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Cyclic Spectral Analysis of OFDM/QAM Modulation Using Stochastic Matrix-Based Method

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Abstract: The cyclostationarity of the OFDM/QAM signals induced by the use of cyclic prefix or pulse shaping is the key to blind synchronization, channel identification and equalization in these systems. Therefore, continuous-time and discrete-time spectral correlation characterization of the pulse shaping OFDM/QAM signals is performed by a matrix-based stochastic method based on Markov chain signal representation. Exact analytical expression for the cyclic spectrum of the continuous-time OFDM/QAM signal is derived applying the proposed method, the characteristic cyclic features according to the cyclic prefix and/or pulse shaping employing are analyzed and some graphed results are presented.

Keywords: Signal processing, cyclic spectral analysis, orthogonal frequency-division multilpexing, OFDM, pulse shaping, spectral correlation, Markov chain.

1 Introduction

Orthogonal frequency-division multiplex (OFDM) is a communications technique that divides the available broadband into many orthogonal subcarriers each one being affected by frequency-flat rather then frequency-selective fading, which simplifies equalization.

In the time-frequency dispersive environments (such as the mobile radio channel), in general, intersymbol interference (ISI) and intercarrier interference (ICI) will be caused due to lack of orthogonality between OFDM

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transmitter and receiver basis functions. The error resulting from both ISI and ICI depends critically on the time-frequency localization of the transmitter basis functions [2], [3].

OFDM systems based on a cyclic prefix (CP) employ a rectangular pulse that has poor frequency localization and requires insertion of spectral guard regions to avoid ICI. By using pulses other then rectangular, the spectrum can be shaped and more well-localized in frequency, which is beneficial from ICI point of view. However, pulse shaping with ideal bandpass filter has poor temporal localization and causes substantial ISI in time-dispersive environments. Therefore, in the time-frequency dispersive environments the optimal solution is employment of a time- frequency well-localized pulse shaping filter.

It is known that time-frequency well-localized orthogonal basis functions exist only for $T\Delta f > 1$ (T- symbol duration, Δf -subcarrier spacing [2], [3]. The OFDM based on quadrature amplitude modulation (OFDM/QAM) does not allow time-frequency well-localized transmitter basis functions for critical time-frequency density $T\Delta f = 1$) where spectral efficiency is maximal [2], [3]. The OFDM/QAM systems with CP has $T\Delta f > 1$. Although employing time-frequency guard region ($T\Delta f > 1$) reduces spectral efficiency, the use of pulse shaping filter with improved time-frequency localization is especially important for reducing out-of-band emission.

The increasing demand for high-date-rate communications through the time-frequency dispersive channels makes blind channel identification and synchronization methods that do not use pilot symbols or training sequences, very interesting. Exploiting second-order cyclostationarity of OFDM/QAM signals is elegant solution for that.

The cyclostationarity of the OFDM/QAM signals can be induced by insertion of CP, by employing pulse shaping, and by the use of different transmit subcarrier powers (subcarrier weighting). The utilization of spectral redundancy of cyclostationary OFDM/QAM signals enables substantial performance improvement in the blind channel identification and synchronization in these systems, parameter estimation, detection and classification of these signals [1], [5]. The cyclic spectrum approach is an interesting way of evoking cyclostationarity of OFDM/QAM signal.

The advances in digital signal processing are achieved by processing signals as cyclostationary [1]. Cyclostationary signal processing techniques exploit the underlying signal periodicities. It exhibits as correlation between shifted spectral components. Exploitation of this inherent spectral redundancy associated with spectral correlation offers significant advantages at performing different signal processing tasks in difficult environments. The cyclic spectrum and cyclic features analysis (spectral correlation characterization) are often key stages in deriving a proper signal processing of the cyclostationary signals.

The pulse shaping, CP and subcarrier weighting OFDM/QAM signals are cyclostationary. In this paper, based on the proposed stochastic matrix-based method [5], [6], a complete spectral correlation characterization of these signals is performed. Their cyclic spectrum, when pulse shaping filter is not necessary orthogonal, is derived and the presence of cyclic features is analyzed. The proposed method does not involve complicated theory and provides a simple, straightforward derivation of cyclic spectrum for the observed OFDM/QAM signals.

