FACTA UNIVERSITATIS (NIŠ)
SER.: ELEC. ENERG. vol. 16, April 2003, xx-xx

Electromagnetic Field Distribution of a Nearby Lightning Discharge Inside an Overground Wire Conductive Structure

Vesna Javor

Abstract: Lightning is a major natural source of electromagnetic (EM) radiation and the most impressive one, but only from a secure distance. Its most dangerous effects happen in the case of a direct strike, but it can also make damages to electronic systems and equipment from the distances of up to 1.5km from the direct strike, i.e. in the case of a nearby discharge. Lightning EM field can be analysed using Fast Fourier Transform (FFT). One simple approximation [3] for external pulse field is used and parameters are calculated for the standard pulse function 1.2/50. EM field of a lightning discharge induces currents along conductive structures and resulting field can be obtained using program SPAN. Computer package SPAN for analysis of conductive structures, consisting of linear cylindrical segments, in external EM field of time harmonic plane wave of arbitrary direction and frequency, was made by the author [1,2]. Time response is obtained performing Inverse Fast Fourier Transform (IFFT) to the results of program SPAN in frequency domain. The results for EM field inside the structure and in the near field out of some cage structures on the ground in the case of a nearby lightning discharge are presented in the paper.

Keywords: Lightning discharge, electromagnetic field, pulse function, Fast Fourier Transform, wire structure.

1 Introduction

Lightning can cause damage by directly attaching to an object or a system (direct effects) or it can cause damage by induction effects while striking somewhere near the system (indirect effects). The extent and nature of

Manuscript received November 28, 2002.
The author is with the University of Nis, Faculty of Electronic Engineering of Nis, Beogradska 14, 18 000 Nis, Serbia (e-mail: vjavor@elfak.ni.ac.yu).

damage due to direct and indirect effects of lightning depend both on the characteristics of lightning and the characteristics of objects. Of course that these characteristics, especially of objects, are difficult to be generalised.

Resulting electromagnetic field inside and out of conductive structure consisting of arbitrary positioned and/or interconnected linear cylindrical segments in external EM field is obtained using program SPAN [1]. Integral equation of Hallen's type is used for determining induced currents along conductive segments and for obtaining receiving antenna response for each frequency of the incident field. Current distributions along conductors are approximated by polynomials with complex coefficients [4]. Collocation Method i.e. Point Matching Method is used for approximate numerical solving of the equations system. The problem of other than just antenna conductive segments important for this consideration has been noticed earlier [5]. Ground is concerned to be perfectly conductive and its influence is taken into account using image in the plane mirror of ground surface.

General computer program package was realised on this theoretical basis, for solving an arbitrary configuration of conductive wire structure in external plane wave field of arbitrary frequency and propagation direction.

2 Conductive Structure in Electromagnetic Field

An arbitrary conductive configuration in external EM field of the plane wave of the frequency f, on condition that all parts can be treated as thin cylindrical linear segments with equivalent radii (if any other cross-section) can be treated as receiving antenna and Hallen's integral equation can be used for solving it as a wire structure.

For fulfilled boundary condition for tangential component of electric field on the conductor surface, the solution for the intensity of magnetic vector-potential on the surface of the m-th segment of the receiving antenna is

$$\underline{A}_{z^{(m)}} = \underline{C}_{1m}\cos(kz^{(m)}) + \underline{C}_{2m}\sin(kz^{(m)}) - \underline{F}(z^{(m)}), \tag{1}$$

where

$$\underline{F}(z^{(m)}) = \int_0^{z^{(m)}} \left[\frac{\partial \underline{A}_{x^{(m)}}}{\partial x^{(m)}} + \frac{\partial \underline{A}_{y^{(m)}}}{\partial y^{(m)}}\right]\Bigg|_{\substack{x^{(m)}=a^{(m)}\\ y^{(m)}=0\\ z^{(m)}=s}} \cos\left[k(z^{(m)}-s)\right]\mathrm{d}s$$
$$-\frac{1}{jc}\int_0^{z^{(m)}} \underline{E}_{z^{(m)}e}\sin\left[k(z^{(m)}-s)\right]\mathrm{d}s,$$

where k is the phase constant for frequency f, $\underline{C}_{1m}$ and $\underline{C}_{2m}$ are the complex constants and $(x^{(m)}, y^{(m)}, z^{(m)})$ is the matching point in Descartes' coordinate system with m-th segment as z-axis.

