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A Statistical Analysis of two Classes of Time-Frequency representations

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Abstract: The aim of this paper is to present a statistical analysis of the continuous linear and bilinear time-frequency representations. Such an analysis can be useful for the estimation of different parameters of not stationary signals corrupted by noise, like the instantaneous frequency or the instantaneous bandwidth. The case of discrete time-frequency representations is also considered. Simulations verify the theoretical results obtained.

Keywords: Nonstationary signals, time frequency analysis, statistical analysis, Wigner-Vile distibution, Discrete Wavelet tranform.

1 Introduction

There are a lot of time-frequency representations: the Short-time Fourier transform, the wavelet transform (linear representations) and the members of the Cohen class (bilinear representations). The definition of a linear time-frequency representation is the following.

If the following conditions are satisfied:

1) $\mathbf{A} \subset \Re^n,\ K : \Re \times \mathbf{A} \to \mathbf{C}$

2) $(\forall)\, a \in \mathbf{A},\ \tau \to K(\tau, a)$ is measurable and $\int_{-\infty}^{\infty} |K(\tau, a)|^2 \mathrm{d}\tau = \mathbf{1}$

3) $(\forall)\, \omega \in \Re,\ a \to \Im\{K(\tau, a)\}(\omega)$ is measurable and $\int_{\mathbf{A}} |\Im\{K(\tau, a)\}(\omega)|^2 \mathrm{d}a = \mathbf{C} < \infty$ then the function: TF_x: $\mathbf{A} \times \Re \to \mathbf{C}$, $TF_x(t, \omega) = \langle x(\tau), K(\tau - t, \omega)\rangle = \int_{-\infty}^{\infty} x(\tau) K^*(\tau - t, \omega) \mathrm{d}\tau$, is named linear time-frequency representation of the finite energy signal $x(\tau)$, [1].

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The two variables function $K(u,v)$ is the kernel of the linear time-frequency representation. For different kernels different time-frequency representations are obtained. The kernel for the Short-Time Fourier Transform, (STFT) is $K_{STFT}(\tau,a) = w(\tau)\, e^{ja\tau}$. If the window, $w(\tau)$, is gaussian then the corresponding time-frequency representation is the Gabor transform, (GT). The kernel for the Continuous Wavelet Transform, (CWT) is $K_{CWT}(\tau,a) = \sqrt{a^{-1}}\psi\left(a^{-1}\tau\right)$, where the function ψ is a mother of wavelets.

The bilinear time-frequency representations of the Cohen's class can be computed for the signal x with the relation:

$$_{C}TF_x(t,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\left(s+\frac{\tau}{2}\right) x^*\left(s-\frac{\tau}{2}\right) e^{-j(\omega\tau - us + ut)} \times f(u,\tau)\, du ds d\tau \tag{1}$$

where $f(u,\tau)$ is another kernel. For the Wigner-Ville time-frequency representation this kernel is unitary.

Important parameters of the analyzed signals, like their instantaneous frequencies or their instantaneous bandwidth, can be estimated in the time-frequency plane, due to the good localization of the ridges of those time-frequency representations. Some of the members of the Cohen's class realize a good localization of the ridges of the analyzed signal. A very good example is the Wigner-Ville distribution, [2]. For the mono component signal not perturbed by noise

$$s(t) = \cos\left(2\pi t^2\right), \tag{2}$$

the associated analytical signal is

$$s_a(t) = e^{-j2\pi t^2}, \tag{3}$$

and the instantaneous frequency is

$$f_i(t) = 2t. \tag{4}$$

The Wigner -Ville representation of the signal is:

$$TF_{s_a}^{W-V}(t,\omega) = 2\pi\delta(\omega - 4\pi t) \tag{5}$$

So, this time-frequency representation is perfectly concentrated on the curve $\omega = 2\pi f_i(t)$ of the instantaneous frequency of the signal . Hence for the estimation of the instantaneous frequency of the signal the better time-frequency representation is the Wigner - Ville distribution.

