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Power-Complementary IIR Filter Pairs with an Adjustable Crossover Frequency

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Abstract: This paper introduces two classes of power-complementary recursive low-pass/high-pass filter pairs with an adjustable crossover frequency in such a way that the stopband attenuation of both filters remains the same. For each class, the filter pair is constructed using two all-pass sub-filters as building blocks. Based on the properties elliptic minimal Q-factors transfer functions, simple expressions are derived for evaluating the coefficients in all-pass sections in order to achieve the desired crossover frequency. The design procedures are developed for synthesizing power-complementary filter pairs implemented as a parallel connection of two all-pass sub-filters and for the tapped cascaded interconnections of two identical all-pass sub-filters. The direct parallel connection has both the power-complementary and all-pass complementary property. The second class of filters constructed using several identical copies of the two all-pass filters possesses the power-complementary property

Keywords: Signal processing, IIR filter, half band filter, filter pair, latice wave digital filter, all pass filter.

1 Introduction

Complementary infinite-impulse response (IIR) filter pairs can be synthesized to generate power-complementary, all-pass complementary, or magnitude-complementary filter pairs (see, e.g., [1]). A very attractive alternative to

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generate a power-complementary low-pass/high-pass IIR filter pair is to use lattice wave digital (LWD) filters (parallel connections of two all-pass filters) [2]-[4]. Usually, the intersection for a low-pass/high-pass filter pair occurs in the middle of the base-band, that is, the crossover frequency is located exactly at 1/4 in terms of the normalized frequency. If the stop-band attenuation of both filters is the same, then the low-pass and high-pass filters are both half-band IIR filters (see, e.g., [4]-[5]).

This paper introduces a technique to change, by means of simple formulae, the location of the crossover frequency of the half-band filter pair to an arbitrary location while still retaining the attenuation properties of the initial half-band filter. For this purpose, elliptic minimal Q-factors (EMQF) transfer functions introduced in [6]-[8] provide directly the desired solution.

In addition to the direct parallel connections of two all-pass filters, the structure constructed as a tapped cascaded interconnection of two identical all-pass filters is considered based on the use of the synthesis schemes described in [9]-[12]. This structure allows one to generate power-complementary filter pairs with an adjustable crossover frequency using very low-order all-pass filters.

2 Use of EMQF Filters for Generating Power-Complementary Filter Pairs with an Adjustable Crossover Frequency

This section shows how the properties of EMQF filters [6]-[8] can be exploited in a very straightforward manner for generating power-complementary IIR filter pairs with an adjustable crossover frequency such that the stopband attenuation remains the same. Due to the fact that EMQF filters comprise a half-band filter as a special case, we start with a half-band filter pair to generate the desired complementary filter pair whose crossover frequency can be arbitrarily chosen.

2.1 Properties of EMQF filters for generating power-complementary filter pairs

Complementary IIR filter pair constructed as a parallel connection of two all-pass sub-filters is shown in Figure 1. For half-band IIR filters, the complementary filter pair is given by (see, e.g., [4], [5])

$$G^{HB}(z) = \frac{1}{2}\left[A_0^{HB}(z) \pm A_1^{HB}(z)\right]$$
$$= \frac{1}{2}\Big[\prod_{i=2,4,\ldots}^{\frac{N+1}{2}} \frac{\beta_i^{HB} + z^{-2}}{1+\beta_i^{HB} z^{-2}} \pm z^{-1} \prod_{i=3,5,\ldots}^{\frac{N+1}{2}} \frac{\beta_i^{HB} + z^{-2}}{1+\beta_i^{HB} z^{-2}}\Big], \beta_i^{HB} < \beta_{i+1}^{HB}. \tag{1}$$

Here, N, the filter order, is restricted to be odd and $G^{HB}(z) = [G_{LP}^{HB}(z)\ G_{HP}^{HB}(z)]^T$ is a vector denoting the low-pass (with the plus sign) and high-pass (with the minus sign) half-band filter transfer functions $G_{LP}^{HB}(z)$ and $G_{HP}^{HB}(z)$. The superscript 'HB' is used to emphasize that the filters under consideration are half-band filters. One of the poles of the above transfer functions is located at the origin and the remaining poles are complex-conjugate pairs being located on the imaginary axis at $z = jr_i$ for $i = 2, 3, \ldots, (N+1)/2$, giving $\beta_i^{HB} = (r_i)^2$. In Eq. (1), the notation $\beta_i^{HB} < \beta_{i+1}^{HB}$ indicates how the poles are shared between the two all-pass sections.