2 Cyclostationary Signal Analysis

In general, a complex signal $x(t)$ is called second-order cyclostationary (CS) if its autocorrelation function

$$\mathcal{R}_{xx}(t,\tau) = \mathrm{E}\{(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2})\} \tag{1}$$

is periodic (one period) or polyperiodic (multiple incommensurate periods) in time (t) for each time shift (τ) [1]. For the cyclostationary or polycyclostationary signals, $\mathcal{R}_{xx}(t,\tau)$ has the Fourier series representation

$$\mathcal{R}_{xx}(t,\tau) = \sum_{\alpha} \mathcal{R}^{\alpha}_{xx}(\tau) e^{j2\pi\alpha t} \tag{2}$$

where the sum ranges over all multiples of one or more fundamental cycle frequencies (reciprocal of incommensurate cyclostationarity periods) α for which cyclic autocorrelation, defined as

$$\mathcal{R}_{xx}(\tau) = \lim_{Z\to\infty} \frac{1}{Z} \int_{-\frac{Z}{2}}^{\frac{Z}{2}} \mathcal{R}_{xx}(t,\tau) e^{-j2\pi\alpha t} \mathrm{d}t \tag{3}$$

exists as function of τ and is not identically zero [1].

The spectral correlation (cyclic spectrum) is the Fourier transform of the cyclic autocorrelation [1], i.e.

$$S^{\alpha}_{xx}(f) = \int_{-\infty}^{\infty} \mathcal{R}^{\alpha}_{xx}(\tau) e^{-j2\pi f\tau} \mathrm{d}\tau \tag{4}$$

The conjugate cyclic autocorrelation $\mathcal{R}^{\alpha}_{xx^*}(\tau)$ and the conjugate cyclic spectrum $S^{\alpha}_{xx^*}(f)$, for a complex signal $x(t)$, are obtained by removing the conjugation operation $*$ in the corresponding above definitions.

The spectral correlation is an important characteristic property of the second-order cyclostationary signals. If the signal $x(t)$ exhibits cyclostationarity with cycle frequency α in time domain, then it also exhibits the spectral correlation at shift α in the frequency domain.

3 OFDM/QAM System

The OFDM/QAM signal $y(t)$ can be expressed as

$$y(t) = \Re\{x(t)e^{j2\pi f_c t}\} \tag{5}$$

where f_c is the carrier frequency and $x(t)$ is the complex envelope (baseband equivalent) given by sum of N parallel subchannel complex QAM modulations. The continuous-time complex envelope of the OFDM/QAM signal, in general, has the form

$$x(t) = \sum_{k=0}^{N-1} x_k(t) = \sum_{k=0}^{N-1} w(k) \sum_{l} \gamma_{k,l} q(t - lT) e^{j2\pi \Delta f k t} \tag{6}$$

where T is duration of a complete OFDM/QAM symbol with CP of duration T_c, N is the number of subcarriers spaced with $\Delta f = 1/(T - T_c)$, $q(t)$ is the transmitter pulse shaping filter impulse response, $w(k)$ is subcarrier weighting function, and $\gamma_{k,l}$ are complex-valued data symbols. It is assumed that information symbol sequence $(\gamma_{k,l})$ is independently and identically distributed (i.i.d.) with variance σ^2, i.e. $\mathrm{E}\{\gamma_{k,l}\gamma^*_{k',l'}\}$ (E denotes expectation operator, and $\delta_{n,k}$ is the Kronecker delta function).

A time-frequency well-localized pulse shaping filter with impulse response $q(t)$ is said to be orthogonal if it guarantees perfect ICI and ISI cancellation [3]. The design of orthogonal filters with bandwidth $\leq 2/T$ is equivalent to design the square-root Nyquist filter [3].

The complex-valued information symbol sequence $\{\gamma_{k,l}\}$ takes M values, i.e. $\gamma_{k,l} \in \{c_m\}_{m=1}^{M}$, where in- phase components, $a_m = \Re\{c_m\}$, and quadrature components, $b_m = \Im\{c_m\}$, of M-ary complex symbols (a_m, b_m) are arranged in a rectangular signal constellation.

