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Decimation by Non-Integer Factor in Software Radio Receivers

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Abstract: The sampling rate conversion is a critical functionality of the software radio receiver. Because the signals of different system standards have incommensurate symbol/sampling rates and a common Analog-to Digital Converter (ADC) is to be used for all supported standards, the decimation factor may become very difficult non-integer number. This paper gives overviews and comparisons of two efficient fractional decimator structures based on Cascaded Integrator-Comb (CIC) filters and low order polynomialbased interpolation filters.

Keywords: Decimation, interpolation, software radio.

1 Introduction

The ability to process signals corresponding to a wide range of frequency bands and channel bandwidths is a critical issue of the 3rd generation cellular multi-standard radios and impacts heavily on the design of both analogue and digital stages of the radio. Because of the need to support different wireless standards, the concept of software radio has arisen. We can define software radio as a single hardware solution adaptable to different system standards by changing software. It is important to develop receiver concepts which use common hardware platform for the same functionality for different standards [1].

Sampling rate reduction (decimation) from a high ADC sampling rate to a small multiple of the symbol rate is a key functionality in any digital

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radio receiver. The standards to be supported by a software radio platform are often based on incommensurate clock and symbol rates. This makes the sample rate adaptation and decimation a critical functionality in multistandard radio design [1]. The software radio concept introduces the following requirements for the decimation system [2], [3]: (i) decimation factor R can be a difficult fractional or even an irrational number, (ii) the multirate filter chain should have good anti-aliasing and anti-imaging properties, (iii) the overall structure should be simple because of possible high sampling rate, and (iv) it is desired to have a unique decimation hardware platform that is programmable and adaptable for different decimation factors and filter requirements. This paper discusses the problem of sampling rate adaptation and decimation by a non-integer in software radio receivers. This paper compares two promising decimator structures in terms of aliasing attenuation, complexity, and programmability.

2 Decimation by Non-Integer Factor

Sampling rate conversion is needed whenever two systems working at different sampling rates are to be interconnected [4]. A good example is a radio receiver, where typically a high sampling rate is used in the ADC due to various reasons [1]. Since the sampling rate conversion can be seen as a process of resampling, in spectral domain the repetitions of the input signal spectrum are expected. If the original signal is not band-limited, the different spectral replicas will overlap after downsampling. This overlapping in spectral domain, known as aliasing, changes the signal irreversibly. Similarly, after interpolation (upsampling) the spectrum repeats itself at the multiples of the sampling rate. These are called image spectra, and the phenomenon itself imaging [4]. The sampling rate conversion system must provide enough attenuation for aliasing and imaging in order to preserve the desired signal.

The conversion between arbitrary sampling rates includes cases where the sampling rate change (decimation) factor is integer, rational, irrational, or even a time-varying number. The overall decimation can be divided into two main parts, decimation by an integer factor and sampling rate conversion by a non-integer factor. The decimation by integer ratio is a very well explored topic and there are many efficient solutions for this task [4].

No generally applicable solution for the problem of sampling rate conversion by a non-integer has been found yet. The type of the used filter depends on the application at hand. Different filter types are used for different input sampling frequencies, whether the non-integer sampling conversion block is placed at the beginning, in the middle, or at the end of the overall decimation chain. However, the placement of the non-integer block at the beginning of the overall decimation chain has several advantages; the filter requirements are relaxed due to the high ratio between input sampling rate and desired signal bandwidth.

There are several different approaches for sampling rate conversion by a non-integer factor [5], [6]. The filters intended for integer and rational sampling rate conversion factors can be considered as time varying with periodically changing coefficients. There are two approaches that can be applied for these filters: (i) rational factor means storage of a certain number of filter coefficients, and the correct set is applied according to timing, and (ii) irrational factor means calculation of a new coefficient set as required for each new sample. Therefore, the required number of coefficients sets becomes infinite, or in practice with finite resolution to represent the coefficients, finite but very large. The required coefficients storage memory can become very large and the implementation of the structure impractical. In order to reduce the storage requirements, it is possible to perform the coefficient design during the filtering process based on the current relative position between input and output samples. Sometimes the complexity of coefficients calculations exceeds the complexity of filtering operations.

Recently, polynomial-based filters have been found as an efficient solution to the problem of non-integer sampling rate conversion. The sampling rate conversion can be considered as a problem of calculating a new output sample between input samples. Therefore, it is possible to look at this problem in a sense of mathematical interpolation [7]. If the impulse response of the polynomial filter can be expressed in each sampling interval by means of the low-order polynomial, it is possible to efficiently implement this filter using the socalled Farrow structure. The main advantage of the Farrow structure is in that it has only one changeable parameter, the fractional interval μ.

