

Two Theorems on Controllability Preserving Decomposition of Complex Symmetry Nonlinear Systems

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Abstract: In this paper the problems on isomorphic decomposition and controllability of a class of nonlinear systems possessing symmetries on basis of quotient systems is studied. The isomorphic decomposition formations of these systems are drawn. Finally, it is shown that controllability of the original systems can be determined by that of the subsystems, which are obtained through isomorphic decomposition. Corresponding sufficient and necessary conditions in terms of two new theorems have been derived.

Keywords: Circuits and systems theory, controllability, isomorphic decomposition, nonlinear systems.

1 Introduction

The differential geometric approach [1, 2, 3] to the study of general nonlinear and other complex systems has enabled the discovery of entirely new insight into the theory of systems and control [4, 5]. Since the seminal paper by Isidori and his co-authors [6] appeared, the early works [7, 8, 9, 10] have paved the way in this field. Following these discoveries, during the recent years, a remarkable progress has been made [11, 12] in the study of systems possessing symmetries in the structure [13] as well as of interconnected and complex systems [4, 5]. In general, general

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non-linear systems are extremely difficult to study compared to the linear ones because of the complexity of the system structure [13, 14, 15], and not surprisingly the design is even much more difficult [5, 16, 17]. Hence a number of system-theoretic analyses have been carried out and this line of research is continuing. By and large, researchers have focused their attention on systems having somewhat special structure, and certain systems possessing symmetries and similarities just represent such classes of systems. In fact, the symmetric structure is rather fundamental in physics and engineering [2, 13], and it can bring about a great convenience for theoretical studies as well as enhance their application. The approach via differential-geometric techniques has been successfully applied by a many scholars [6]-[12],[10, 17], albeit beginning with the linear case first [12].

The concept of symmetry for general non-linear controlled systems was first presented by Grizzle and Marcus [7] in 1985. They have dealt with some problems on symmetric systems also including their local and global decompositions. Since then studies in this area have been more expanded. The specific features of controllability of systems possessing symmetries has hardly been studied in the past couple of decades. Zhao and Zhang [15] first used the concept of general symmetry and discussed the problem of controllability. However, they did not include into their study the controllability issue by system decomposition. The information about subsystems was not therefore used to its full.

On the grounds of the results in [11, 15] and the general theoretical study by Zhang [12], in this paper a concept referred to as solvable general symmetric systems is presented first and then exploited in the sequel for controllability preserving, isomorphic decomposition. The relationship between isomorphic decomposition and controllability has been investigated. It has been explored from the observation angle of quotient systems and, in particular, studied from the viewpoint of feedback quotient systems. Corresponding sufficient and necessary conditions between original and decomposed systems are derived via these two points of view.

2 Problem Statement and Relevant Mathematical Definitions

Consider the following complex system represented by the general model of non-linear, controlled dynamic systems

$$\dot{x} = f(x, u) \quad (1)$$

where $x \in M$, $u \in U$ is a smooth manifold with n -dimensions, U is a manifold of admissible controls of appropriate dimension, and system function is differentiable with respect to its variables. The system mapping and functions are assumed to be smooth and related by commutative diagram in Figure 1; the latter property is

prerequisite for system (1) to have a symmetry. In this paper, smooth will always mean the class of functions C^∞ .

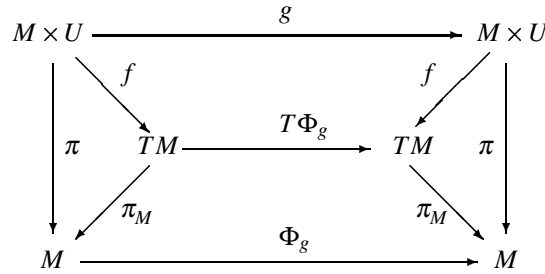


Fig. 1. The commutative diagram of system mappings.

A left action (or G -action) of a connected Lie group G (with k -dimensions) on is a smooth mapping $\Phi: G \times M \rightarrow M$ such that

- (i) for all $x \in M$, $\Phi(e, x) = x$;
- (ii) for each $g, h \in G$, $\Phi(g, \Phi(h, x)) = \Phi(gh, x)$ for all $x \in M$.