4 Spectral Correlation Characterization of OFDM Signal

The cyclic spectrum of pulse shaping OFDM/QAM signal $y(t)$, given by (5), if the complex envelope $x(t)$ is CS, can be expressed as [5], [6]

$$\begin{aligned} S_{yy}^{\alpha}(f) =& \frac{1}{4}[S_{xx}^{\alpha}(f-f_c) + S_{xx}^{-\alpha}(-f-f_c)^* \\ &+ S_{xx^*}^{\alpha-2f_c}(f) + S_{xx^*}^{-\alpha-2f_c}(-f_c)^*], \end{aligned} \tag{7}$$

where $S_{xx}^{\alpha}(f)$ and $S_{xx^*}^{\alpha}(f)$ are the cyclic spectrum and the conjugate cyclic spectrum of the complex envelope $x(t)$, respectively. It can be seen that the cyclic spectrum $S_{xx}^{\alpha}(f)$ and the conjugate cyclic spectrum $S_{xx^*}^{\alpha}(f)$ of the complex envelope $x(t)$ determine parts of the cyclic spectrum $S_{yy}^{\alpha}(f)$ of the digital carrier-modulated signal $y(t)$ at cycle frequencies associated with the symbol rate and associated with the doubled carrier frequency, respectively [5].

The complex envelope $x(t)$ of the OFDM/QAM signal can be represented as the sum of N statistically independent subchannel QAM modulated signals $\{x_k(t)\}_{k=0}^{N-1}$, i.e.

$$x(t) = \sum_{k=0}^{N-1} x_k(t), \qquad x_k(t) = w(k)e^{j2\pi\Delta fkt}v(t) \tag{8}$$

where

$$v(t) = \sum_{l} \gamma_{k,l}q(t-lT) \tag{9}$$

is the complex envelope of QAM signals.

It can be easily proved that autocorrelation, cyclic autocorrelation and cyclic spectrum of the complex QAM signals $\{x_k(t)\}_{k=0}^{N-1}$, (8, 9), are given as

$$\begin{aligned} \mathcal{R}_{x_kx_k}(t,\tau) =& w^2(k)e^{j2\pi\Delta fk\tau}\mathcal{R}_{vv}(t,\tau) \\ \mathcal{R}_{x_kx_k}^{\alpha}(\tau) =& w^2(k)e^{j2\pi\Delta fk\tau}\mathcal{R}_{vv}^{\alpha}(\tau) \\ S_{x_kx_k}^{\alpha}(f) =& w^2(k)S_{vv}^{\alpha}(f-k\Delta f) \end{aligned} \tag{10}$$

Because all of QAM signals $\{x_k(t)\}_{k=0}^{N-1}$ are independent of each other, corresponding functions for composite complex envelope $x(t)$ of the

OFDM/QAM signal are

$$
\begin{aligned}
\mathcal{R}_{xx}(t,\tau) &= \sum_{k=0}^{N_1} \mathcal{R}_{x_k x_k}(t,\tau) = \mathcal{R}_{vv}(t,\tau) \sum_{k=0}^{N-1} w^2(k) e^{j2\pi\Delta f k\tau} \\
\mathcal{R}_{xx}^{\alpha}(\tau) &= \sum_{k=0}^{N_1} \mathcal{R}_{x_k x_k}^{\alpha}(t,\tau) = \mathcal{R}_{vv}^{\alpha}(\tau) \sum_{k=0}^{N-1} w^2(k) e^{j2\pi\Delta f k\tau} \\
S_{xx}^{\alpha}(f) &= \sum_{k=0}^{N-1} w^k(k) S_{vv}^{\alpha}(f - k\Delta f)
\end{aligned}
\tag{11}
$$

Thus, the problem of the cyclic spectral analysis of the OFDM/QAM signal is reduced to cyclic spectral analysis of the complex envelope $v(t)$ of the QAM signal.

In the discrete-time signal model, a critically sampled version of $x(t)$ is obtained by taking samples at times $t = nT_s = n/(N\Delta f) = n(T - T_c)/N$. Then, delays equal to half the sampling increment are not allowed and asymmetric definition of autocorrelation is used for discrete-time signals: $\hat{\mathcal{R}}(n,\tau) = \mathrm{E}\{x(n)x^*(n-\tau)\}$, where τ is an integer lag parameter and $x(n)\underline{\underline{\Delta}}x[n/(N\Delta f)]$. Applying this discrete-time definitions on sampled version of complex envelope $x(t)$ of OFDM/QAM signal, the following discrete-time version of autocorrelation is obtained

$$
\hat{\mathcal{R}}_{xx}(n,\tau) = \sigma_\gamma^2 \sum_{k=0}^{N-1} w^2(k) e^{j\frac{2\pi}{N}k\tau} \sum_l q(n - lP) q(n - \tau - lP) \tag{12}
$$

where P is the complete OFDM/QAM symbol length ($T = PT_s$). In general, from (12) it follows, in discrete-time version, that autocorrelation of complex envelope of the OFDM/QAM signal is periodic with period P in time n for every τ. It is, in continuous-time version, equivalent to periodicity with period T in time t. Therefore, in general, the OFDM/QAM signal is CS with period $T(P)$. It is easily seen that discrete-time OFDM/QAM signal is stationary only in the case of no time-frequency guard regions ($T_c = 0$; i.e. $N = P$), no pulse shaping (i.e. $q(t)$ is a rectangular pulse) and no subcarrier weighting (i.e. $|w(k)| = 1$, $k = 1, \ldots, N-1$) [3], [4]. However, insertion of a CP, subcarrier weighting or pulse shaping (nonoverlapping or overlapping) induces cyclostationarity of OFDM/QAM signals [2].

