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Structure Optimization of Electrical Power Network Using Ant Colony Approach

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Abstract: This paper describes and uses an ant colony meta-heuristic optimization method to solve the redundancy optimization problem. This problem is known as total investment-cost minimization of series-parallel power system configuration. Redundant components are included to achieve a desired level of availability. System availability is represented by a multi-state availability function. The power systems components are characterized by their performance (capacity), availability and cost. These components are chosen among a list of products available on the market. The proposed meta-heuristic seeks to the best minimal cost power system configuration with desired availability. To estimate the series-parallel power system availabiliti, a fast method based on universal moment generating function (UMGF) is suggested. The ant colony approach is used as an optimization technique. An example of electrical power system is presented.

Keywords: Power engineering, ant colony, redundancy optimization, multi-state systems, universal generating function (UMGF).

1 Introduction

One of the most important problems in many industrial applications is the redundancy optimization problem. This latter is well known combinatorial optimization problem where the design goal is achieved by discrete choices

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made from components available on the market. The natural objective function is to find the minimal cost configuration of a series-parallel power system under availability constraints. The system is considered to have a range of performance levels from perfect working to total failure. In this case the system is called a multi-state system (MSS). Let consider a multi-state system containing n subsystems $C_i (i = 1, 2, \ldots, n)$ in series arrangement. For each subsystem C_i there are various versions, which are proposed by the suppliers on the market. Components are characterized by their cost, performance and availability according to their version. For example, these components can represent machines in a manufacturing system to accomplish a task on product in our case they represent the whole of electrical power system (generating units, transformers and electric carrying lines devices). Each subsystem C_i contains a number of components connected in parallel. Different versions of components may be chosen for any given subsystem. Each subsystem can contain components of different versions as sketched in Fig.1.

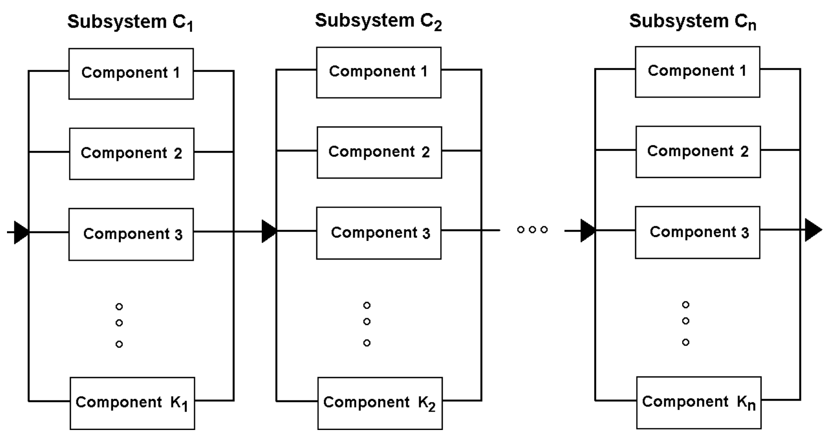

Fig. 1. Series-parallel power system structure.

A limitation can be undesirable or even unacceptable, where only identical components are used in parallel (i.e. homogeneous system) for two reasons. First, by allowing different versions of the devices to be allocated in the same system, one can obtain a solution that provides the desired availability or reliability level with a lower cost than in the solution with identical parallel devices. Second, in practice the designer often has to include additional devices in the existing system. It may be necessary, for example, to modernize a system according to a new demand levels from customers or according to new reliability requirements.

The vast majority of classical reliability or availability analysis and optimization assume that components and system are in either of two states (i.e., complete working state and total failure state). However, in many real life situations we are actually able to distinguish among various levels of performance for both system and components. For such situation, the existing dichotomous model is a gross oversimplification and so models assuming multi-state (degradable) systems and components are preferable since they are closer to reliability. Recently much works treat the more sophisticated and more realistic models in which systems and components may assume many states ranging from perfect functioning to complete failure. In this case, it is important to develop MSS reliability theory. In this paper, an MSS reliability theory will be used, where the binary state system theory is extending to the multi-state case. As is addresses in recent review of the literature for example in [1] or [2]. Generally, the methods of MSS reliability assessment are based on four different approaches:

1. The structure function approach.
2. The stochastic process (mainly Markov) approach.
3. The Monte-Carlo simulation technique.
4. The universal moment generating function (UMGF) approach.