3 Efficient Filter Structure

If the non-integer sampling rate conversion system is placed at the beginning of the overall decimation chain, the filter requirements are relaxed due to the high oversampling ratio. Therefore, in this case, it is possible to use simpler filter structures reducing the workload and power consumption significantly.

Cascaded Integrator-Comb (CIC) filters have a simple regular structure without multipliers. CIC decimation filter (see [8]) consists of N cascaded digital integrator stages operating at high input sampling rate F_in, followed by N cascaded comb or differentiator stages operating at low sampling rate F_{in}/R. Its frequency response is given by

$$H_{CIC}(e^{j\omega}) = e^{-j\frac{\omega N(R-1)}{2}} \left(\frac{\sin \frac{\omega R}{2}}{R \sin \frac{\omega}{2}} \right)^N , \tag{1}$$

where $\omega = 2\pi f / F_{in}$ is the normalized input frequency. When the decimation factor is an irrational number one solution is to use polynomial-based interpolation filters. Among them, linear interpolation filter has a simple implementation structure, only one multiplication per output sample is needed. Because interpolation is basically a reconstruction problem, polynomial-based interpolation can be analyzed using the hybrid analog/digital model shown in Fig. 1. In this model, the interpolated output samples $y(l)$ are obtained by sampling the reconstructed signal $y_a(t)$ at the time instants $t = (n_l + \mu_l)T_{in}$. Here n_l is an integer, $\mu_l \in [0, 1)$ is the adjustable fractional interval, and T_{in} is the sampling interval of the input signal $x(n)$. For linear interpolation, the impulse response of the reconstruction filter $h_a(t)$ is a triangular function, and thus, its frequency response is given by

$$H_a(f) = \left(\frac{\sin(\frac{\pi f}{F_{in}})}{\frac{\pi f}{F_{in}}} \right)^2 \tag{2}$$

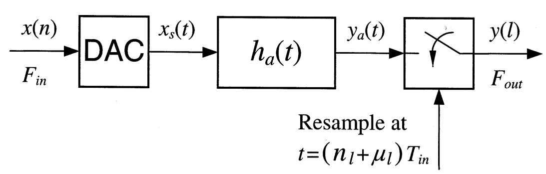

Fig. 1. The hybrid analog/digital model.

4 Novel Fractional Decimators (FD)

4.1 FD based on CIC filter and linear interpolation

Because we consider here a system that can be used also for irrational decimation, the overall decimation factor is determined by

$$R = \frac{F_{in}}{F_{out}} = R_{int} + \varepsilon \tag{3}$$

where $F_{in} = 1/T_{in}$ and $F_{out} = 1/T_{out}$ are the input and output sampling frequencies, whereas R_{int} is the integer part and $\varepsilon \in [0, 1)$ is the decimal part of the overall decimation factor. Figure 2 illustrates the proposed structure for the decimation filter [2]. The input signal $x(n)$ is divided into polyphase components $x_k(m)$ for $k = 0, 1, \ldots, R_{int} - 1$ by using delay line and parallel CIC filters having the decimation factor of R_{int}. Only few of these R_{int} parallel CIC filters are to work at the same time, hence, the number of parallel CIC filters is reduced. The sampling rate at the output of the CIC filters is F_{in}/R_{int}. The final, possibly irrational decimation by $1 + \varepsilon/R_{int}$ is done using linear interpolation between some of the two signal pairs $x_k(m)$ and $x_{k\oplus 1}(m)$, where $\oplus$ denotes a modulo R_{int} summation.