The good concentration around the instantaneous frequency law properties of the linear time-frequency representations of signals with double modulation of the form:

$$s(t) = A(t)\, e^{jb(t)}, \tag{6}$$

is also known, [3]. It is recognized that the Gabor time-frequency representation realizes the better localization in the time-frequency plane (see the Heisenberg principle). The parameter estimation process is more difficult when the acquired signal is perturbed by a noise. This is the reason why in the following a statistical analysis of different time-frequency representations of this noise is presented. In the second section of this paper are studied the continuous linear time-frequency representations. In the third section is considered the case of continuous bilinear time-frequency representations. The forth paragraph is dedicated to discrete linear time-frequency representations. The discrete bilinear time-frequency representations are analyzed in the fifth section. The last paragraph is dedicated to conclusions.

2 The statistical analysis of continuous linear time-frequency representations

This type of time-frequency representations is already defined. We suppose that the signal to be represented in the time-frequency plan, $n(t)$, is a stationary noise. The linear time-frequency representation of the noise $n(t)$ is:

$${}_lTF_n(t,\omega) = \int_{-\infty}^{\infty} n(\tau)\, K^*(\tau - t,\omega)\, \mathrm{d}\tau \tag{7}$$

The system for the computation of this time-frequency representation is a time invariant linear system with the impulse response $K^*(-t,\omega)$, where the frequency, ω, represents a parameter. At any frequency, ω, this system responds to the input signal, $n(t)$, with the signal $n_0(t)$, the linear time-frequency representation, ${}_lTF_n(t,\omega)$, computed at that frequency.

This is a random process with the mean

$$E\{{}_lTF_n(t,\omega)\} = M_n \int_{-\infty}^{\infty} K^*(\tau - t,\omega)\, \mathrm{d}\tau \tag{8}$$

where M_n represents the mean of the noise $n(t)$. If this noise has a zero mean then the mean of its continuous linear time-frequency representation is also null. The correlation function of the time-frequency representation from the relation (7) is:

$$E\left\{{}_lTF_n\left(t_1,\omega_1\right){}_lTF_n\left(t_2,\omega_2\right)\right\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K^*\left(\tau_1-t_1,\omega_1\right)K^*\left(\tau_2-t_2,\omega_2\right) \times R_n\left(\tau_1-\tau_2\right)\mathrm{d}\tau_1\mathrm{d}\tau_2 \qquad (9)$$

where $R_n(\tau)$ represents the correlation of the noise $n(t)$. For a zero mean white noise with standard deviation σ the last relation becomes:

$$E\left\{{}_lTF_n\left(t_1,\omega_1\right){}_lTF_n\left(t_2,\omega_2\right)\right\} = \sigma^2\int_{-\infty}^{\infty} K^*\left(\tau_1-t_1,\omega_1\right)K^*\left(\tau_1-t_2,\omega_2\right)\mathrm{d}\tau_1 = R_{{}_lTF_n}\left(t_1-t_2,\omega_1-\omega_2\right) \qquad (10)$$

So, any continuous linear time-frequency representation correlates the input noise. In Figure 1 is represented the two dimensional power spectral density of the Gabor time-frequency representation of a zero mean white noise with standard deviation equal with 0.0305.

The correlation effect can be observed because the two dimensional power spectral density isn't constant. This correlation can be avoided only for discrete linear time-frequency representations. It is well known the whitening effect of the discrete wavelet transform, [4].

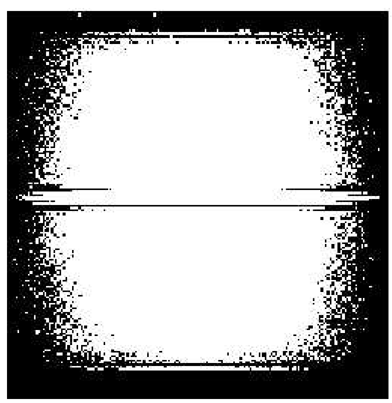

Fig. 1. The power spectral density of the Gabor time-frequency representation of a white noise.