In the arbitrary point on the surface of one of the conductors $\underline{A} = \sum_m \underline{A}(z^{(m)})$, for $m = 1, \ldots, n$, where n is the number of conductive cylindrical line segments of equivalent radii $a^{(m)}$ and

$$\underline{A}(z^{(m)}) = i_{z^{(m)}} \frac{\mu}{4\pi} \int_0^{h^{(m)}} \underline{I}_{z^{(m)}}(z^{(m)\prime}) \frac{e^{-jkr_{m,m}}}{r_{m,m}} \mathrm{d}z^{(m)}. \tag{2}$$

As the following relations exist

$$\underline{A}_{x^{(m)}} = \sum_{\substack{l \\ l \neq m}} \sin(\psi_{l,m}) \sin\theta_{l,m} \underline{A}(z^{(l)}) \tag{3}$$

$$\underline{A}_{y^{(m)}} = \sum_{\substack{l \\ l \neq m}} -\cos(\psi_{l,m}) \sin\theta_{l,m} \underline{A}(z^{(l)}) \tag{4}$$

$$\underline{A}_{z^{(m)}} = \underline{A}(z^{(m)}) + \sum_{\substack{l \\ l \neq m}} \cos\theta_{l,m} \underline{A}(z^{(l)}) \tag{5}$$

from equating (5) with (1) the following is obtained

$$\underline{F}(z^{(m)}) = -\frac{1}{jc} \int_0^{z^{(m)}} \underline{E}_{z^{(m)}e} \sin[k(z^{(m)} - s)]\mathrm{d}s + \frac{\mu}{4\pi} \sum_{\substack{l \\ l \neq m}} \int_0^{h^{(l)}} \underline{I}_{z^{(l)}}(z^{(l)\prime}) \tag{6}$$
$$\times \int_0^{z^{(m)}} \cos[k(z^{(m)} - s)] \left[\frac{\partial}{\partial z^{(l)\prime}} + \cos\theta_{l,m} \frac{\partial}{\partial z^{(m)}}\right] \frac{e^{-jkr_{l,m}}}{r_{l,m}} \Bigg|_{\substack{x^{(m)}=a^{(m)} \\ y^{(m)}=0 \\ z^{(m)}=s}} \mathrm{d}s\mathrm{d}z^{(l)\prime}.$$

Using boundary conditions on the conductors surfaces and Kirchhoff's laws in the points of their conjunctions and from (1), (2) and (6), the following equation system is obtained, for determining unknown complex constants and the coefficients of the polynomial approximations of the segments currents

$$\sum_{l=1}^{n} \int_0^{h^{(l)}} \underline{I}_{z^{(l)}}(z^{(l)\prime}) \underline{K}_{m,l}(z^{(m)}, z^{(l)\prime}) \mathrm{d}z^{(l)\prime} - \frac{4\pi \underline{C}_{1m}}{\mu} \cos(kz^{(m)})$$
$$- \frac{4\pi \underline{C}_{2m}}{\mu} \sin(kz^{(m)}) = \frac{4\pi}{jc\mu} \int_0^{z^{(m)}} \underline{E}_{z^{(m)}e} \sin[k(z^{(m)} - s)]\mathrm{d}s \tag{7}$$

for $m = 1, 2, \ldots, n$.