The other left-action is a smooth mapping $\theta: G \times M \times U \rightarrow M \times G$, $(g, x, u) \mapsto \theta_g(x, u)$ $\{\theta: G \times M \times U \rightarrow M \times U, (g, x, u) \mapsto \theta_g(x, u)\}$ Mapping Φ is free and proper. So M/G and $Gx = \{\Phi_g x: g \in G\}$ are $n - k$ -dimensional and k -dimensional manifolds respectively. Suppose further that $p: M \rightarrow G/M$ admits a cross section σ . Set $R_{(1)}(x)$ denotes a reachable collective (of system (1)) at point x , and $d_0 \in R_{(1)}(\sigma)$ means that each $x \in \sigma$ can reach d_0 .

Below the necessary the definitions and concepts [12] for dealing with system decomposition problem, some compiled from the literature[7, 12] and some novel ones introduced, are presented. Concepts such as manifolds, bundles, diffeomorphisms and distributions are not defined in the paper as they are now standard in systems and control literature [6]-[12]; some of standard mathematical references are [1, 3]. The following definitions (see Figure 1) are needed in here.

Definition 2.1 [7]: Let θ and Φ be actions of G on $M \times U$ and M respectively. Then the system (1) has symmetry (G, θ, Φ) if the associated mappings satisfy the commutative diagram for all $g \in G$, where $T\Phi_g$ is the tangent map of Φ_g , π is a smooth fiber bundle and π_M is the natural projection of TM on M .

Definition 2.2 [7] (G, Φ) is a state-space symmetry of system (1) if (G, θ, Φ) is a symmetry of system (1) for $\theta_g = (\Phi_g, IDU): (x, u) \mapsto (\Phi_g(x), u)$.

Definition 2.3 [12] (G, Φ, q) is a general symmetry of system (1) if there exists a smooth mapping $q: G \times U \rightarrow U$, $(g, u) \mapsto q(u, g)$ such that

$$(\Phi_g)_* f(x, u) = f(\Phi_g x, q(u, g)) \quad (2)$$

Definition 2.4 [12] A smooth mapping $q: G \times U \rightarrow U$, $(g, u) \mapsto q(u, g)$ is solvable if there exists $u' = q^*(u, g) \in U$ such that

$$q(u', g) = u, \quad \forall u \in U, \quad g \in G. \quad (3)$$

Definition 2.5 [12] System (1) is solvable general symmetry if (G, Φ, q) is general symmetry of the system (1) and q is solvable.

Definition 2.6 The quotient system of the system (1) is the system

$$\dot{y} = \tilde{f}(y, u) = p_* f(\sigma(y), u) \quad (4)$$

defined on manifold M/G for $u \in U$.

Definition 2.7 The feedback quotient system of the system (1) is the system

$$\dot{y} = \tilde{f}^l(y, u) = p_* f(\sigma(y), \alpha(\sigma(y), v)) \quad (5)$$

which is defined on manifold M/G for all $v \in V$, $\sigma(y) \in M$, where $u = \alpha(\sigma(y), v)$ is feedback law and V is a permitted control manifold.

3 Isomorphic Decomposition of Systems with Solvable General Symmetries

In this section, the concrete formations of isomorphic decomposition of systems possessing solvable symmetries are given. The presentation begins with the following lemma.

Lemma 3.1 [11]: Assume that the system (1) is a general symmetry system. Then

- (a) $p(x(t))$ is an integral curve (starting at point $p(x_0)$) of system (4) if $x(t)$ is a integral curve (starting at point x_0) of system (1),
- (b) there exists an integral curve $x(t)$ (starting at point x_0) of system (1) such that $y(t) = p(x(t))$ if is an integral curve (starting at point $p(x_0)$) of system (4).

Using this lemma, it is possible to derive and prove the following main concluding result.

Theorem 3.1 *Suppose system (1) is a control system with general symmetry (G, Φ, q) , Φ is free and proper, q is solvable, and $p : M \rightarrow M/G$ admits a cross section σ . Then system (1) is isomorphic to the system*

$$\dot{y} = \tilde{f}(y(t), u'(t)) = p_* f(\sigma(y(t)), u'(t)) \quad (6a)$$

$$\dot{g}(t) = (T_e L_{g(t)})(T_e \tilde{\Phi}_{\sigma(y)})^{-1}(f(\sigma(y(t)), u'(t)) - (T_{y(t)} \sigma) f(\sigma(y(t)), u'(t))) \quad (6b)$$

where $u' = q^*(u, g)$. The proof is given in the Appendix.

Theorem 3.1 shows that systems possessing solvable general symmetries, under certain conditions, can be isomorphic to two subsystems. So, it is natural one to question what a relationship there may exist between the former and later system structures. It is this idea precisely that has led us to the result presented in the subsequent section.

4 Controllability of Systems with Solvable General Symmetries

This section deals with controllability of this class of systems with the symmetry property as mentioned above. First the following lemma is introduced.