The power of the output signal of the system for the computation of the continuous linear time-frequency representation is:

$$P_{n_0} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |N_0(\omega)|^2 \, \mathrm{d}\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} |N(\omega)|^2 \left|\Im\left\{K^*(-t,\omega)\right\}(\omega)\right|^2 \mathrm{d}\omega$$

$$\leq \frac{1}{2\pi}\int_{-\infty}^{\infty} |N(\omega)|^2 \, \mathrm{d}\omega \int_{-\infty}^{\infty} \left|\Im\left\{K^*(-t,\omega)\right\}(\omega)\right|^2 \mathrm{d}\omega \ ^{3)}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} |N(\omega)|^2 \, \mathrm{d}\omega = P_n \tag{11}$$

So, at any frequency, the power of the signal $n_0(t)$ is inferior to the power of the signal $n(t)$. *Hence, the continuous linear time-frequency representation realizes a spreading of the noise in the time-frequency plane.*

In conclusion there are two useful effects in the time-frequency plane: the concentration of the useful signal and the spreading of the noise.

3 A statistical analysis of the continuous bilinear time-frequency representations

The members of the Cohen class are described in the relation (1). Its mean is

$$E\left\{{}_{C}TF_n(t,\omega)\right\} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_n(\tau)\, e^{-j(\omega\tau - us + ut)} f(u,\tau)\, \mathrm{d}u\mathrm{d}s\mathrm{d}\tau \tag{12}$$

If $n(t)$ is a zero mean white noise with standard deviation σ then the mean of its time-frequency representation becomes:

$$E\left\{{}_{C}TF_n(t,\omega)\right\} = \sigma^2 f(0,0) \tag{13}$$

The Cohen's class time-frequency representations are not zero mean random processes. The mean of the Wigner-Ville time-frequency representation for a zero mean white noise with standard deviation σ is equal with the power of this noise, σ^2. *So, a very good estimation of the input noise's power can be realized by statistical averaging of few Wigner-Ville time-frequency representations of different realizations of this noise.* The computation of the correlation of a Cohen's class time-frequency representation for the signal $n(t)$ is more difficult.

$$
\begin{aligned}
&E\left\{{}_C TF_n\left(t_1,\omega_1\right){}_C TF_n\left(t_2,\omega_2\right)\right\} = \frac{1}{4\pi^2}\\
&\times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} {}_4R_n\left(\tau_1,\tau_2,s_1,s_2\right)\\
&\times f\left(u_1,\tau_1\right) f\left(u_2,\tau_2\right) e^{-j(\omega_1\tau_1-u_1s_1+u_1t_1+\omega_2\tau_2-u_2s_2+u_2t_2)}\\
&\times \mathrm{d}u_1\mathrm{d}s_1\mathrm{d}\tau_1\mathrm{d}u_2\mathrm{d}s_2\mathrm{d}\tau_2 \qquad (14)
\end{aligned}
$$

where ${}_4R_n\left(\tau_1,\tau_2,s_1,s_2\right)$ represents the forth order correlation of the input signal. This function can be computed with the relation

$$
\begin{aligned}
{}_4R_n\left(\tau_1,\tau_2,s_1,s_2\right) &= E\left\{n\left(t_1\right)n\left(t_2\right)n\left(t_3\right)n\left(t_4\right)\right\}\\
&= cum_4\left\{n\left(t_1\right)n\left(t_2\right)n\left(t_3\right)n\left(t_4\right)\right\} + R_n(t_1,t_2)R_n(t_3,t_4)\\
&+ R_n(t_1,t_3)R_n(t_2,t_4) + R_n(t_1,t_4)R_n(t_2,t_3) \qquad (15)
\end{aligned}
$$

where the forth order cumulant can be computed using the relation:

$$
cum_4\left\{n\left(t_1\right)n\left(t_2\right)n\left(t_3\right)n\left(t_4\right)\right\} = E\left\{n^4\right\} - 3E\left\{n^2\right\} \qquad (16)
$$