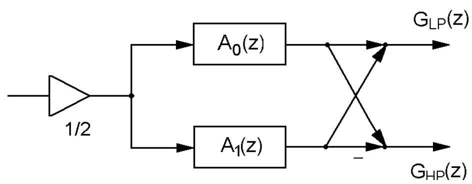

Fig. 1. Double-complementary IIR filter pair constructed as a parallel connection of two all-pass filters.

This filter pair when generated by means of an odd-order elliptic filter is characterized by the following attractive properties. First, the sum of the squared-magnitude responses of the two filters is identically equal to unity. Hence, the filter pair has the power-complementary property. Second, the 3-dB crossover frequency (the frequency where the squared-magnitude responses of both $G_{LP}^{HB}(z)$ and $G_{HP}^{HB}(z)$ achieve the value of 1/2) is located at $f_{3dB} = 1/4$ in terms of the normalized frequency. Third, the passband and stopband edges, denoted by f_p and f_s in terms of the normalized frequency, satisfy $f_s = 1/2 - f_p$, that is, the distances of the edges from the center of the base-band are the same. Fourth, the attenuation of both the low-pass and high-pass half-band filters is the same. Since the sum of the transfer functions $G_{LP}^{HB}(z)$ and $G_{HP}^{HB}(z)$ is $A_0^{HB}(z)$, the resulting filter pair is also an all-pass complementary filter pair, thereby called as a double-complementary filter pair [1].

The low-pass/high-pass EMQF filter pair is generated according to the half-band filter pair, as given by Eq (1), as follows

$$G^{HB}(z) = \frac{1}{2}\left[A_0^{HB}(z) \pm A_1^{HB}(z)\right]$$
$$= \frac{1}{2}\Big[\prod_{i=2,4,\ldots}^{\frac{N+1}{2}} \frac{\beta_i + \alpha(1+\beta_i)z^{-1} + z^{-2}}{1 + \alpha(1+\beta_i)z^{-1} + \beta_i z^{-2}} \tag{2}$$
$$\pm \frac{\alpha_1 + z^{-1}}{1 + \alpha_1 z^{-1}} \prod_{i=3,5,\ldots}^{\frac{N+1}{2}} \frac{\beta_i + \alpha(1+\beta_i)z^{-1} + z^{-2}}{1 + \alpha(1+\beta_i)z^{-1} + \beta_i z^{-2}}\Big], \beta_i < \beta_{i+1}.$$

where $G(z) = [G_{LP}(z)\ G_{HP}(z)]^T$ is again a vector denoting the resulting low-pass and high-pass filter transfer functions $G_{LP}(z)$ and $G_{HP}(z)$ that can be implemented as shown in Figure 1. The simple formulae for converting the delay term z^{-1} and the second-order all-pass sections in Eq. (1) to the first-order and the second-order sections in Eq. (2) will be given later on in this section. It should be pointed out that when using LWD filters for the implementation, the adapter coefficient is $\gamma = -\alpha_1$ for the first-order all-pass section. For the second-order all-pass sections, the corresponding adapter coefficients are given by $\gamma_1 = \beta_i$ and $\gamma_2 = -\alpha$ (see, e.g., [4]). There exists the following three very attractive properties of EMQF filters guaranteeing that for the resulting EMQF filter pair, as given by Eq. (2), the crossover frequency can be changed while keeping the attenuation of the low-pass and high-pass filters the same as for the start-up half-band filter pair. Furthermore, the double-complementary property is preserved. First, these filters are generated using half-band IIR filters as start-up solutions [6]-[8]. Second, instead of directly applying the well known low-pass-to-low-pass transformation or low-pass-to-high-pass transformation, the filters with the given pass-band and stop-band edges are generated using simple formulae. Third, if a low-pass or a high-pass filter is generated first, then it is guaranteed that the filters in the resulting low-pass/high-pass filter pair have exactly the same attenuation in the stop-band regions. This is due to the fact the start-up filter is an IIR half-band filter.