In ref [1], a comparison between these four approaches highlights that the UMGF approach is fast enough to be used in the optimization problems where the search space is sizeable. The problem of total investment-cost minimization, subject to reliability or availability constraints, is well known as the redundancy optimization problem (ROP). The ROP is studied in many different forms as summarized in [3], and more recently in [4]. The ROP for the multi-state reliability was introduced in [5]. In [6] and [7], genetic algorithms were used to find the optimal or nearly optimal power system structure.

This work uses an *ant colony* optimization approach to solve the ROP for multi-state power system. The idea of employing a colony of cooperating agents to solve combinatorial optimization problems was recently proposed in [8]. The ant colony approach has been successfully applied to the classical travelling salesman problem in [9], and to the quadratic assignment problem in [10]. Ant colony shows very good results in each applied area. It has been recently adapted for the reliability design of binary state systems in [11]. The ant colony has also been adapted with success to other combinatorial optimization problems such as the vehicle routing problem in [12]. The ant colony method has not yet been used for the redundancy optimization of multi-state systems.

1.1 Approach and outlines

The problem formulated in this paper lead to a complicated combinatorial optimization problem. The total number of different solution to be examined is very large, even for rather small problems. An exhaustive examination of all possible solutions is not feasible given reasonable time limitations. Because of this, the ant colony optimization (or simply ACO) approach is adapted to find optimal or nearly optimal solutions to be obtained in a short time. The newer developed meta-heuristic method has the advantage to solve the ROP for MSS *without* the limitation on the diversity of versions of components in parallel. Ant colony optimization is inspired by the behavior of real ant colonies that exhibit the highly structured behavior. Ants lay down in some quantity an aromatic substance, known as *pheromone*, in their way to food. An ant chooses a specific path in correlation with the intensity of the pheromone. The pheromone trail evaporates over time if no more pheromone in laid down by others ants, therefore the best path has more intensive pheromone and higher probability to be chosen.

During the optimization process, artificial ants will have to evaluate the availability of a given selected structure of the series-parallel system (electrical network). To do this, a fast procedure of availability estimation is developed. This procedure is based on a modern mathematical technique: the $\mathcal{Z}$-transform or UMGF which was introduced in [13]. It was proven to be very effective for high dimension combinatorial problems: see e.g., in [2]. The universal moment generating function is an extension of the ordinary moment generating function (UGF) in [14]. The method developed in this paper allows the availability function of reparable series-parallel MSS to be obtained using a straightforward numerical procedure.

The rest of this paper is outlined as follows. We start in section 2 with the formulation of the redundancy optimization problem. Next, we develop the availability estimation of a series-parallel multi-state system method in Section 3. In Section 4, we describe the ant colony optimization approach to solve the redundancy optimization problem. In Section 5, illustrative examples and numerical results are presented in which the optimal choice of components in a system is found. Conclusions are drawn in Section 6.

2 Formulation of the Redundancy Optimization Problem

Let consider a series-parallel power system containing n subsystems $C_i (i = 1, 2, \dots, n)$ in series arrangement as represented in Fig.1. Every subsystem C_i contains a number of different components connected in parallel. For each subsystem i, there are a number of component versions available in the market. For any given system component, different versions and number of components may be chosen. For each subsystem i, components are characterized according to their version v by their cost (C_{iv}), availability (A_{iv}) and performance (Σ_{iv}). The structure of subsystem i can be defined by the numbers of parallel components (of each version) k_{iv} for $i \leq v \leq V_i$, where V_i is a number of versions available for component of type i. Fig.2 illustrates these notations for a given subsystem i. The entire system structure is defined by

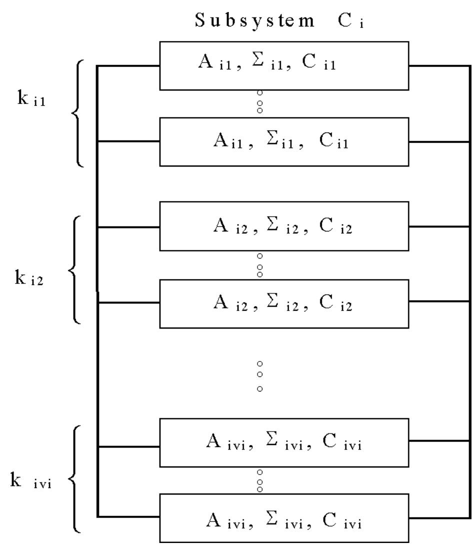

Fig. 2. Detailed structure of a given subsystem.