Lemma 4.1 [11]: $\Phi_g s_t(x, u)$ is a integral curve (starting at $\Phi_g x$) of system (1) corresponding to control $q(u, g)$, that is, $\Phi_g s_t(x, u) = s_t(\Phi_g x, q(u, g))$ if $s_t(x, u)$ is a integral curve (starting at x) of the system (1) corresponding to control $u(t)$.

The next main result of this paper, given in terms of the Theorem 4.1, follows from the discussion insofar and the necessity part of its proof follows from Theorem 3.1 above.

Theorem 4.1 *Suppose that the system (1) has general symmetry (G, Φ, q) , Φ is free and proper, q is solvable, and $p : M \rightarrow M/G$ admits a cross section σ . Hence $x_0 (\in M)$ is changed into q_0, y_0 through isomorphic decomposition. Further, suppose that the system (1) is weakly controllable on σ . Then by means of set $g\sigma = \{\Phi(g, x) : x \in G\}$, $g \in G$ one can obtain that sufficient and necessary conditions of the system (1) being globally controllable at point x_0 are determined by:*

- (a) subsystem (6 a) is globally controllable at point y_0 ;
- (b) subsystem (6 b) is globally controllable at point g_0 .

The proof is given in the Appendix.

On the grounds of Theorem 4.1 (under certain conditions stated and described in the presentation above) it is possible to decompose the original system into two,

and by repeated decomposition thereafter each of them can be further decomposed provided the conditions of this theorem are still satisfied. The respective subsystem dimensions are decreased accordingly while the controllability property of the original system remains preserved.

5 Conclusion

In this paper, the original system has been decomposed through quotient system and feedback quotient system. Each of two ways has its own advantages and disadvantages. For example, when it is very difficult for one to find an appropriate feedback law, one may decompose the original system through quotient system such as solvable general symmetry systems.

It has been shown that complex non-linear control systems possessing general symmetries, under technical conditions of smooth mappings, do admit isomorphic decompositions in terms of lower dimensional subsystems and feedback loops. Furthermore, controllability between the original system and the subsystems is equivalent under certain conditions. Therefore, this can be exploited further when analyzing the controllability of complex non-linear systems and using their decomposed form in terms of symmetric subsystems.

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Appendix

Proof of Theorem 3.1

Note $\sigma(y)$ is uniquely determined by y because Φ is free and proper and $p : M \rightarrow M/G$ admits a cross section σ . So, $p_*f(\sigma(y(t)), u'(t))$ is unique tangent vector of system (6 a) at point $y \in M/G$ and system (6 a) is uniquely defined. Next recall Lemma 3.1 and consider the following statement [A].

Statement [A]: Let $x_0 \in M$, $u(t)$ be a continuous time function, $x(t)$ the integral curve of system (1) corresponding to $u(t)$. Then $y(t) = p(x(t))$ is the corresponding integral curve of system (6 a) having $y(0) = p(x(0))$.

Let, according to $p : M \rightarrow M/G$ admitting a cross section σ , define a differentiable curve $d(t) \in M$ by means of $d(t) = \sigma(y(t))$. Since $p(d(t)) = p(x(t))$ and Φ

is free and proper, one can write $x(t) = \Phi_{g(t)}(d(t))$ for a uniquely defined differentiable curve $g(t) \in G$.

Proof of Statement [A]: Since system (1) has general symmetry (G, Φ, q) , q is solvable, and $x(t)$ is the integral curve of system (1) corresponding to $u(t)$. Noting that $x(t)$ and $\sigma(p(x))$ have the same orbit, one can get

$$\begin{aligned} \dot{y}(t) &= p_* f(x(t), u(t)) \\ &= p_* f(\Phi_{g(t)} \sigma(p(x(t))), u(t)) \\ &= p_* f(\Phi_{g(t)} \sigma(p(x(t))), q(u'(t), g(t))) \end{aligned}$$

that is,

$$\begin{aligned} \dot{y}(t) &= p_*(\Phi_{g(t)})_* f(\sigma(p(x(t))), u'(t)) \\ &= (p \circ \Phi_{g(t)})_* f(\sigma(p(x(t))), u'(t)) \\ &= p_* f(\sigma(p(x(t))), u'(t)) \\ &= p_* f(\sigma(y(t)), u'(t)) \end{aligned}$$

where $u' = q^*(u(t), g(t))$. The goal now is to find a differential equation for $g(t)$.