2.2 Design procedure

Based on the properties of EMQF filters, a power-complementary filter pair with the given odd order N, the given 3-dB crossover frequency f_{3dB}, and the given minimum stop-band attenuation A_s in decibels can be carried out in the following steps

1. Synthesize the start-up power-complementary half-band IIR filter pair, as given by Eq. (1), such that the minimum attenuation for both the low-pass and high-pass filter transfer functions $G_{LP}^{HB}(z)$ and $G_{HP}^{HB}(z)$ is exactly A_s. Let the corresponding normalized cutoff frequencies be f_p^{HB} and $f_s^{HB} = 1/2 - f_p^{HB}$.

2. Determine the so-called selectivity factor $\xi = \tan(\pi f_s^{HB})/\tan(f_p^{HB})$.

3. Synthesize the desired power-complementary IIR filter pair, as given by Eq. (2), by determining its parameters according to the formulae given in Table 1.

Table 1. Parameters for a power-complementary filter pair with the given f_{3dB} cutoff frequency.

First-order Section	$\alpha_1 = -\dfrac{1-\tan(\pi f_{3dB})}{1+\tan(\pi f_{3dB})}$	
Second-order Section	$\alpha = -\dfrac{1-\tan^2(\pi f_{3dB})}{1+\tan^2(\pi f_{3dB})}$	$\beta_i = \dfrac{\beta_i^{HB}+\alpha_1^2}{\beta_i^{HB}\alpha_1^2+1}$

2.3 Properties of the resulting filter pairs

The above procedure results in the power-complementary filter pair with the following properties:

1. The crossover frequency being related to the common constant α in Eq. (2) or in Table 1 as follows:

$$f_{3dB} = \frac{\cos^{-1}(-\alpha)}{2\pi} \tag{3}$$

   is located exactly at the given frequency point.

2. The normalized cutoff frequencies are given by

$$\begin{aligned} f_p &= \frac{1}{\pi}\tan^{-1}\left[\frac{\tan(\pi f_{3dB})}{\sqrt{\xi}}\right] \\ f_s &= \frac{1}{\pi}\tan^{-1}\left[\sqrt{\xi}\tan(\pi f_{3dB})\right] \end{aligned} \tag{4}$$

3. The minimum stop-band attenuation for both $G_{LP}(z)$ on the interval $[f_s, 1/2]$ and $G_{HP}(z)$ on the interval $[0, f_p]$ is exactly A_s.

4. The parameter α is the common constant for all the second-order all-pass sections in Eq. (2).

2.4 Numerical example

Consider power-complementary filter pairs of order $N = 7$ and having $A_s =$ 60 dB. The orders of the all-pass sections are thus 4 and 3. Figure 2 shows the responses of the overall bank for some values of the crossover frequency. The thick line is used for the start-up half-band filter pair. For each new filter pair, only 5 constant values have to be computed using the expressions of Table 1.

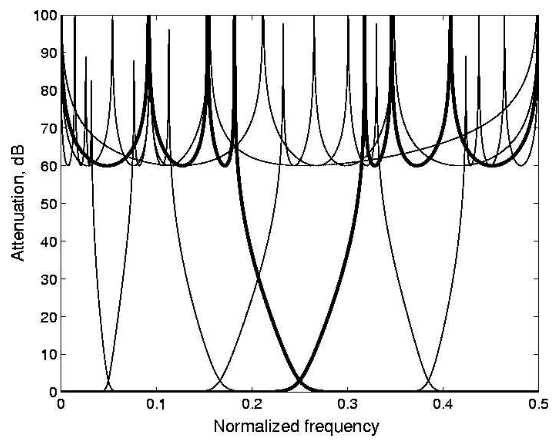

Fig. 2. Family of seventh-order power-complementary filter pairs for $A_s = 60$ dB.

3 Generating Power-Complementary IIR Filter Pairs Using a Tapped Cascaded Interconnection of Two Identical All Pass Filters

This section shows how the synthesis techniques described in [9], [11] can be used for generating power-complementary IIR filter pairs with an adjustable crossover frequency. In this case, the overall structure is generated as a tapped cascaded interconnection of two identical all-pass filters and again the properties of EMQF filters are exploited.

