}\Big[\frac{1}{1+z_0(\frac{r_i}{r_0})^{-4}}\Big]^j \end{aligned} \tag{14}$$

Then, averaged conditional capture probability is calculated as

$$\overline{p_c^L}(z_0,n) = 1-\sum_{j=0}^{L}\binom{L}{j}(-1)^j\int[I_j(r_0,z_0)]^n h(r_0)\mathrm{d}r_0, \tag{15}$$

where the integral $I_j(\cdot)$ is expressed as

$$\begin{aligned} I_j(r_0,z_0) &= \int_0^1 \frac{h(r_i)\mathrm{d}r_i}{\Big[1+z_0(\frac{r_i}{r_0})^{-4}\Big]^j} \\ &= \frac{1}{1+2j}\Big(\frac{1}{r_0^4 z_0}\Big)^j {}_2F_1(j,\frac{1}{2}+j;\frac{3}{2}+j;-\frac{1}{r_0^4 z_0}), \end{aligned} \tag{16}$$

where ${}_2F_1(\cdot)$ is the hypergeometric function. Given $j=1$, $I_1(r,z_0)$ attains the form of (6). The averaged conditional capture probabilities $\overline{p_c}(z_0,n)$ for three capture models in function of the capture ratio z_0 given $N(=n+1)=10$ contending stations is depicted in Fig. 1. It is obvious that for CM 2, the approximate theoretical solution closely approaches accurate values (determined by Monte Carlo simulations) as z_0 increases above 10 dB.

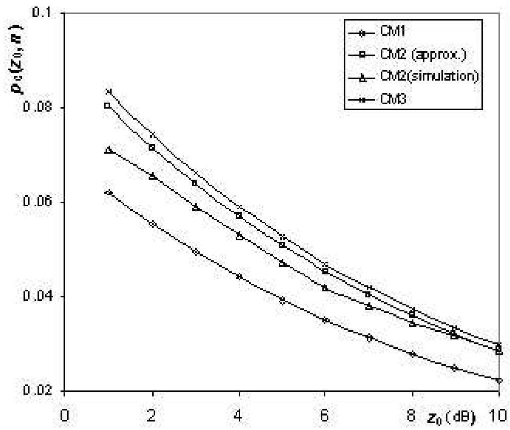


Fig. 1. The comparison of the three capture models depending on the capture ratio ($N = 10$).

4 Saturation Throughput of IEEE 802.11b

We used the results from [7] and [8] in estimating the saturation throughput of IEEE 802.11b Distributed Coordination Function (DCF) under capture with L-fold antenna diversity. Thus, the saturation throughput S_{max} of IEEE 802.11b WLAN (operating in Basic or RTS/CTS access mode) in ideal channel conditions (e.g. absence of capture) is expressed as [7]

$$S_{max} = \frac{P_{tr} P_{suc} L}{(1 - P_{tr})\sigma + P_{tr} P_{suc} T_s + P_{tr}(1 - P_{suc}) T_c} \tag{17}$$

In (17), L is frame payload size, P_{tr} is the probability of at least one transmission in observed time slot, P_{suc} is the probability of a successful transmission assuming at least one station is transmitting, and σ is duration of an empty slot time. T_s is the average time the channel is sensed busy by each station because of a successful transmission, and T_c is the average time the channel is sensed busy during a collision. Values of T_s and T_c differ depending on the network access mode (Basic or RTS/CTS access, as expressed in [8]) and the additional network operating parameters defined in IEEE 802.11b specification [3].

Assuming total of N stations contend for the channel during one same initial slot, the probability of at least one transmission in that slot P_{tr} can be expressed through the probability τ of a station transmitting in a randomly chosen (transmitting, colliding or empty) slot

$$P_{tr} = 1 - (1 - \tau)^N, \tag{18}$$

while the probability of a successful transmission P_{suc} is

$$P_{suc} = \frac{N\tau(1-\tau)^{N-1} + P_c}{P_{tr}}. \tag{19}$$

Eq. (19) indicates that, given at least one station is transmitting, probability of successful transmission P_{suc} is formed by adding the capture probability P_c to the probability of transmission of exactly one station $N\tau(1-\tau)^{N-1}$.

In a saturated IEEE 802.11 WLAN, probability τ depends on number of contending stations N, initial contention window W, and the number of retransmissions m after which the maximal value of the initial backoff timer is frozen. Refer to [8] for analytical calculation of τ. Given $W = 8$ and $m = 5$, τ vs. N is shown in Fig. 2.