the vectors $\boldsymbol{k}_i = \{k_{iv}, (1 \leq i \leq n, 1 \leq v \leq V_1)\}$. For a given set of vectors $\boldsymbol{k}_1, \boldsymbol{k}_2, \ldots, \boldsymbol{k}_n$ the total cost of the system can be calculated as

$$C = \sum_{i=1}^{n} \sum_{i=1}^{V_i} k_{iv} C_{iv}. \tag{1}$$

2.1 Availability of reparable multi-state power systems

The series-parallel system is composed of a number of failure prone components, such that the failure of some components leads only to a degradation of the system performance. This system is considered to have a range of performance levels from perfect working to complete failure. In fact, the system failure can lead to decreased capability to accomplish a given task, but not to complete failure. An important MSS measure is related to the ability of the system to satisfy a given demand.

In electric power systems, reliability is considered as a measure of the ability of the system to meet the load demand (D), i.e., to provide an adequate supply of electrical energy (Σ). This definition of the reliability index is widely used in power systems: see e.g., in [14], [15], [16] and in [6]-[7]. The Loss of Load Probability index (LOLP) is usually used to estimate the reliability index in [17]. This index is the overall probability that the load demand will not be met. Thus, we can write $R = Probab(\Sigma \geq D)$ or $R = 1 - LOLP$ with $LOLP = Probab(\Sigma < D)$. This reliability index depends on consumer demand D.

For reparable MSS, a multi-state steady-state availability E is used as $Probab(\Sigma \geq D)$ after enough time has passed for this probability to become constant in [16]. In the steady-state the distribution of states probabilities is given by equation (2), while the multi-state stationary availability is formulated by equation (3)

$$P_j = \lim_{t \to \infty} [Probab(\Sigma(t) = \Sigma_j)] \tag{2}$$

$$E = \sum_{\Sigma_j \geq D} P_j. \tag{3}$$

If the operation period T is divided into intervals (with duration's $T_1, T_2, \ldots, T_M$) and each interval has a required demand level ($D_1, D_2, \ldots, D_M$, respectively), then the generalized MSS availability index A is

$$A = \frac{1}{\sum_{j=1}^{M} T_j} \sum_{j=1}^{M} Probab(\Sigma \geq D_j) T_j. \tag{4}$$

We denote by $\boldsymbol{D}$ and $\boldsymbol{T}$ the vectors $\{D_j\}$ and $\{T_j (1 \leq j \leq M)\}$, respectively. As the availability A is a function of $\boldsymbol{k}_1, \boldsymbol{k}_2, \ldots, \boldsymbol{k}_n$, D and T, it will be written $A(\boldsymbol{k}_1, \boldsymbol{k}_2, \ldots, \boldsymbol{k}_n, \boldsymbol{D}, \boldsymbol{T})$. In the case of a power system, the vectors $\boldsymbol{D}$ and $\boldsymbol{T}$ define the cumulative load curve (consumer demand). In reality the load curves varies randomly; an approximation is used from random curve to discrete curve see in [18]. In general, this curve is known for every power system.

2.2 Optimal design problem formulation

The multi-state power system redundancy optimization problem of electrical power system can be formulated as follows: find the minimal cost system configuration $\boldsymbol{k}_1, \boldsymbol{k}_2, \ldots, \boldsymbol{k}_n$, such that the corresponding availability exceeds or equal the specified availability A_0. That is, minimize

$$C = \sum_{i=1}^{n} \sum_{v=1}^{V_1} k_{iv} C_{iv}, \tag{5}$$

subject to

$$A(\boldsymbol{k}_1, \boldsymbol{k}_2, \ldots, \boldsymbol{k}_n, \boldsymbol{D}, \boldsymbol{T}) \geq A_0. \tag{6}$$

The input of this problem is the specified availability and the outputs are the minimal investmentcost and the corresponding configuration determined. To solve this combinatorial optimization problem, it is important to have an effective and fast procedure to evaluate the availability index for a series-parallel power system. Thus, a method is developed in the next section to estimate the value of $A(\boldsymbol{k}_1, \boldsymbol{k}_2, \ldots, \boldsymbol{k}_n, \boldsymbol{D}, \boldsymbol{T})$.