Matching points are chosen equidistantly along m-th segment. The electric field in arbitrary field point (x, y, z) is of different phase in different matching points, and $\underline{E}_{z^{(m)}e}$ is its tangential component in corresponding matching point on the surface of the m-th segment, for $z^{(m)} = s$, in (1), (6), (7) and (13). For $l \neq m$, the Kernels of these integrals are of the forms

$$\underline{K}_{m,l}(z^{(m)}, z^{(l)\prime}) = \cos\theta_{l,m} \frac{e^{-jkr_{l,m}}}{r_{l,m}}\Bigg|_{\substack{x^{(m)}=a^{(m)}\\ y^{(m)}=0}} \tag{8}$$

$$- \int_0^{z^{(m)}} \cos\left[k(z^{(m)} - s)\right]\left[\frac{\partial}{\partial z^{(l)\prime}} + \cos\theta_{l,m}\frac{\partial}{\partial z^{(m)}}\right]\frac{e^{-jkr_{l,m}}}{r_{l,m}}\Bigg|_{\substack{x^{(m)}=a^{(m)}\\ y^{(m)}=0\\ z^{(m)}=s}} \mathrm{d}s \tag{9}$$

and for $l = m$

$$\underline{K}_{m,l}(z^{(l)}, z^{(l)\prime}) = \frac{e^{-jkr_{l,l}}}{r_{l,l}}\Bigg|_{\substack{x^{(l)}=a^{(l)}\\ y^{(l)}=0}}.$$

The distance between the matching point on the m-th segment and the element with current of the l-th segment is, for $l \neq m$

$$\begin{aligned} r_{l,m} = \Big\{ &\left[x^{(m)} - x_{pl}^{(m)} - z^{(l)\prime}\sin\psi_{l,m}\sin\theta_{l,m}\right]^2 \\ &+ \left[y^{(m)} - y_{pl}^{(m)} - z^{(l)\prime}\cos\psi_{l,m}\sin\theta_{l,m}\right]^2 \\ &+ \left[z^{(m)} - z_{pl}^{(m)} - z^{(l)\prime}\cos\theta_{l,m}\right]^2 \Big\}^{\frac{1}{2}} \end{aligned} \tag{10}$$

where the point $(x_{pl}^{(m)}, y_{pl}^{(m)}, z_{pl}^{(m)})$ is the beginning of the l-th segment with respect to the co-ordinate system of m-th segment, and for $l = m$

$$r_{l,l} = \sqrt{x^{(l)2} + y^{(l)2} + (z^{(l)} - z^{(l)\prime})^2}. \tag{11}$$

In equation (10) $\theta_{l,m}$, $\psi_{l,m}$ and $\varphi_{l,m}$ are the mutual Euler's angles for the corresponding Descartes' co-ordinate systems of l-th segment and m-th segment.

Polynomial approximations for the segments currents are

$$\underline{I}_{z^{(m)}}(z^{(m)\prime}) = \sum_{t=0}^{p_m} \underline{B}_{mt}\left[\frac{z^{(m)\prime}}{h^{(m)}}\right]^t, \quad \text{for} \quad m = 1, 2, \ldots, n, \tag{12}$$

where $h^{(m)}$ is the length of the m-th conductive segment, $z^{(m)\prime}$ is the co-ordinate of the point along current source, $\underline{B}_{mt}$ is the complex coefficient and p_m is the polynomial degree for the current approximation along the m-th segment of the structure.

Kirchhoff's current law for the node p is

$$\sum_{r=1}^{n}\sum_{s=0}^{n_r}\underline{B}_{rs}\left[\frac{z^{(r)}}{h^{(r)}}\right]^s = 0, \tag{13}$$

where r is the number of incident conductors to the node p, $h^{(r)}$ is the length of the r-th segment, $z^{(r)}$ is the co-ordinate of the incident beginning/end of the r-th segment in the corresponding co-ordinate system, $\underline{B}_{rs}$ is the complex polynomial coefficient of the current approximation of n_r-degree along r-th segment.

The potential on either beginning or end of the m-th segment is

$$\begin{aligned}\underline{V}_{z^{(m)}} &= jc\Big[\underline{C}_{1m}\sin(kz^{(m)}) + \underline{C}_{2m}\cos(kz^{(m)})\Big] \\ &+ \int_0^{z^{(m)}} \underline{E}_{z^{(m)}e}\cos[k(z^{(m)}-s)]\mathrm{d}s \\ &+ jc\int_0^{z^{(m)}}\left[\frac{\partial\underline{A}_{x^{(m)}}}{\partial x^{(m)}} + \frac{\partial\underline{A}_{y^{(m)}}}{\partial y^{(m)}}\right]\Bigg|_{\substack{x^{(m)}=a^{(m)}\\ y^{(m)}=0\\ z^{(m)}=s}} \sin\big[k(z^{(m)}-s)\big]\mathrm{d}s\end{aligned} \tag{14}$$

what equates with the potentials of all segments ends/beginnings in the conjunction of node p.