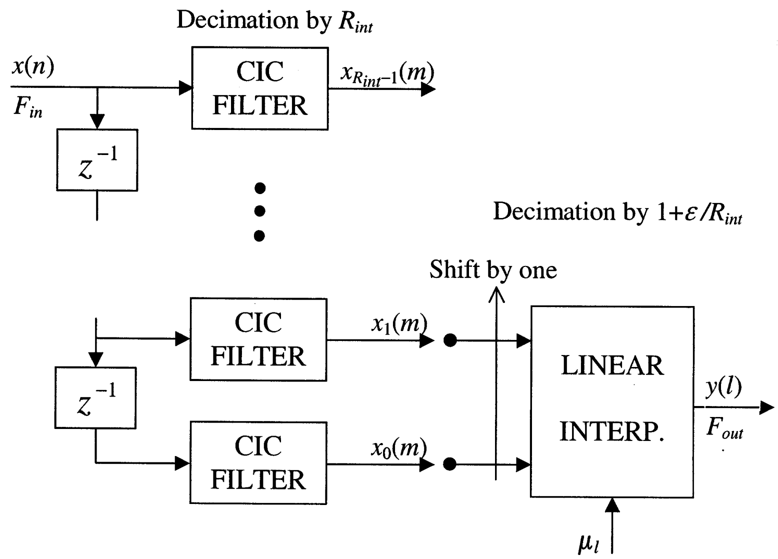

Fig. 2. Model of proposed decimation filter.

The linear interpolation block in Fig. 2 is shifted by one branch when needed, according to a certain condition (see [2] for details). Because of the modulus R_{int} summation mentioned above, the next signal pair for linear interpolation after $x_{R_{int}-2}(m)$ and $x_{R_{int}-1}1(m)$ is $x_{R_{int}-1}(m)$ and $x_0(m)$. The fractional interval μ_l is re-calculated for each output sample $y(l)$ for $l = 0, 1, 2, \ldots$. The time interval between samples $x_k(m)$ and $x_{k\oplus 1}(m)$ equals to T_{in} and, thus, the linear interpolation is done at the high input sampling frequency F_{in}. This means better image attenuation.

The overall frequency response of the decimation filter structure in Fig. 2 is a product of the frequency responses of the parallel CIC filters and linear interpolation filter. Note that the former response is a periodical whereas the latter is not. Since the linear interpolation is done at the higher input rate F_{in}, its frequency response is given by Eq. (2). Consequently, the overall zero-phase frequency response of the proposed decimation filter, relative to the input sampling frequency, is given by

$$H_t(\omega) = H_{CIC}(\omega) H_a\left(\frac{\omega F_{in}}{2\pi}\right) = \left(\frac{sin\left(\frac{\omega R_{int}}{2}\right)}{R_{int} sin\left(\frac{\omega}{2}\right)}\right)^N \left(\frac{sin\left(\frac{\omega}{2}\right)}{\frac{\omega}{2}}\right)^2 \quad (4)$$

where $\omega = 2\pi f/F_{in} = 2\pi f/(RF_{out})$.

The actual implementation of the proposed decimator structure may be based on the block diagram shown in Fig. 3. In the general case, for any

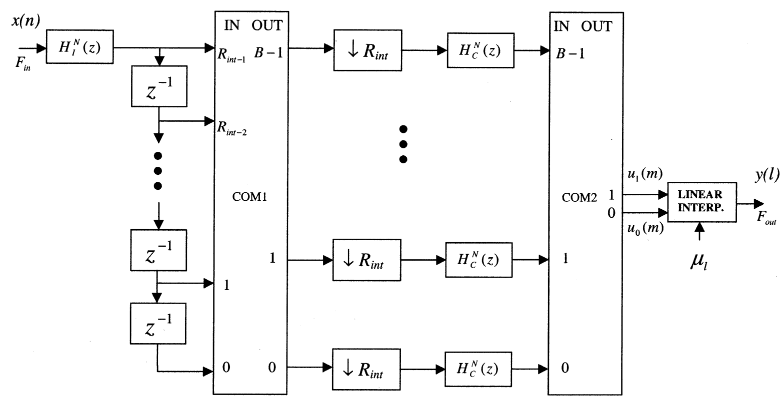

Fig. 3. Efficient implementation structure for N^{th} order CIC filter for any non-integer decimation rate R. $H_I^N(z)$ denotes integrator stages, and $H_C^N(z)$ denotes comb stages of the CIC filter. The commutators COM1 and COM2 are used to select the correct input branch for the B comb sections and for the linear interpolation, respectively.

e, the number of needed comb branches is $B = N + 2$. Two of them are used for the actual interpolation and the remaining N branches are used for initializing the state-variables of the branches needed later. The details of the control logic are given in [2]. Here we stress that the linear interpolation works at the output sampling rate. However, virtually it works as if it was at the beginning of the overall structure having high input sampling rate. The main advantage of this lies in that only one multiplication is needed per each output sample. The linear interpolation filter contains one multiplier while the CIC part is multiplierless. However, this structure is not easily programmed as the attenuation of the aliasing bands severely depends on the decimal part of the sampling factor ε.