3 Multi-State System Availability Estimation Method

The procedure used in this paper is based on the universal z-transform, which is a modern mathematical technique introduced in [13]. This method, convenient for numerical implementation, is proved to be very effective for high dimension combinatorial problems. In the literature, the universal z-transform is also called universal moment generating function (UMGF) or simply u-function or u-transform. In this paper, we mainly use the acronym UMGF. The UMGF extends the widely known ordinary moment generating function in [14].

3.1 Definition

The UMGF of a discrete random variable Σ is defined as a polynomial

$$u(z) = \sum_{j=1}^{J} P_j z^{\Sigma_j}, \tag{7}$$

where the variable Σ has J possible values and P_j is the probability that Σ is equal to Σ_j.

The probabilistic characteristics of the random variable Σ can be found using the function $u(z)$. In particular, if the discrete random variable Σ is the MSS stationary output performance, the availability E is given by the probability $Probab(\Sigma \geq D)$ which can be defined as follows

$$Probab(\Sigma \geq D) = \Psi[u(z) z^{-D}] \tag{8}$$

where Ψ is a distributive operator defined by expressions (9) and (10)

$$(Pz^{\sigma-D}) = \begin{cases} P, & \text{if } \sigma \geq D \\ 0, & \text{if } \sigma < D \end{cases} \tag{9}$$

$$\left(\sum_{j=1}^{J} z^{\Sigma_j - D}\right) = \sum_{j=1}^{J} \Psi\left(P_j z^{\sigma_j - D}\right) \tag{10}$$

It can be easily shown that equations (7)-(10) meet condition $Probab(\Sigma \geq D) = \sum_{\Sigma_j \geq D} P_j$. By using the operator Ψ, the coefficients of polynomial $u(z)$ are summed for every term with $\Sigma_j \geq D$, and the probability that Σ is not less than some arbitrary value D is systematically obtained. Consider single components with total failures and each component i has nominal performance Σ_i and availability A_i. Then, $Probab(\Sigma = \Sigma_i) = A_i$ and $Probab(\Sigma = 0) = 1 - A_i$. The UMGF of such an component has only two terms can be defined as

$$\begin{aligned} u_i(z) &= (1 - A_i)z^0 + A_i z^{\Sigma_i} \\ &= (1 - A_i) + A_i z^{\Sigma_i} \end{aligned} \tag{11}$$

To evaluate the MSS availability of a series-parallel system, two basic composition operators are introduced. These operators determine the polynomial $u(z)$ for a group of components.

3.2 Parallel components

Let consider a subsystem m containing J_m components connected in parallel. As the performance measure is related to the system productivity, the total performance of the parallel subsystem is the *sum* of performances of all its components. In power systems engineering, the term capacity is usually used to indicate the quantitative performance measure of an component in [6]. It may have different physical nature. Examples of components capacities are: generating capacity for a generator, pipeline capacity for a water circulator, carrying capacity for an electric transmission line, etc. The capacity of an component can be measured as a percentage of nominal total system capacity. In a electrical network, components are generators, transformers and electrical lines. Therefore, the total performance of the parallel component is the sum of performances in [19].

The u-function of MSS subsystem m containing J_m parallel components can be calculated by using the Γ operator

$$u_p(z) = \Gamma[u_1(z), u_2(z), \ldots, u_n(z)] \tag{12}$$

where $\Gamma(g_1, g_2, \ldots, g_n) = \sum_{i=1}^{n} g_i$.

Therefore for a pair of components connected in parallel

$$\begin{aligned}\Gamma[u_1(z), u_2(z), \ldots, u_n(z)] &= \Gamma\Big(\sum_{i=1}^{n} P_i z^{a_i}, \sum_{j=1}^{m} Q_j z^{b_j}\Big) \\ &= \sum_{i=1}^{n}\sum_{j=1}^{m} P_i Q_j z^{a_i+b_j}\end{aligned} \tag{13}$$

Parameters a_i and b_j are physically interpreted as the respective performances of the two components. n and m are numbers of possible performance levels for these components. P_i and Q_j are steady-state probabilities of possible performance levels for components.