Electric field in the points in the near zone of the structure is the result of external field $\underline{E}_e$, and of field $\underline{E}$, that is the consequence of induced currents, which components can be determined from the results for magnetic vector potential, according to the equation

$$\begin{aligned}\underline{E} &= \mathrm{grad}\underline{\varphi} - j\omega\underline{A} \\ &= -j\frac{c}{k}\mathrm{graddiv}\underline{A} - j\omega\underline{A}.\end{aligned} \tag{15}$$

3 Computer Package SPAN

Program package SPAN [1,2] is realised such that it solves system of equations (7) and gives the results for currents along the conductive segments, treating structure as receiving antenna. Field, potential and other important parameters can also be obtained. Program is written in FORTRAN language and double precision is preferable. Ending points of segments are also included in matching points. Kirchhoff's laws are used for the nodes where segments are interconnected and condition that the current is zero at free ends of segments. Boundary condition for tangential component of electric field is used, so (7) is of Electric Field Integral Equations (EFIE).

4 FFT of Lightning Pulse Function

In the case of pulse field of lightning discharge current, one simple analytical approximation of pulse function representing external electric field is chosen [3],

$$\frac{y(t)}{y_{max}} = \begin{cases} (\tau e^{1-\tau})^a, & 0 \le \tau \le 1 \\ (\tau e^{1-\tau})^b, & 1 \le \tau < \infty \end{cases} \tag{16}$$

for $\tau = t/t_m$, which, for the standard pulse 1.2/50, that decreases to half of the maximum value in 50μs, with rising time 1.2μs, has the obtained parameters values $a = 4$ and $b = 0.0312596735$, while $t_m = 1.906398381\mu$s. Time t_m is the time in which pulse function achieves maximum value and after that the intensity decreases to null (Fig. 1). In order to analyse conductive structure's response to pulse electromagnetic field of a lightning discharge, FFT is performed in 8192 points, because of specific pulse excitation function with these characteristics: short upward and long lasting downward part of the function representing this field.

For sampling of normalised function $y(t)/y_{max}$ from Fig. 1, the interval $\Delta T = 19.06398381$ns is chosen. This sampling interval and the number of points $N = 8192$ for FFT correspond to transforming the function $y(t)/y_{max}$ from time domain, from interval $[0, T]$, where $T = N\Delta T = 156.17215537152\mu$s, to the frequency domain and to interval $[-f/2, f/2]$. Program FAS [6] is used for both FFT and IFFT. Sampling frequency is $f = 1/\Delta T \cong 52.455$MHz and corresponding interval in frequency domain is $\Delta f = (N\Delta T)^{-1} = (T)^{-1} \cong 6.4$kHz. Real and imaginary parts of FFT for pulse function presented in Fig. 1 are presented on Figs. 2 and 3. It is not necessary to take into account all of the points, in order to obtain IFFT, which is very useful for decreasing computation time while calculating in frequency domain.

5 Examples of Cage Structures in Electromagnetic Field

One conductive structure shaped as parallelepiped is chosen, with basic dimensions $a = 9$m, $b = 12$m and of height $c = 6$m, case (a). Radii of all con-

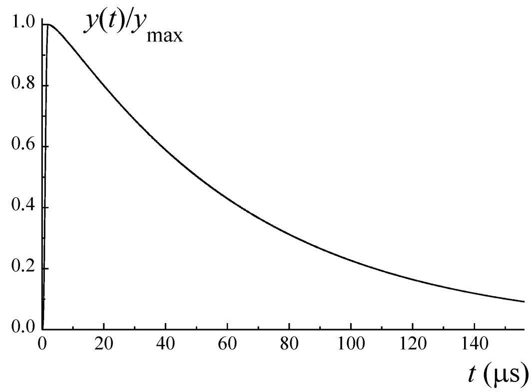

Fig. 1. Normalised value of pulse electric field.