4.2 Fractional programmable CIC decimation filter

Even though it provides enough attenuation for aliasing components and it has low power consumption, the structure presented above is not programmable enough. By using noninteger delay in the feed-forward branch of comb stage and polynomial interpolation filter between integrator and comb stages of the CIC filter, we achieve improved attenuation for the aliasing frequency components. The fractional programmable CIC decimation filter structure [3], shown in Fig. 4, consists of N integrator stages operating at input rate F_{in}, polynomial interpolation filter (PIF in Fig. 4) $h_a(t)$, resampler, and N comb stages operating at output sampling rate F_{out}. The role of

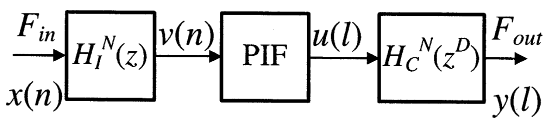

Fig. 4. The fractional programmable CIC filter.

polynomial interpolation filter is to provide the samples to the input of the comb stage. It should be noted that the interpolation filter does not work all the time, as it prepares the samples for the output stages that work at the output sampling rate. The control logic of the interpolation filter is very similar to one presented in [3]. In this way the workload is reduced as the interpolation filter in practical realization works at the output sampling rate. In the case of non-integer decimation factor R, we can realize the frequency response (3) by placing a non-integer delay D in the feed-forward branches of the comb stages. D is determined by the desired length of the moving average (CIC) filter K and overall decimation ratio R as $D = {}^K/_R$. The moving average filter length K has influence on the frequency response of the overall structure. The value of K determines the positions of the zeros in the overall frequency response. The frequency response of the overall system is a product of two frequency responses of the systems in cascade, that is

$$H_{ov}(e^{j2\pi f/F_{in}}) = H(e^{j2\pi f/F_{in}})H_a(jf), \tag{5}$$

The main complexity of the structure is related to the noninteger delay filter, which contains a certain number of multipliers. As non-integer delay approximation we use the FIR fractional delay filter designed using Lagrange interpolation method. In this method the delay z^{-D} is approximated by

$$H_D(z) = z^{-D} \approx \sum_{n=0}^{L} h_D(n)z^n \tag{6}$$

where L is the filter length, and filter coefficients $h_D(n)$ have the explicit form.The transfer function of the comb stage is expressed as the transfer function of a FIR filter as follows

$$H_C(z) = 1 - z^{-D} \approx 1 - h_D(0) - \sum_{n=1}^{L} h_D(n) z^{-n}. \tag{7}$$

The overall frequency response becomes

$$H_{ov}\left(e^{j\frac{2\pi f}{F_{in}}}\right) = e^{j\arg[H_{ov}(e^{j\frac{2\pi f}{F_{in}}})]} \left[\frac{|H_C(e^{j\frac{2\pi f}{F_{in}}})|}{K\sin\left(\frac{\pi f}{F_{in}}\right)}\right]^N \left[\mathrm{sinc}(\frac{\pi f K}{F_{in}})\right]^2 \tag{8}$$

The fractional programmable CIC filter has relatively simple structure. The polynomial filter between integrator and comb stages works at the output sampling rate as it prepares samples for the later stages that operates at the low output rate. The FIR delay filter has L multipliers working also at the output sampling rate. In the case of N^{th} order CIC filter and linear interpolation filter total number of multiplications per output sample is $N_L + 1$. The main advantages of the fractional programmable CIC are high flexibility and possible programmability as the position of zeros in frequency response of the overall structure can be easily adjusted.

5 Case Studies

In this example, the bandwidth of the input signal is $f_p = 0.001F_{in}$ and decimation factor $R = 34^1/_{34}$. It is required that the aliasing frequency bands are attenuated at least by $A_s = 80$ dB and the passband distortion is less than δ_p =0.01 (0.086 dB). These requirements are met by a fractional decimator filter having the CIC filter of order $N = 3$ and linear interpolation filter. Fig. 5 presents the bands that cause aliasing to the desired band. As it can be seen, the minimum attenuation of these bands is 84.4 dB. Because the over-sampling factor is still high after decimation, the worst case passband distortion caused by the proposed filter structure is only 0.06 dB. The same requirements are met by the fractional programmable CIC filter with $N = 3$, $K = 69$, thus $D = 2.027658$ (Lagrange FIR fractional delay filter of length $L = 6$), using the linear interpolation filter. The aliasing bands of this structure are shown in Fig. 6. In this case, the worst case aliasing attenuation is 87.2 dB, and the passband distortion is the same.