If $n(t)$ is a zero mean white noise with standard deviation σ then

$$
cum_4\left\{n\left(t_1\right)n\left(t_2\right)n\left(t_3\right)n\left(t_4\right)\right\} = 0 \qquad (17)
$$

and

$$
\begin{aligned}
{}_4R_n\left(t_1,t_2,t_3,t_4\right) &= \sigma^4(\delta\left(t_1-t_2\right)\delta\left(t_3-t_4\right) + \delta\left(t_1-t_3\right)\delta\left(t_2-t_4\right))\\
&+ \sigma^4\left(\delta\left(t_1-t_4\right)\delta\left(t_2-t_3\right)\right)
\end{aligned}
$$

and the correlation of its Cohen's class time-frequency representation becomes

$$
\begin{aligned}
&E\left\{{}_C TF_n\left(t_1,\omega_1\right){}_C TF_n\left(t_2,\omega_2\right)\right\}\\
&\quad = -\frac{\sigma^4}{2\pi}\Im_2\left\{f\left(u_2,\tau_1\right)f\left(-u_2,\tau_1\right)\right\}\left(\omega_1+\omega_2,t_1+t_2\right) + \sigma^4 f\left(0,0\right)\\
&\quad + \frac{\sigma^4}{\pi}\Im_2\left\{f\left(u_2,\tau_1\right)f\left(-u_2,\tau_1\right)\right\}\left(\omega_2-\omega_1,t_2-t_1\right) \qquad (18)
\end{aligned}
$$

where $\Im_2$ represents the two-dimensional Fourier transform. For the case of the Wigner - Ville time-frequency distribution, the last relation becomes

$$E\left\{TF_n^{W-V}(t_1,\omega_1)\,TF_n^{W-V}(t_2,\omega_2)\right\} = 4\pi\sigma^4\delta(\omega_2-\omega_1)\,\delta(t_2-t_1) + \sigma^4 - 2\pi\sigma^4\delta(\omega_2+\omega_1)\,\delta(t_2+t_1) \quad (19)$$

So, the Wigner - Ville time-frequency representation of a zero mean white noise with standard deviation is a two dimensional random process, very close to a two dimensional white noise. For the noise considered in the experiment in Figure 1, the two dimensional power spectral density of its continuous Wigner - Ville time-frequency representation is presented in Figure 2.

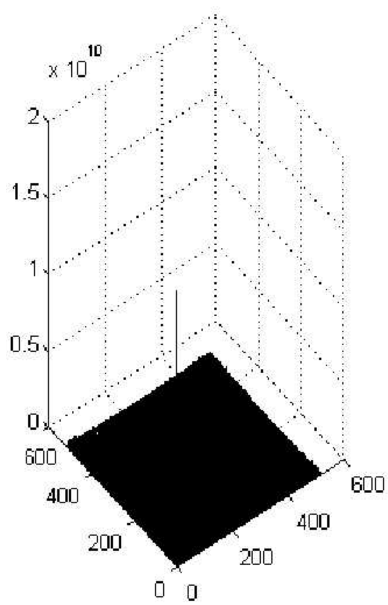

Fig. 2. The two-dimensional power spectral density of the Wigner - Ville representation of a white noise.

This function is practically constant everywhere. So, generally, the Cohen's class time-frequency representations correlates the input signal (see the relation (18)) but the Wigner-Ville time-frequency representation is an exception. Because the Wigner - Ville distribution is a spectro-temporal density of energy that don't correlates the input noise, it posses the noise power's spreading in the time-frequency plane property. This is the reason why this time-frequency representation could be a good solution for the estimation of some parameters of a not stationary signal, in the time-frequency plane. Unfortunately it posses some interference terms. The presence of those terms makes the reading of the representation in the time-frequency plane very difficult.

4 A statistical analysis of the discrete linear time-frequency representations

The Discrete Short Time Fourier Transform (DSTFT) of a discrete in time noise, $n[o]$, has the expression

$$TF_n^{DSTFT}[o,k] = \sum_{l=-\infty}^{\infty} n[l]\, w[l-o]\, e^{-jk\frac{2\pi}{N}l} \quad k=\overline{0,N-1} \tag{20}$$

Its mean is

$$E\left\{TF_n^{DSTFT}[o,k]\right\} = E\{n\} \sum_{l=-\infty}^{\infty} w[l-o]\, e^{-jk\frac{2\pi}{N}l} \tag{21}$$