One can see that the Γ operator is simply a product of the individual u-functions. Thus, the subsystem UMGF is

$$u_p(z) = \prod_{j=1}^{J_m} u_j(z). \tag{14}$$

Given the individual UMGF of components defined in equation (11), we have:

$$u_p(z) = \prod_{j=1}^{J_m} (1 - A_j + A_j z^{\Sigma_i}). \tag{15}$$

3.3 Series components

When the components are connected in series, the component with the least performance becomes the bottleneck of the system. This component therefore defines the total system productivity. To calculate the u-function for system containing n subsystems connected in series, the operator η should be used

$$u_s(z) = \eta[u_1(z), u_2(z), \ldots, u_m(z)], \tag{16}$$

where $\eta(g_1, g_2, \ldots, g_m) = \min\{g_1, g_2, \ldots, g_m\}$, so that

$$\begin{aligned}\eta[u_1(z), u_2(z)] &= \eta(\sum_{i=1}^{n} P_i z^{a_i}, \sum_{j=1}^{m} Q_j z^{b_j}\Big) \\ &= \sum_{i=1}^{n}\sum_{j=1}^{m} P_i Q_j z^{\min\{a_i, b_j\}}\end{aligned} \tag{17}$$

Applying composition operators Γ and η consecutively, one can obtain the UMGF of the entire series-parallel system. To do this we must first determine the individual UMGF of each component.

3.4 Components with total failures

Let consider the usual case where only total failures are considered ($K = 2$) and each component of type i and version v_i has nominal performance Σ_{iv} and availability A_{iv}. In this case, we have: $Probab(\Sigma = \Sigma_{iv}) = A_{iv}$ and $Probab(\Sigma = 0) = 1 - A_{iv}$. The UMGF of such an component has only two terms and can be defined as in equation (11) by $u_i^*(z) = (1 - A_{iv})z^0 + A_{iv}z^{\Sigma_{iv}} = 1 - A_{iv} + A_{iv}z^{\Sigma_{iv}}$.

Using the Γ operator, we can obtain the UMGF of the i-th system component containing k_i parallel components as $u_i(z) = [u_i^*(z)]^{k_i} = (A_{iv}z^{\Sigma_{iv}} + 1 = A_{iv})^{k_i}$.

The UMGF of the entire system containing n components connected in series is

$$u_s(z) = \eta[(A_{1v}z^{\Sigma_{1v}} + 1 - A_{1v})^{k_1}, (A_{2v}z^{\Sigma_{2v}} + 1 - A_{2v})^{k_2}, \ldots, (A_{nv}z^{\Sigma_{nv}} + 1 - A_{nv})^{k_n}]. \qquad (18)$$

To evaluate the probability $Probab(\Sigma \geq D)$ for the entire system, the operator Γ is applied to equation (12)

$$Probab(\Sigma \geq D) = [u_s(z)z^{-D}] \qquad (19)$$

The above procedure was implemented and tested on a PC computer and shown to be effective and fast. The UMGF method, convenient for numerical implementation, is efficient for the high dimension combinatorial problem formulated in this work. In our optimization technique to solve this problem, artificial ants will evaluate the availability of given selected structures of the series- parallel power system. To do this, the fast implemented procedure of availability estimation will be used by the optimization program.

The next section presents the ant colony meta-heuristic optimization method to solve the redundancy optimization problem for multi-state power systems.

4 The Ant Colony Optimization Approach

The problem formulated in this paper is a complicated combinatorial optimization problem. The total number of different solutions to be examined is very large, even for rather small problems. An exhaustive examination of the enormous number of possible solutions is not feasible given reasonable time limitations. Thus, because of the search space size of the ROP for MSS, a new meta-heuristic is developed in this section. This meta-heuristic consists in an adaptation of the ant colony optimization method.

4.1 The ACO principle

Recently, in [8] introduced a new approach to optimization problems derived from the study of any colonies, called "Ant System". Their system inspired by the work of real ant colonies that exhibit the highly structured behavior. Ants lay down in some quantity an aromatic substance, known as pheromone, in their way to food. An ant chooses a specific path in correlation with the intensity of the pheromone. The pheromone trail evaporates over time if no more pheromone in laid down by others ants, therefore the best paths have more intensive pheromone and higher probability to be chosen. This simple behavior explains why ants are able to adjust to changes in the environment, such as new obstacles interrupting the currently shortest path.