3.1 Proposed filter pair and efficient implementation

In this case, the low-pass and high-pass transfer functions are given by

$$H_{LP}(z) = \sum_{m=0}^{M} a[m][A_0(z)]^m[A_1(z)]^{M-m} \qquad (5)$$

and

$$H_{HP}(z) = \sum_{m=0}^{M} (-1)^m a[M-m][A_0(z)]^m [A_1(z)]^{M-m}, \tag{6}$$

where M is an odd integer and $A_0(z)$ and $A_1(z)$ are the all-pass filters in Eq. (1) or (2), that is, $G_{LP}(z) = [A_0(z) + A_1(z)]/2$ is a low-pass filter of an odd order N. The above power-complementary filter pair can be implemented using the lattice structure shown in Figure 3 [11]. The details on how to convert the $a[m]$'s to the k_l's can be found, e.g., in [11].

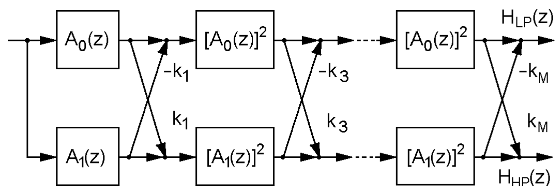

Fig. 3. Lattice structure for the proposed power-complementary filter pair.

3.2 Design procedure

Given the minimum stopband attenuation A_s, the overall synthesis problem is to find the $M+1$ coefficients $a[m]$ and the Nth-order low-pass half-band filter[1] $G_{LP}^{HB}(z) = [A_0^{HB}(z) + A_1^{HB}(z)]/2$, as given by Eq. (1), such that the following conditions are met:

Condition 1: The minimum attenuation for both the low-pass and high-pass filter transfer functions $H_{LP}(z)$ and $H_{HP}(z)$ is exactly A_s.

Condition 2: The normalized cutoff frequencies f_p^{HB} and f_s^{HB} are related via $f_s^{HB} = 1/2 - f_p^{HB}$ and f_p^{HB} is maximized.

The desired coefficients $a[m]$ can be found as follows:

1. Determine $\Delta = 10^{-A_s/10}$.
2. Optimize the $a[m]$'s to maximize θ_p such that the amplitude response of $F(z) = \sum_{m=0}^{M} a[m]z^{-m}$ stays within the limits 1 and $\sqrt{1-\Delta}$ in the normalized pass-band region $[0, \theta_p]$ and the maximum amplitude value is $\sqrt{\Delta}$ in the normalized stop-band region $[1/2 - \theta_p, 1/2]$.

Note that $F(z)$ is the transfer function of a minimum-phase finite-impulse response (FIR) filter. The above procedure can be accomplished by slightly modifying the design scheme described in [9].

[1] In this case, the all-pass sections in Fig. 3 are $A_0^{HB}(z)$ and $A_1^{HB}(z)$.

What is left is to design the power-complementary half-band filter pair, as given by Eq. (1), such that the above-mentioned two conditions are met and, then, to transfer the crossover frequency f_{3dB} to the desired frequency according to the discussion of Subsection 2.2. This can be accomplished by using the design procedure described in Subsection 2.2. The main difference is that now, due to the use of several copies of the two identical all-pass sections, the required minimum stop-band attenuation is, instead of the original A_s, given as follows (see [9] for details):

$$\hat{A}_s = -10\log_{10}\{\cos[\pi(\frac{1}{2} - \theta_p)[\}. \tag{7}$$

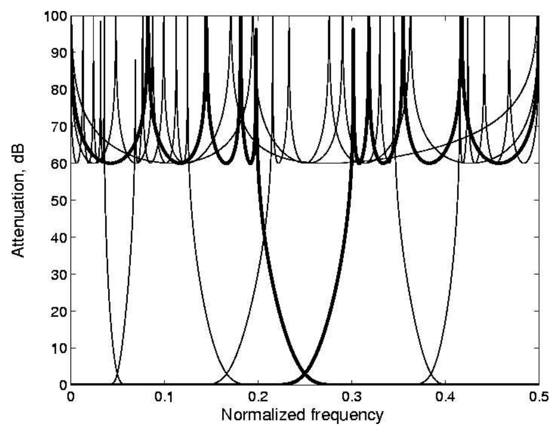

Fig. 4. Power-complementary filter pairs constructed using 5 identical copies of second-order and first-order all-pass sections. $A_s = 60$ dB.