The minimum attenuation of the aliasing bands occurs at the edge of the first aliased band, and it depends on CIC filter order N, decimal part

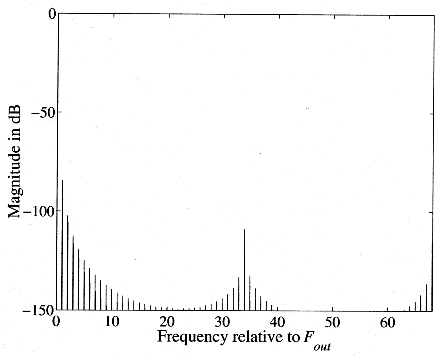

Fig. 5. Aliasing bands amplitude responses of the fractional decimator filter.

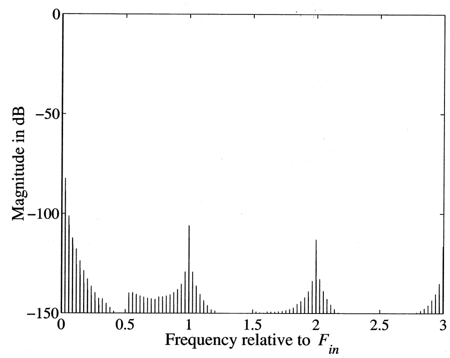

Fig. 6. Aliasing bands amplitude response of the fractional programmable CIC filter.

of the decimation factor, and relative bandwidth of the desired input signal. Table 1 gives values of the minimum attenuation of the aliasing bands for the case of the fractional decimator. These results are given for the integer part of decimation factor $R_{int} = 34$. We can compare these to the data given in Table 2 for the fractional programmable CIC filter. We can notice that the fractional programmable CIC filter attenuates more aliasing frequency components, and more important, the attenuation does not depend on decimal part of decimation factor e. However, the complexity of the fractional programmable CIC filter is considerably higher than complexity of the fractional decimator, in terms of multiplications per output sample.

Table 1. The minimum attenuation of the aliasing bands (in dB) for the fractional decimation filter.

Decimal part of the decimation factor	The passband edge, normalized F_{in}	CIC filter order			
		$N = 1$	$N = 2$	$N = 3$	$N = 4$
$\varepsilon = 0.05$	$f_p = 0.001$	28.7	57.4	86.1	114.8
	$f_p = 0.002$	22.6	45.2	67.8	90.4
	$f_p = 0.005$	14.1	28.2	42.3	56.4
	$f_p = 0.01$	7.5	14.9	22.3	29.8
$\varepsilon = 0.5$	$f_p = 0.001$	25.9	51.8	77.7	103.6
	$f_p = 0.002$	21.0	42.1	63.1	84.1
	$f_p = 0.005$	13.4	26.8	40.2	53.6
	$f_p = 0.01$	7.1	14.2	21.3	28.3

Table 2. The minimum attenuation of the aliasing bands (in dB) for the fractional progammable CIC filter.

Decimal part of the decimation factor	The passband edge, normalized to the input sampling rate	CIC filter order			
		$N=1$	$N=2$	$N=3$	$N=4$
$\varepsilon = 0.05$	$f_p = 0.001$	29.1	58.2	87.2	116.3
	$f_p = 0.002$	22.8	45.6	68.4	91.2
	$f_p = 0.005$	14.2	28.4	48.2	56.7
	$f_p = 0.01$	7.5	14.9	22.4	29.9
$\varepsilon = 0.5$	$f_p = 0.001$	29.0	57.9	86.9	115.8
	$f_p = 0.002$	22.7	45.4	68.01	90.71
	$f_p = 0.005$	14.1	28.1	42.1	56.2
	$f_p = 0.01$	7.6	14.7	22.0	29.4

6 Conclusions

We have overviewed two structures for the sampling rate conversion by a non-integer factor. Both structures are developed as the first stage in multirate decimation chain of a multistandard radio receiver. The fractional decimator structure composed of CIC filter and linear interpolation filter has very simple structure, and thus lower power consumption. However, since the stopband attenuation depends heavily on the decimal part of the decimation factor this structure does not provide high reconfigurability, which is a very important issue of any software radio component. The fractional programmable CIC filter provides higher attenuation of the aliasing components, which is less dependent on the value of the decimal part of the decimation factor.

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