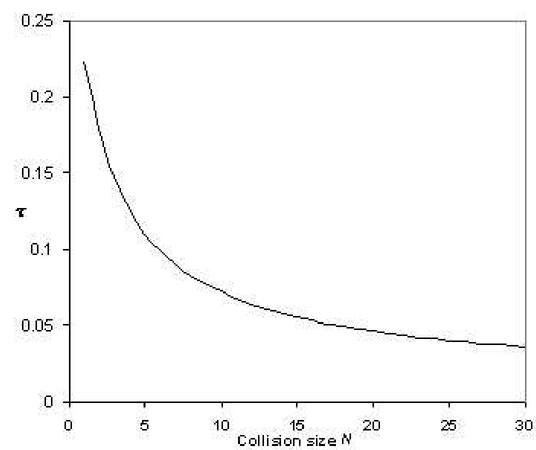


Fig. 2. Transmission probability of each station in random time slot.

Probability τ also impacts the overall probability of capture P_c as the following

$$P_c = \sum_{i=1}^{N-1} R_i \overline{p_c^L}(z_0, i), \tag{20}$$

where R_i is the probability of i interfering frames being generated in the observed time slot,

$$R_i = \binom{N}{i+1} \tau^{i+1}(1-\tau)^{N-i-1}. \tag{21}$$

The frame success probability P_{suc}, estimated according to (19), vs. N is displayed in Fig. 3 for CM 1, CM 2 and CM 3 (and by using probabilities ? from Fig. 2). The ideal selection capture model CM 3 produces highest improvement of success probability, while the capture model without antenna diversity - the lowest.

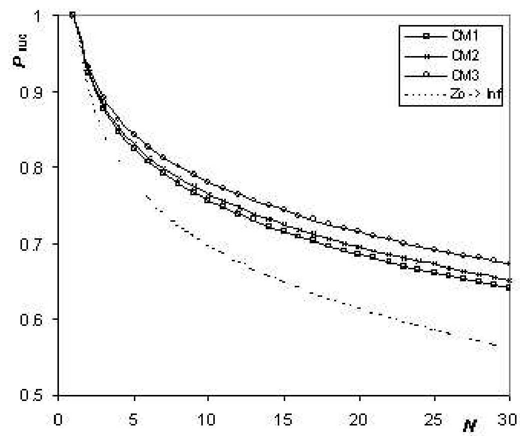


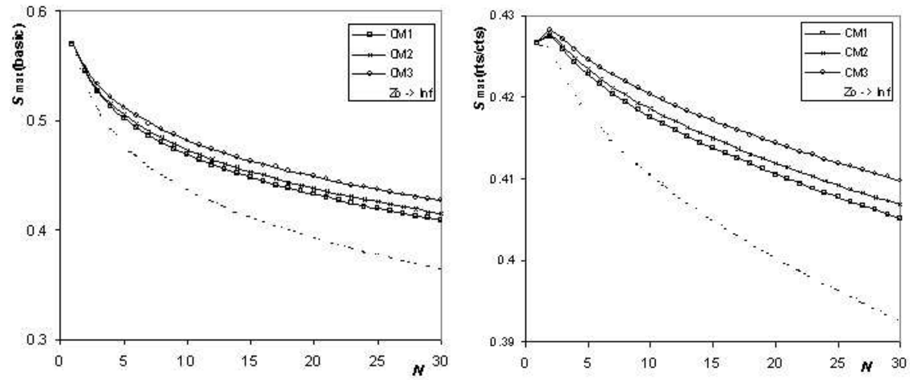
Fig. 3. Improvement of success probability P_{suc} for the three capture models ($z_0 = 10$ dB).

By substituting (18) and (19) into (17), one is able to calculate the theoretical saturation throughput under capture in Basic and RTS/CTS access modes. Given $z_0 = 10$ dB, the plot of S_{max} vs. N for the three capture models is depicted in Fig. 4.