4.1 Markov chain QAM signal model

Previously described complex envelope $v(t)$ of subcarrier QAM signal can be expressed in matrix form as [5], [6]

$$v(t) = \sum_k \boldsymbol{\varepsilon}_k \boldsymbol{g}^T(t-kT) = \sum_k \boldsymbol{g}(t-kT)\boldsymbol{\varepsilon}_k^T \tag{13}$$

where $(\boldsymbol{\varepsilon}_k)$ is an aperiodic homogeneous Markov stationary vector-valued chain which takes values from M-dimensional unit-basis vector space, i.e. $\boldsymbol{\varepsilon}_k \in \{\boldsymbol{e}_i\}_{i=1}^m$, $\boldsymbol{e}_i = [\delta_{i1}, \delta_{i2}, \ldots, \delta_{iM}]$. $\boldsymbol{g}(t)$ is the state vector pulse (the signaling pulse vector) whose components are signaling waveforms $\{g_i(t)\}_{i=1}^M$ associated with each state in which chain remains for T seconds. Thus, Markov chain $(\boldsymbol{\varepsilon}_k)$ is a vector indicator of signaling waveforms $\{g_i(t)\}_{i=1}^M$ associated with information symbols $\gamma_{k,l} \in \{c_m\}_{m=1}^M$.

Analyzing QAM rectangular signal constellation, the values of the complex symbols $\gamma_{k,l}$, and the form of complex envelope $v(t)$ it can be noticed that the signaling pulse vector $\boldsymbol{g}(t)$ can be represented in the form

$$\boldsymbol{g}(t) = [\boldsymbol{p}(t), -\boldsymbol{p}^*(t), -\boldsymbol{p}(t), \boldsymbol{p}^*(t)]. \tag{14}$$

Dimensions of the vector pulses $\boldsymbol{g}(t)$ and $\boldsymbol{p}(t)$ are $1 \times M$ and $1 \times (M/4)$, respectively. The vector pulse $\boldsymbol{p}(t)$ corresponds to I-quadrant signal points of QAM signal space and has the form

$$\boldsymbol{p}(t) = q(t)[c_1, c_2, \ldots, c_{\frac{M}{4}}]. \tag{15}$$

This Markov QAM signal model can be completely described by $1 \times M$-dimensional initial state probability vector $\boldsymbol{w}^{(0)} = [w_i^{(0)}] = [\Pr\{\boldsymbol{\varepsilon}_0 = \boldsymbol{e}_i\}]$, $M \times M$-dimensional state probability transition matrix $\boldsymbol{P} = [\Pr\{\varepsilon_{n+1} = \boldsymbol{e}_j/\boldsymbol{\varepsilon}_n = \boldsymbol{e}_j\}]$, and the state vector pulse $\boldsymbol{g}(t)$ given by (14). The appropriate state transition probability matrix $\boldsymbol{P}$ and the initial state probability vector $\boldsymbol{w}^{(0)}$, for the statistically independent and equiprobable symbols $\gamma_{k,l}$ are given by

$$\boldsymbol{P} = \frac{1}{M}[\mathbf{1}_{M\times m}], \qquad \boldsymbol{w}^{(0)} = \frac{1}{M}[\mathbf{1}_{1\times M}], \tag{16}$$

where $\mathbf{1}_{n\times m}$ denotes $n \times m$- dimensional matrix (vector) having all elements equal to unity. The stationary state probability vector $\boldsymbol{w} = [w_i]_{1\times M} = \Pr[\{\boldsymbol{\varepsilon}_n = \boldsymbol{e}_i\}] = \lim_{k\to\infty} \boldsymbol{w}^{(0)}\boldsymbol{P}^k$ is identical to $\boldsymbol{w}^{(0)}$. The joint probability matrix of $(\boldsymbol{\varepsilon}_k)$ is given as $\boldsymbol{W}_k = [\Pr\{\boldsymbol{\varepsilon}_n = \boldsymbol{e}_i, \boldsymbol{\varepsilon}_{n+k} = \boldsymbol{e}_i\}]_{M\times M} = \boldsymbol{W}_0\boldsymbol{P}^k$,