Artificial ants used in ant system are agents with very simple basic capabilities mimic the behavior of real ants to some extent. This approach provides algorithms called ant algorithms. The Ant System approach associates pheromone trails to features of the solutions of a combinatorial problem, which can be seen as a kind of adaptive memory of the previous solutions. Solutions are iteratively constructed in a randomized heuristic fashion biased by the pheromone trails, left by the previous ants. The pheromone trails, τ_{ij}, are updated after the construction of a solution, enforcing that the best features will have a more intensive pheromone. An Ant algorithm presents the following characteristics. It is a natural algorithm since it is based on the behavior of ants in establishing paths from their colony to feeding sources and back. It is parallel and distributed since it concerns a population of agents moving simultaneously, independently and without supervisor. It is cooperative since each agent chooses a path on the basis of the information, pheromone trails, laid by the other agents with have previously selected the same path. It is versatile that can be applied to similar versions the same problem. It is robust that it can be applied with minimal changes to other combinatorial optimization problems. The solution of the travelling salesman problem (TSP) was one of the first applications of ACO.

Various extensions to the basic TSP algorithm were proposed, notably by Dorigo and Gambardella in [9]. The improvements include three main aspects: the state transition rule provides a direct way to balance between exploration of new edges and exploitation of a priori and accumulated knowledge about the problem, the global updating rule is applied only to edges which belong to the best ant tour and while ants construct solution, a local pheromone updating rule is applied. These extensions have been included in the algorithm proposed in this paper.

4.2 ACO-based solution approach

In our reliability optimization problem, we have to select the best combination of parts to minimize the total cost given a reliability constraint. The parts can be chosen in any combination from the available components. Components are characterized by their reliability, capacity and cost. This problem can be represented by a graph (Fig.3) in which the set of nodes comprises the set of subsystems and the set of available components (i.e. max (M_j), $j = 1, \ldots, n$) with a set of connections partially connect the graph (i.e. each subsystem is connected only to its available components). An additional node (blank node) is connected to each subsystem.

In Fig.3, a series-parallel power system is illustrated. At each step of the construction process, an ant uses problem-specific heuristic information, denoted by η_{ij} to choose the optimal number of components in each subsystem. An imaginary heuristic information is associated to each blank node. These new factors allow us to limit the search surfaces (i.e. tuning factors). An ant positioned on subsystem i chooses a component j by applying the rule given by

$$j = \begin{cases} \angle \max\limits_{m \in AC_1} ([\tau_{im}]^\alpha [\eta_{im}]^\beta) & \text{if } q \leq q_o \\ J & \text{if } q > q_0, \end{cases} \tag{20}$$

and J is chosen according to the probability

$$p_{ij} = \begin{cases} \dfrac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta}{\sum\limits_{m \in AC_i} [\tau_{im}]^\alpha [\eta_{im}]^\beta} & \text{if } j \in AC_i \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

where α is the relative importance of the trail, β is the relative importance of the heuristic information η_{ij}, AC_i is the set of available components choices for subsystem i , and q is random number uniformly generated between 0 and 1.

The heuristic information used is $\eta_{ij} = 1/(1 + c_{ij})$, where c_{ij} represents the associated cost of component j for subsystem i. A "tuning" factor $t_i = \eta_{ij} = 1/(1 + c_{i(M_i+1)})$ is associated to blank component $(M_i + 1)$ of

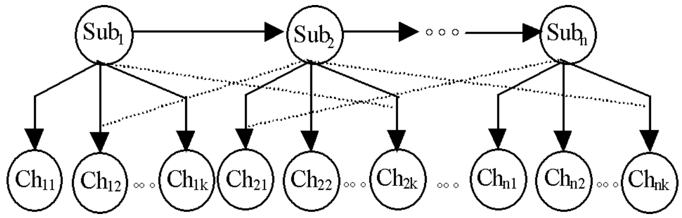

Fig. 3. Definition of a series-parallel system with n subsystems into a graph.

subsystem i. The parameter q_0 determines the relative importance of exploitation versus exploration: every time an ant in subsystem i have to choose a component j, it samples a random number $0 \leq q \leq 1$. If $q \leq q_0$ then the best edge, according to (14), is chosen (exploitation), otherwise an edge is chosen according to (15) (biased exploration).