If Basic access scheme is utilized (Fig. 4a), capture effect generates obvious throughput improvement (as compared to the absence of capture, $z_0 \to \infty$), which is further increased by the use of antenna diversity. Conversely, the use of the RTS/CTS access method (Fig. 4b) contributes significantly to the throughput robustness of the IEEE 802.11b WLAN under capture. Variation of the saturation throughput in RTS/CTS mode for both capture models over is bounded within an interval of 4% for $N < 30$. Actual value of capture ratio z_0 to apply in order to determine the actual throughput increase depends on the receiver design. Note that throughput graphs refer to a single IEEE 802.11b BSS where all stations operate at 11 Mbit/s and generate frames with fixed-size payload of $L = 1000$ octets.

5 Spatial Fairness

Spatial fairness is a measure of the ability of a station within a single BSS cell to successfully capture the radio channel in function of its location. Our measure of spatial fairness is the probability of successful transmission P_{suc} in function of the distance r_0 of successfully transmitting station to AP.



(a) Basic access. (b) RTS/CTS acccess.

Fig. 4. Theoretical saturation throughput of DCF under capture ($z_0 = 10$ dB)

For our capture models, P_{suc} vs. r_0 can be expressed by modifying (19) and (20). For the three capture models, the conditional capture probability of a test station at distance r_0 from the AP can be expressed as

$$\begin{aligned} \overline{p_c^l}(z_0, n|r_0) &= [I(r_0, z_0]^n, \\ \overline{p_c^L}(z_0, n|r_0) &= \sum_{i=0}^{L-1} \binom{L}{i+1} (-1)^i [I(r_0, (i+1)z_0]^n, \\ \overline{p_c^L}(z_0, n|r_0) &= 1 - \sum_{i=0}^{L-1} \binom{L}{i} (-1)^i [I_i(r_0, (i+1)z_0]^n, \end{aligned} \tag{22}$$

for CM 1, CM 2 and CM 3, respectively.

Now, we modify (20) and obtain the spatial distribution of the capture probability as follows

$$P_c(r_0) = \sum_{i=1}^{N-1} R_i \overline{p_c}(z_0, i|r_0). \tag{23}$$

Finally, the distance distribution of the success probability can be solved in closed-form by using (19)

$$P_{suc}(r_0) = \frac{N\tau(1-\tau)^{N-1} + P_c(r_0)}{P_{tr}} \tag{24}$$

Fig. 5 depicts spatial distribution of the frame success probability with respect to the AP located in the BSS center for $z_0 = 2$ dB and 10 dB, and $N = 10$.

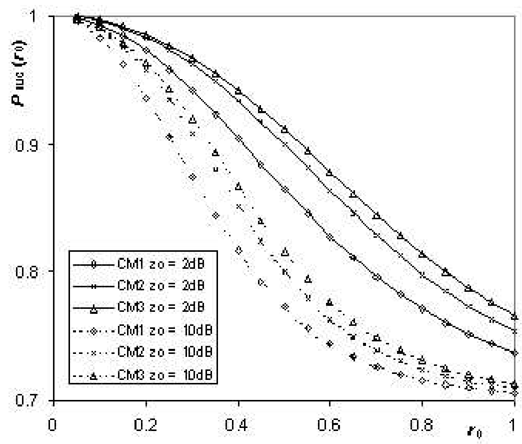


Fig. 5. Spatial fairness for channel acquisition for the three capture models ($N = 10$).

Note that P_{suc} and its spatial distribution are insensitive to the access mode. As expected, the success probability is higher for lower distances between the transmitting station and the AP, i.e. the stations closer to the BSS center receive better performance compared to those at BSS edges. We emphasize the graphs refer to a traffic-saturated network. However, as the traffic load decreases, variation in the frame success probability with location becomes less significant. As we increase the number of contending stations N, closer stations are also favored to some extent.

6 Conclusion

In this paper, we provide an analysis of the capture effect when antenna diversity with L antennas is employed at the base station. We were able to obtain an exact closed-form solution for the conditional capture probability in case of ideal selection diversity, and an approximate closed-form solution for the conditional capture probability in case of maximum selection diversity in a Rayleigh-faded channel. Then we applied obtained analytical results for the maximum selection and the ideal selection strategy in order to theoretically predict the saturation throughput of a single IEEE 802.11b BSS at 11 Mbit/s exposed to the capture effect. Compared to the capture model without diversity, the two capture models with diversity of 2 antennas introduce additional increase of the IEEE 802.11b throughput.

We particularly emphasize that the obtained analytical expressions for capture probabilities with diversity have general significance and can be applied over any other multiple access wireless network.

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