$k \geq 1$, $\boldsymbol{W}_{-k} = \boldsymbol{W}_k^T$ where $\boldsymbol{W}_0 = \mathrm{diag}[(\boldsymbol{w}]$ is the diagonal matrix of stationary state probabilities $\{w_i\}_{i=1}^M$. It can be shown [6] that the mean and autocorrelation of Markov chain $(\boldsymbol{\varepsilon}_k)$ are given by $\boldsymbol{\mu}_\varepsilon = \mathrm{E}\{\boldsymbol{\varepsilon}\} = \boldsymbol{w}$ and $\mathcal{R}_\varepsilon(k) = \mathrm{E}\{\varepsilon_n^T \varepsilon_{n+k}\} = \boldsymbol{W}_k$, respectively.

4.2 Cyclic spectrum of QAM signal

Applying definitions in Section 2 on described Markov chain model of the complex envelope $v(t)$ of QAM signal (described in subsection 4.1) and following the procedure as in [5], [6], cyclic spectrum $S_{vv}^\alpha(f)$ and conjugate cyclic spectrum $S_{vv^*}^\alpha(f)$ can be obtained

$$S_{vv}^\alpha(f) = \begin{cases} \frac{1}{MT} Q^*(f - \frac{\alpha}{2}) Q(f + \frac{\alpha}{2}) \sum\limits_{m=1}^{M} c_m^* c_m, & \alpha = \frac{n}{T} \\ 0, & \alpha \neq \frac{n}{T} \end{cases} \tag{17}$$

$$S_{vv^*}^\alpha(f) = \begin{cases} \frac{1}{MT} Q(-f + \frac{\alpha}{2}) Q(f + \frac{\alpha}{2}) \sum\limits_{m=1}^{M} c_m^2 & \alpha = \frac{n}{T} \\ 0, & \alpha \neq \frac{n}{T} \end{cases} \tag{18}$$

where $Q(f)$ is the Fourier transform of shaping pulse $q(t)$. For the QAM, the last factor in eqn. (18) is $\sum_{m=1}^M c_m^* c_m = M\sigma_\gamma^2$ and, because of a rectangular QAM signal point space the sum $\sum_{m=1}^M c_m^2$ in eqn. (18) differs from zero only for $M = 2$. Therefore, considering eqn. (7), QAM signals ($M \geq 4$) does not exhibit spectral correlation at frequencies associated with the doubled carrier frequency $\alpha = \pm 2f_c + n/T$, but only at $\alpha = n/T$, instead.

4.3 Cyclic spectrum of pulse shaping OFDM/QAM signal

Substitution of the above results into eqn. (11) yields explicit formula for the cyclic spectrum of the complex envelope $x(t)$ of pulse shaping OFDM/QAM signal

$$S_{xx}^\alpha(f) = \begin{cases} \frac{\sigma_\gamma^2}{T} \sum\limits_{k=1}^{N-1} w^2(k) Q(f - \frac{k}{T - T_c} + \frac{\alpha}{2}) Q^*(f - \frac{k}{T - T_c} - \frac{\alpha}{2}), & \alpha = \frac{n}{T} \\ 0, & \alpha \neq \frac{n}{T} \end{cases} \tag{19}$$

Finally, based on expression (7), the cyclic spectrum of pulse shaping

OFDM/QAM signal $y(t)$, given by (5), is obtained in the explicit form

$$S_{yy}^{\alpha}(f)=\frac{\sigma_{\gamma}^{2}}{4T}\sum_{k=0}^{N-1}w^{2}(k)\Big[Q(f-\frac{k}{T-T_c}-f_c+\frac{\alpha}{2})Q^{*}(f-\frac{k}{T-T_c}-f_c-\frac{\alpha}{2}) \\ +Q(f+\frac{k}{T-T_c}+f_c+\frac{\alpha}{2})Q^{*}(f+\frac{k}{T-T_c}+f_c+\frac{\alpha}{2})\Big],\alpha=\frac{n}{T} \tag{20}$$

where $Q(f)$ is the Fourier transform of shaping pulse $q(t)$.