If the mean of the noise n[o] is equal with zero then the mean of its DSTFT time-frequency representation is also equal with zero. The correlation of this time-frequency representation is

$$E\left\{TF_n^{DSTFT}[o_1,k_1]\, TF_n^{DSTFT}[o_2,k_2]\right\} = \sum_{l_1=-\infty}^{\infty} \sum_{l_2=-\infty}^{\infty} w[l_1-o_1]\, w[l_2-o_2] \times e^{-jk_1\frac{2\pi}{N}l_1} e^{-jk_2\frac{2\pi}{N}l_2} R_n[l_1,l_2] \tag{22}$$

If $n[o]$ is a zero mean white noise with standard deviation σ, then the last relation becomes

$$E\left\{TF_n^{DSTFT}[o_1,k_1]\, TF_n^{DSTFT}[o_2,k_2]\right\} = \sigma^2 \sum_{l_1=-\infty}^{\infty} w[l_1-o_1]\, w[l_1-o_2] \times e^{-j(k_1+k_2)\frac{2\pi}{N}l_1} \tag{23}$$

For the Discrete Gabor (DG) time-frequency representation the last relation becomes

$$E\left\{TF_n^{DG}[o_1,k_1]\, TF_n^{DG}[o_2,k_2]\right\} = \sigma^2 e^{-\pi\left(\frac{n_1-n_2}{2}\right)^2 - j\frac{\pi}{N}(o_1+o_2)(k_1+k_2)} \times \Im_d\left(e^{-\pi p^2}\right)\left(\frac{2\pi}{N}(k_1+k_2)\right) \tag{24}$$

where $\Im_d$ represents the operator of the Discrete Fourier Transform. Tacking into account the relation (23) and (24) it can be observed that the DSTFT with its particular case the DG correlates their input signals.

In the following is analyzed the Discrete Wavelet Transform (DWT). For the case of the CWT, the relation (8) becomes:

$$E\left\{CWT_n\left(t,\omega\right)\right\} = M_n \int_{-\infty}^{\infty} \sqrt{\omega}\psi^*\left(\omega\tau\right)\mathrm{d}\tau \tag{25}$$

Tacking into account the relation between the coefficients of the DWT and the CWT

$$DWT_n\left[o,k\right] = CWT_n\left(t,\omega\right)\Big|_{\substack{t=0,\\ \omega=k2^m}} \tag{26}$$

the mean of the coefficients of DWTn can be computed

$$E\left\{DWT_n\left[o,k\right]\right\} = \sqrt{k2^m} \int_{-\infty}^{\infty} \psi^*\left(k2^m\tau\right)\mathrm{d}\tau \tag{27}$$

This mean is equal with zero only if the mean of the input noise, M_n, is equal with zero.

A complete statistical analysis of this discrete time-frequency representation is presented in [5]. This transform de-correlates the input noise. In Figure 3 is presented the power spectral density of a colored noise (top) and the power spectral density of its DWT (bottom).

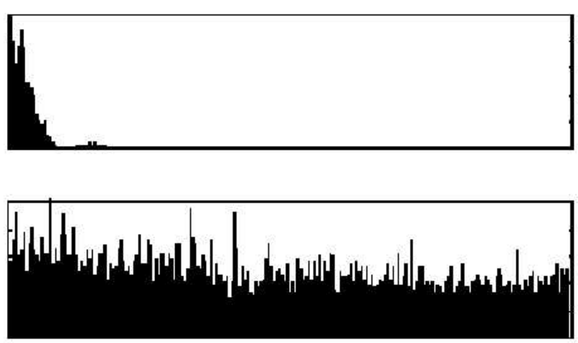

Fig. 3. A statistical analysis of Discrete Wavelet Transfrm.