By means of the chain rule of differentiation, one can find

$$\begin{aligned} f(x(t), u(t)) = \dot{x} &= \frac{d}{dt} \Phi(g(t), d(t)) \\ &= T_{d(t)} \Phi_{g(t)} \dot{d}(t) + T_{g(t)} \Phi_{d(t)} \dot{g}(t). \end{aligned} \quad (7)$$

The next step is to rewrite the second term. Note that $\dot{g}(t) \in T_{g(t)} G$. Let $\xi_g \in T_g G$ and denote by $\xi = T_g L_{g^{-1}}(\xi_g) \in T_e G$, where L_h is the left translation operator on G . Then, for a given point $m \in M$, it follows

$$\begin{aligned} T_g \Phi_m(\xi_g) &= (T_g \Phi_m)(T_e L_g)(\xi) = T_e(\Phi_m \circ L_g)(\xi) \\ &= T_e(\Phi_g \circ \Phi_m)(\xi) = (T_m \Phi_g)(T_e \Phi_m)(\xi) \end{aligned} \quad (8)$$

But, it is also valid

$$T_e \Phi_m(\xi) = \frac{d}{dt} \Phi_m(e^{t\xi}) \Big|_{t=0} = \xi_M(m) \quad (9)$$

the infinitesimal generator of Φ corresponding to ξ . Hence,

$$T_g \Phi_m(\xi_g) = T_m \Phi_g(\xi_M(m)) = T_m \Phi_g((T_g L_{g^{-1}})_M(m)) \quad (10)$$

and substitution of (10) into (7) gives

$$f(x(t), u(t)) = T_{d(t)} \Phi_{g(t)} \dot{d}(t) + T_{d(t)} \Phi_{g(t)} (T_{g(t)} L_{g^{-1}(t)} \dot{g}(t))_M(d(x)) \quad (11)$$

Therefore q being solvable gives

$$f(x(t), u(t)) = f(x(t), q(u'(t), g(t))) \quad (12)$$

which satisfies

$$u'(t) = q^*(u(t), g(t)). \quad (13)$$

Thus, the use of (12) in (11) will result in

$$f(x(t), q(u'(t), g(t))) = T_{d(t)} \Phi_{g(t)} \dot{d}(t) + T_{d(t)} \Phi_{g(t)} (T_{g(t)} L_{g^{-1}(t)} \dot{g}(t))_M(d(t)). \quad (14)$$

System (1) having general symmetry (G, Φ, q) gives

$$T_m \Phi_g f(m, u'(t)) = f(\Phi_g(m), q(u'(t), g(t))), \quad (15)$$

by substituting (15) into (14). Meanwhile by replacing m in $d(t)$ one obtains

$$T_{d(t)} \Phi_{g(t)} f(d(t), u(t)) = T_{d(t)} \Phi_{g(t)} \dot{d}(t) + T_{d(t)} \Phi_{g(t)} (T_{g(t)} L_{g^{-1}(t)} \dot{g}(t))_M(d(t)), \quad (16)$$

since $\Phi_g : M \rightarrow M$ is a diffeomorphism for all $g \in G$ and $T_{d(t)} \Phi_{g(t)}$ is non-singular. Hence,

$$f(d(t), u'(t)) = \dot{d}(t) + (T_{g(t)} L_{g^{-1}(t)} \dot{g}(t))_M(d(t)). \quad (17)$$

Now, let it be set

$$\xi_M(d(t)) = (T_{g(t)} L_{g^{-1}(t)} \dot{g}(t))_M(d(t)) \quad (18)$$

From (17),

$$\xi_M(d(t)) = f(d(t), u'(t)) - \dot{d}(t) \quad (19)$$

Thus, by applying (9), one obtains

$$T_e \Phi_{d(t)}(\xi(t)) = \xi_M(d(t)) + T_{d(t)} \Phi_{g(t)} (T_{g(t)} L_{g^{-1}(t)} \dot{g}(t))_M(d(t)) \quad (20)$$

Φ being free and proper implies that $\Phi_m : G \rightarrow M$ is a diffeomorphism onto its range. Hence, (20) can be solved uniquely for $\xi(t)$ to give

$$\xi(t) = (T_e \tilde{\Phi}_{d(t)})^{-1} \xi_M(d(T)) \quad (21)$$

or

$$T_{g(t)} L_{g^{-1}(t)} \dot{g}(t) = (T_e \tilde{\Phi}_{d(t)})^{-1} \xi_M(d(T)) \quad (22)$$

where $\tilde{\Phi}_m : G \rightarrow Gm$ by $g \mapsto \Phi(g, m)$. Hence since L_g is a diffeomorphism for all g .