5 Simulation results

The following figures show some results of continuous-time cyclic spectral analysis of pulse shaping OFDM/QAM. The cyclic spectrum magnitude for the complex envelope of OFDM/QAM signal ($N = 16$) in the case of no pulse shaping, no subcarrier weighting and with CP ($T_c = 0.2T$) is shown in Fig. 1, and without CP ($T_c = 0$) in Fig. 2. Fig. 3 and Fig. 4 show the cyclic spectrum magnitude for OFDM/QAM signal with pulse shaping square-root Nyquist filter with rolloff factor $r = 0.5$ and no CP, and for general case of employing CP ($T_c = 0.2T$) and the same pulse shaping square-root Nyquist filter, respectively. On these figures one can notice the above mentioned characteristic cyclic features of OFDM signals.

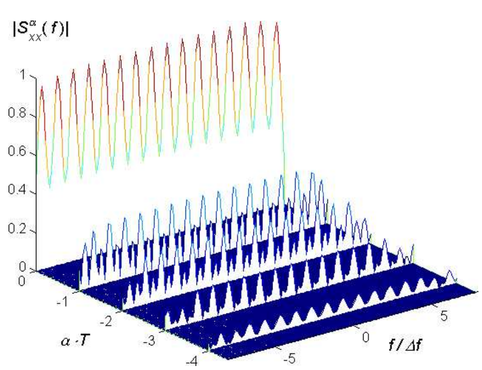

Fig. 1. Cyclic spectrum for the complex envelope of no pulse shaping OFDM/QAM signal ($N = 16$) with cyclic prefix.

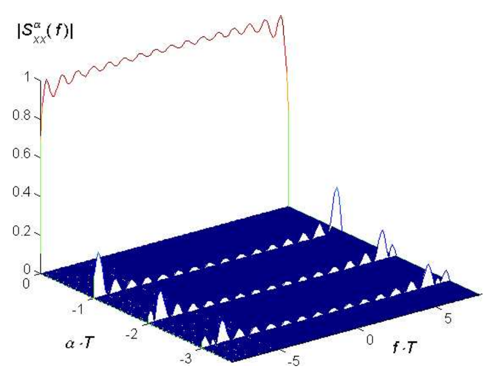

Fig. 2. Cyclic spectrum for the complex envelope of no pulse shaping OFDM/QAM signal ($N = 16$) without cyclic prefix.

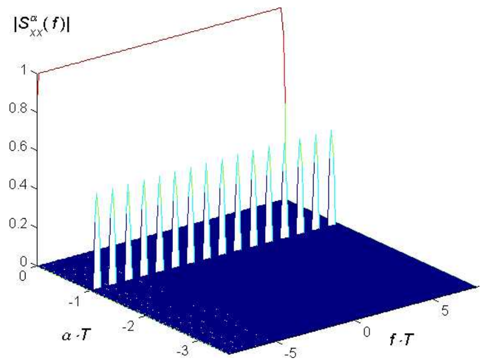

Fig. 3. Cyclic spectrum for the complex envelope of square-root Nyquist filter pulse shaping OFDM/QAM signal ($N = 16$) without cyclic prefix.

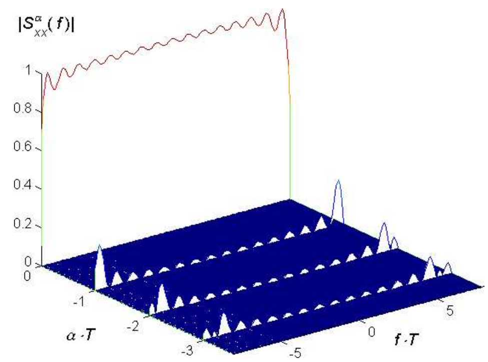

Fig. 4. Cyclic spectrum for the complex envelope of square-root Nyquist filter pulse shaping OFDM/QAM signal ($N = 16$) with cyclic prefix.

6 Conclusion

The continuous-time and discrete-time spectral correlation characterization of the pulse shaping OFDM/QAM signals are performed by the proposed matrix-based stochastic method based on Markov chain signal representation. This cyclic spectral analysis shows that discrete-time OFDM/QAM signal is stationary only in the case of no time-frequency guard regions, no pulse shaping and no subcarrier weighting. Even in this case, the continuous-time OFDM/QAM signal exhibits cyclostationarity, although it's cyclic features are very suppressed. However, insertion of a cyclic prefix, subcarrier weighting or pulse shaping always induces cyclostationarity of OFDM/QAM signals. Some similar results were obtained, by other means, in [2].

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