The pheromone update consists of two phases: local and global updating. While building a solution of the problem, ants choose components and change the pheromone level on subsystem- component edges. This local trail update is introduced to avoid premature convergence and effects a temporary reduction in the quantity of pheromone for a given subsystem-component edge so as to discourage the next ant from choosing the same component during the same cycle. The local updating is given by

$$\tau_{ij}^{\text{new}} = (1-\rho)\tau_{ij}^{\text{old}} + \rho\tau_0, \tag{22}$$

where ρ is a coefficient such that $(1-\rho)$ represents the evaporation of trail and τ_0 is an initial value of trail intensity. It is initialized to the value $(nTC_{nn})^{-1}$ with n is the size of the problem (i.e. number of subsystem and total number of available components) and TC_{nn} is the result of a solution obtained through some simple heuristic.

After all ants have constructed a complete system, the pheromone trail is then updated at the end of a cycle (i.e. global updating), but only for the best solution found. This choice, together with the use of the pseudo-random-proportional rule given by (14) and (15), is intended to make the search more directed: ants search in a neighborhood of the best solution found up to the current iteration of the algorithm. The pheromone level is updated by applying the following global updating rule

$$\tau_{ij}^{\text{new}} = (1-\rho)\tau_{ij}^{\text{old}} + \rho\Delta\tau_{ij} \tag{23}$$

$$\Delta\tau_{ij} = \begin{cases} \frac{1}{TC_{best}} & \text{if } (i,j) \in besttour \\ 0 & \text{otherwise.} \end{cases} \tag{24}$$

4.3 The algorithm

An ant-cycle algorithm is stated as follows. At time zero an initialization phase takes place during wish $NbAnt$ ants select components in each subsystem according to the Pseudo-random- proportional transition rule given by (14) and (15). When an ant selects a component, a local update is made to the trail for that subsystem-component edge according to equation (16). In this equation, ρ is a parameter that determines the rate of reduction of the pheromone level. The pheromone reduction is small but sufficient to lower the attractiveness of precedent subsystem- component edge. At the end of a cycle, for each *ant* k, the value of the system's reliability A_k and the total cost TC_k are computed. The best feasible solution found by ants (i.e. total cost and assignments) is saved. The pheromone trail is then updated for the best solution obtained according to (17) and (18). This process is iterated until the tour counter reaches the maximum number of cycles NC_{max} or all ants make the same tour (stagnation behavior).

5 Illustrative Example

In order to illustrate the proposed ant colony algorithm, a numerical example is solved by use of the data given in Table 1. Each component of the subsystem is considered as a unit with total failures. Table 2 contains the data of cumulative demand.

The maximum numbers of components Ch_{max} in parallel are set to (4,5,4,6,4). The number of ants used to find the best solution is 50. The simulation results depend greatly on the values of the coefficients α and β. Different t_i values (tuning factors associated to blank components) were tested and shown to influence greatly the algorithm. The best found values of t_i are ($t_1 = -0.13$, $t_2 = -0.04$, $t_3 = 2.3$, $t_4 = -0.35$, $t_5 = 0.35$). Several simulations are made for $\alpha = 5$ and $\beta = 1$ and the best solution is obtained in 500 cycle . Table 3 presents the obtained configuration.

5.1 Description of the system to be optimized

The electrical power station system which supplies the consumers is designed with five basic subsystems (stations) as depicted in Figures 4 and 5. The Fig.4 showed the detailed process of the electrical power station system distribution.

The process of electrical power system distribution follows as: The electrical power is generated from the station units (Subsystem 1). Then transformed for high voltage (HT) by the HT transformers (Subsystem 2) and carried by the HT lines (Subsystem 3). A second transformation in HT/MT transformers (Subsystem 4) which supplies the MT load by the MT lines (Subsystem 5). Each component of the system is considered as unit with total failures.

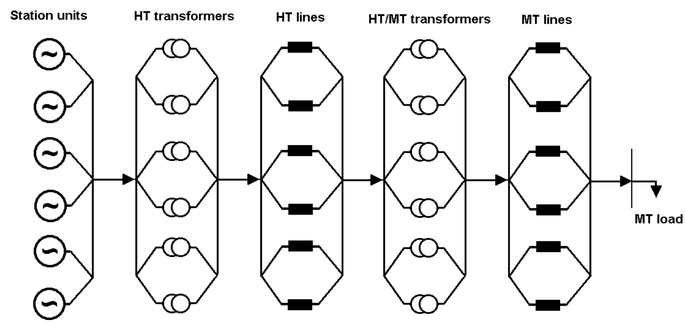

Fig. 4. Detailed electrical generating power station system.