$$\dot{g}(t) = (T_e L_{g(t)}) (T_e \tilde{\Phi}_{\sigma(y(t))})^{-1} [f(\sigma(y(t)), u'(t)) - (T_{y(t)} \sigma) \tilde{f}(y(t), u'(t))]. \quad (23)$$

Finally, using the fact that $d(t) = \sigma(y(t))$, one gets

$$\dot{g}(t) = (T_e L_{g(t)}) (T_e \tilde{\Phi}_{\sigma(y(t))})^{-1} [f(d(t), u'(t)) - (T_{y(t)} \sigma) \tilde{f}(y(t), u'(t))]. \quad (24)$$

and substitution of (13) and (6a) into (24) gives (6b). And along with Lemma 3.1 to this end, both the necessity and sufficiency of Theorem 3.1 have been proved completely.

Proof of Theorem 4.1

Since system (1) has general symmetry, q is solvable, and $p : \rightarrow M/G$ admits a cross section σ , it is apparent that system (1) is isomorphic to system (1) in Theorem 3.1. Hence the respective conditions apply.

The necessity of Theorem 4.1: Proof of necessity is obvious since it is clearly seen from its statement and from the above point.

Prior to completion of the proof of sufficiency, let us point out the next statement.

Statement [B]: System (1) is weakly controllable on $g\sigma$, $\forall g \in G$, that is $x_1 \in R_{(1)}(g\sigma)$, and $x_2 \in R_{(1)}(x_1)(\forall x_1, x_2 \in g\sigma)$.

Proof of Statement [B]: For $\forall x_2 \in g\sigma$, there exists $x'' \in \sigma$ such that $x_2 = \Phi_g x''$. Set $x_1 = \Phi_g(x')$ for $x' \in \sigma$. Since $x' \in R_{(1)}(\sigma)$, hence $x' \in R_{(1)}(x'')$. From Lemma 4.1, one can easily get $x_1 = \Phi_g(x') \in R_{(1)}\Phi_g(x'') = R_{(1)}(x_2)$. For $x_2 \in g\sigma$ is at will, we have $x_1 \in R_{(1)}(g\sigma)$. The reasons of $x_2 \in R_{(1)}(\forall x_1, x_2 \in g\sigma)$ is similar. Hence, system (1) is weakly controllable on $g\sigma$.

The sufficiency of Theorem 4.1: Let set $z \in M$ and $z \notin g_0\sigma$ (if $z \in g_0\sigma$, one can easily have $z \in R_{(1)}(x_0)$ from the conclusion having been proved just now). According to the condition (a) of Theorem 3.1, there exists a integral curve $y(t)$ (starting at y_0) of system (6a) such that $y(t_1) = p(z)$. And then from Lemma 3.1, there exists a integral curve $x(t)$ (starting at x_0) of the system (1) such that $p(x(t_1)) = y(t_1) = p(z)$. It is not difficult for one to find that $x(t_1)$ and z lie on the same orbit. Therefore, there exists $\bar{g} \in G$ satisfying $z = \Phi_{\bar{g}}(x(t_1))$. From Lemma 4.1, one can get $z = \Phi_{\bar{g}}(x(t_1)) \in R_{(1)}(\Phi_{\bar{g}}(x_0))$.

Let $g = \bar{g}g_0$. From the condition (b) of Theorem 4.1, there at least exists a point $x^*(x \in g\sigma) \in R_{(1)}(x_0)$ (note, otherwise, for $\forall x \in g_0\sigma$ can not reach $x_1(\forall x_1 \in g\sigma)$ since the system (1) is weakly controllable on $g_0\sigma$ and $g\sigma$). And, then one draws a contradictory conclusion that system (6 b) can not satisfy g being reachable for g_0 . x^* can obviously be expressed as $x^* = \Phi_g(x') = \Phi_{\bar{g}g_0}(x') = \Phi_{\bar{g}}(\Phi_{g_0}(x'))(x' \in \sigma)$. Since system (1) is weakly controllable for x_0 on $g_0\sigma$, we easily know $x_0 \in R_{(1)}(\Phi_{g_0}(x'))$. And then we have $\Phi_{\bar{g}} \in R_{(1)}(\Phi_{\bar{g}}(\Phi_{g_0}(x'))) = R_{(1)}(x^*)$ from Lemma 4.1. Hence $z = \Phi_{\bar{g}}(x(t_1)) \in R_{(1)}(\Phi_{\bar{g}}(x_0))$. Finally, we obtain that the system (1) is globally controllable at point x_0 according to $z(\in M)$ being at will.

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