5 A statistical analysis of the discrete bilinear time-frequency representations.

A recent paper, [6], presents a statistical analysis of the discrete time-frequency representations from the Cohen's class. A discrete Cohen's class time-frequency representation of the signal n[o] is:

$$TF_n^C[o,k] = \sum_{l_1=-\infty}^{\infty} \sum_{l_2=-\infty}^{\infty} f(l_1,l_2)\, n[o+l_1+l_2]\, n^*[o-l_1+l_2] \times e^{-2jk\frac{2\pi}{N}l_1} \tag{28}$$

Its mean is

$$\begin{aligned} E\left\{TF_n^C[o,k]\right\} &= \sum_{l_1=-\infty}^{\infty} \sum_{l_2=-\infty}^{\infty} f(l_1,l_2)\, E\left\{n[o+l_1+l_2]\, n^*[o-l_1+l_2]\right\} \\ &\times e^{-2jk\frac{2\pi}{N}l_1} \\ &= \sum_{l_1=-\infty}^{\infty} \sum_{l_2=-\infty}^{\infty} f(l_1,l_2)\, R_n[2l_1]\, e^{-2jk\frac{2\pi}{N}l_1} \end{aligned} \tag{29}$$

If $n[o]$ is a zero mean white noise with standard deviation σ then the mean of its Wigner-Ville time-frequency representation (obtained for $f(l_1,l_2)=1$) is equal with σ^2.

The correlation of the discrete Cohen's class time-frequency representation of the noise $n[o]$ is:

$$\begin{aligned} E\left\{TF_n^C[o_1,k_1]\,TF_n^C[o_2,k_2]\right\} &= \sum_{l_1=-\infty}^{\infty} \sum_{l_2=-\infty}^{\infty} \sum_{l_3=-\infty}^{\infty} \sum_{l_4=-\infty}^{\infty} f(l_1,l_2)\, f(l_3,l_4) \\ & E\left\{n[o_1+l_1+l_2]\, n^*[o_1-l_1+l_2]\, n[o_2+l_3+l_4]\, n^*[o_2-l_3+l_4]\right\} \\ & e^{-2j(k_1l_1+k_2l_3)\frac{2\pi}{N}} \end{aligned} \tag{30}$$

where $E\left\{n[o_1+l_1+l_2]\, n^*[o_1-l_1+l_2]\, n[o_2+l_3+l_4]\, n^*[o_2-l_3+l_4]\right\}$ represents the forth order correlation of the input noise. This correlation can be computed using the relation (15) and (16). If the input noise is white with zero mean and standard deviation σ, then a result similar to the result presented in the relation (17) can be obtained. Using this result an expression of the correlation of the considered member of the Cohen's class similar to (18) can be obtained.

6 Conclusions

There are two classes of time-frequency representations, the class of linear distributions and the class of bilinear time-frequency representations. These are continuous representations. Only their discrete equivalents can be computed with computers. This is the reason why in this paper are analyzed the continuous and discrete linear and bilinear time-frequency representations. The simulations presented in the figures 1,2 and 3 are obtained using discrete time-frequency representations. Some new results concerning the mean and the correlation of different time-frequency representations are reported in the relation (8), (9), (10), (12), (13), (18), (19), (21), (22), (24), (25), (27), (29) and (30). These relations can be used in applications where signals corrupted by noise are analyzed in the time-frequency plane. The statistical analysis already made proves the advantages of the use of time-frequency representations for the estimation of parameters of not stationary signals corrupted by noise. The time-frequency representations are concentrated on the useful signal and realize a spreading of the noise in the time-frequency plane, [7]. The higher concentration on the useful signal is realized by the Wigner-Ville time-frequency representation, but it contains interference terms. Good concentrations can also be obtained using the Gabor time-frequency representation. This is the reason why a new instantaneous frequency estimation method based on the conjoint use of those two time-frequency representations is proposed in [8]. Using this method the interference terms of the Wigner-Ville time-frequency representation can be reduced. This is a new method for the reduction of interference terms. Other methods of this type are reported in [9] and [10].

Generally the time-frequency representations correlate the noise. There are two exceptions: the Wigner-Ville representation and the Discrete Wavelet Transform. So these two time-frequency representations can be used for denoising. Such an application of the DWT is reported in [11]. The basic idea of a denoising procedure is to realize a filtering in the domain of a transform. If that transform de-correlates the perturbation noise, then in its domain can be used a filtering method appropriate for the reduction of the white noise. There are few very well known methods for the reduction of the white noise. This is the reason why the transform that de-correlates the perturbation noise are very useful in denoising applications. The use of the Wigner-Ville time-frequency representation in denoising applications will be a future preoccupation of the authors of this paper.

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