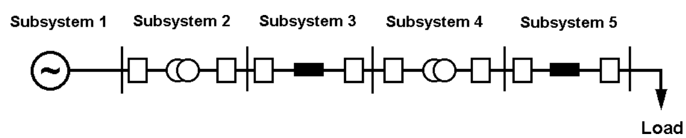

Fig. 5. Synoptic of the detailed electrical generating power station system.

The characteristics of the products available on the market for each type of device are presented in Table 1. This table shown for each subsystem availability A, nominal capacity Σ and cost per unit C. With out loss of generality both the component capacity and the demand levels Table 2 can be measured as a percentage of the maximum capacity.

5.2 Optimal design solution and result discussion

Our natural objective function is to define the minimal cost power system configuration which provides the requested level of availability. The whole of the results obtained by the proposed ant algorithm for different given values of A_0 are illustrated in Table 3. This latter also shows the computed availability index A, the cost C of the system and their corresponding structures. Three different solutions for $A_0 = 0.975$, $A_0 = 0.985$, $A_0 = 0.995$ are represented. In these experiments the values parameters of the ACO algorithm are the set of the following values: $\alpha = 5$, $\beta = 1$, τ_0= 0.05 and ρ= 0.080. The choice of these value affect strongly the solution. These value were obtained

Table 1. Data examples.

Subsystem		Version	Availability A	Cost C	Capacity Σ
Pover Units	**1**	1	0.980	0.590	120
		2	0.877	0.535	100
		3	0.982	0.470	85
		4	0.978	0.420	86
Transformer HT	**2**	1	0.995	0.205	100
		2	0.996	0.189	92
		3	0.997	0.056	53
		4	0.997	0.420	28
		5	0.998	0.042	21
HT lines	**3**	1	0.971	7.525	100
		2	0.973	4.720	60
		3	0.971	3.509	40
		4	0.976	2.420	20
Transformer HT/MT	**4**	1	0.977	0.180	115
		2	0.978	0.160	100
		3	0.978	0.150	91
		4	0.983	0.121	72
		5	0.981	0.102	72
		6	0.971	0.096	72
MT lines	**5**	1	0.984	7.525	100
		2	0.983	4.720	60
		3	0.987	3.509	40
		4	0.981	2.420	20

Table 2. Parameters of the cumulative demand curve.

Demand level %	100	80	50	20
Duration (h)	4203	788	1228	2536
Probability	0.479	0.089	0.140	0.289

by a preliminary optimisation phase. The ACO algorithm is tested well for quite a range of these values. In the ACO algorithm 20 ants are used in each iteration. The stopping criterion is when the number of iterations attempt 500 cycles. The space search visited by the 50 ants is composed of 25000 solutions (50×500 cycles) and the huge space size of an exhaustive search (combinatorial algorithm) about 10^{45}. Indeed, a large comparison between the ACO and an exhaustive one, clearly the goodness of the proposed ACO meta- heuristic which respect to the calculating time.

6 Conclusion

An new algorithm for choosing an optimal series-parallel power structure configuration is proposed which minimizes total investment cost subject to

Table 3. Optimal solutions obtained by ant colony algorithm.

A_0	Optimal structure	Comuted Availability A	Computed Cost C
0.975	Subsystem **1**: Components **1-2-3-4** Subsystem **2**: Components **1-2-3-4-5** Subsystem **3**: Components **2-3-4-4** Subsystem 4: Components **1-2-3-4-5-6** Subsystem **5**: Components **2-3-4-4**	0.986	18.822
0.985	Subsystem **1**: Components **1-3-3-4** Subsystem **2**: Components **1-2-3-4-5** Subsystem **3**: Components **2-3-4-4** Subsystem 4: Components **1-2-3-4-5-6** Subsystem **5**: Components **2-3-4-4**	0.986	18.771
0.995	Subsystem **1**: Components **2-3-4-4** Subsystem **2**: Components **1-3-4-4-5** Subsystem **3**: Components **2-3-3-4** Subsystem 4: Components **1-2-3-4-5-6** Subsystem **5**: Components **1-2-3-4**	0.998	20.199

availability constraints. This algorithm seek and selects components among a list of available products according to their availability, nominal capacity (performance) and cost. Also defines the number and the kind of parallel components in each subsystem. The proposed method allows a practical way to solve wide instances of reliability optimization problem of multi-state power systems without limitation on the diversity of versions of components put in parallel. A combination is used in this algorithm is based on the universal moment generating function and an ant colony optimization algorithm.

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