3.3 Numerical example

Consider power-complementary filter pairs that are constructed using $M = 5$ identical copies of two all-pass filters of order 2 and 1, that is, $N = 3$. The desired attenuation is $A_s = 60$ dB. Figure 4 displays the responses of the overall bank for some values of the crossover frequency. The thick line is used for the start-up filter pair. For each new filter pair, only 3 constant values have to be computed. The values of the additional tap coefficients stay unchanged. The required attenuation of the parallel connection of the first-order and second-order all-pass filters is only 19.37 dB.

3.4 Comments

This paper has concentrated on designing power-complementary filter pairs. It has shown in [13] that magnitude complementary (all-pass-complementary) filter pairs with an adjustable crossover frequency can be designed in a similar manner by adopting the design approach described in [10].

4 Conclusion

This paper has introduced a new approach for designing complementary IIR filter with an adjustable crossover frequency. The start-up complementary filters are low-pass/high-pass half-band filter pairs constructed as parallel connections of two all-pass filter sections (lattice wave digital filters). Exploiting the properties of EMQF (elliptic minimal Q-factors) transfer functions, simple formulae for the direct calculation of the coefficients have been derived. This allows one to implement programmable complementary filter banks with an adjustable crossover frequency in a very simple and straightforward manner.

The particular benefits have been obtained for the realization structures based on the tapped cascaded interconnection of identical all-pass sub-filters leading to power-complementary filter pairs. For these structures, the all-pass sub-filters are of a very low order, thereby making the adjustment of the crossover frequency significantly easier.

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References

[1] S. K. Mitra, *Digital Signal Processing, A Computer-Based Approach.* New York: McGraw-Hill, 2001.

[2] R. Nouta, "The Jaumann structure in wave-digital filters," *Int. J. Circuit Theory Applicat.*, vol. 2, pp. 163–174, June 1974.

[3] A. Fettweis, H. Levin, and A. Sedlmeyer, "Wave digital lattice filters," *Int. J. Circuit Theory Applicat.*, vol. 2, pp. 203–211, June 1974.

[4] L. Gazsi, "Explicit formulas for lattice wave digital filters," *IEEE Trans. Circuits and Syst.*, vol. CAS-32, pp. 203–211, January 1985.

[5] M. Renfors and T. Saramäki, "Recursive nth-band digital filters - part I: Design and properties, part II: Design of multistage decimators and interpolators," *IEEE Trans. Circuits and Syst.*, vol. CAS-34, pp. 24–51, January 1987.

[6] M. D. Lutovac, D. V. Tosić, and B. L. Evans, *Filter Design for Signal Processing Using MATLAB and Mathematica.* Upperside River, NJ: Prentice-Hall, 2000.

[7] M. D. Lutovac and L. D. Milić, "Design of computationally efficient elliptic IIR filters with a reduced number of shift-and-add operations in multipliers," *IEEE Trans. Signal Processing*, vol. 45, Oct. 1997.

[8] L. D. Milić and M. D. Lutovac, "Design of multiplierless elliptic IIR filters with a small quantization error," *IEEE Trans. Signal Processing*, vol. 47, Feb. 1999.

[9] T. Saramäki and M. Renfors, "A novel approach for the design IIR filters as a tapped cascaded interconnection of identical allpass subfilters," in *Proc. 1987 IEEE Int. Symp. Circuits Syst*, (Philadelphia, Pennsylvania), pp. 629–632, May 1987.

[10] H. Johansson and T. Saramäki, "A class of complementary IIR filters," in *Proc. 1999 IEEE Int. Symp. Circuits Syst*, (Orlando, Florida), pp. 299–302, July 1999.

[11] H. Johansson and L. Wanhammar, "High-speed recursive filter structures composed of identical all-pass sub-filters for interpolation, decimation, and QMF banks with perfect magnitude reconstruction," *IEEE Trans. Circuits and Systems-II: Analog and Digital Signal Processing*, pp. 16–28, Jan. 1999.

[12] L. Milić and M. Lutovac, "High speed IIR filters for qmf banks," in *Proc. Int. Symp. TELSIKS'2001*, (Niš, Yugoslavia), pp. 171–174, 2001.

[13] L. Milić and T. Saramäki, "Three calsses og IIR complementary filter pairs with an adjustable crossover frequency," in *Proc. 2003 Int. Symp. Circuits and System, ISCAS'03*, (Bankok, Thailand), pp. 145–148, May 2003.