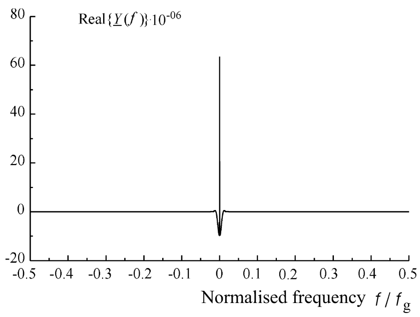

Fig. 2. Real part of FFT for pulse excitation (Fig. 1).

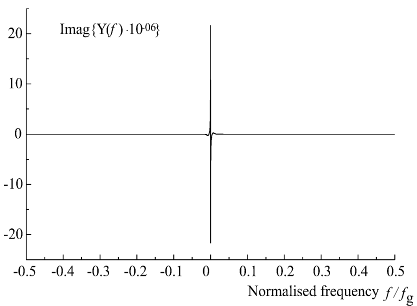

Fig. 3. Imaginary part of FFT for pulse excitation (Fig. 1).

ductive segments (these can be of grounding installations) are $a^{(i)} = 0.03$m, for $i = 1, 2, \dots, n$. For this object on ground surface, its plane mirror figure has to be added, while ground half-space excluded and taken into account by reflected field added to the incident field. Cage structure as in Fig. 4 has to be treated in frequency domain, with minimum of 12 segments for such configuration.

Pulse electric field is given in the first node (as in Fig. 4), with its components $E_\theta = E(t)$, $E_\psi = 0$, $\theta = \pi/2$, $\psi = 0$, where θ and ψ are cylindrical coordinates. For maximum value of the incident field $E_{max} =$ 1V/m, all of the results are presented in the form of graphics. So, all of the values of the resulting field have to be multiplied by factor MPF= 6000 for the distances about 200m and with MPF= 1800 for the distances about 500m from the lightning current of maximum value $I = 10$kA, as in Ref. [7]. It is sufficient to take $p_m = 2$ as the polynomial degree for the approximation of currents. Program SPAN is modified so that it calculates the response for

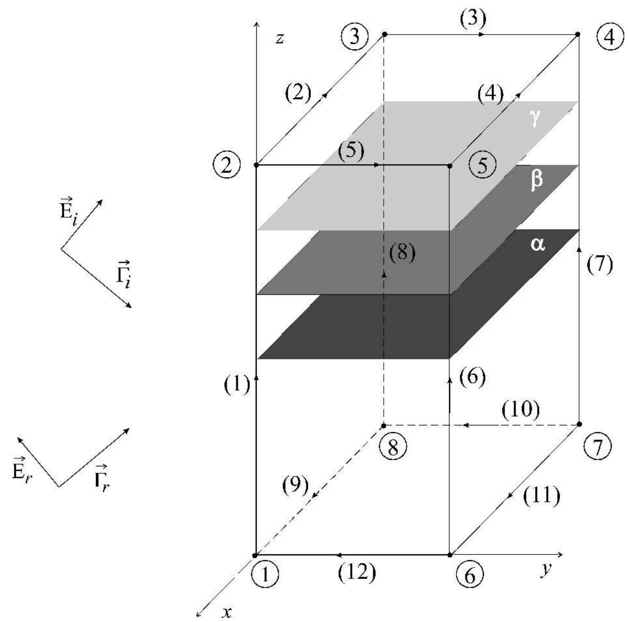

Fig. 4. Conductive structure in pulse electromagnetic field.

all of the frequencies for FFT, i.e. in all points of discretization in frequency domain. IFFT is used to determine time domain response on the basis of obtained SPAN results. Electric field inside the conductive structure is of pulse shape, but amplitudes are different in different points inside the structure, depending on co-ordinates.

6 Results

Plane α, for $z = 6$m, is the plane of ground surface and of zero potential. In the case of a parallelepiped object, positioned on the ground, it represents plane mirror surface. Chosen points in which electric field is calculated are inside the object: A(–6m,4.5m,8m), B(–4m,3m,8m), C(–4m,8m,8m), D(–10m,3m,8m) and E(–10m,8m,8m), in the plane β, for $z = 8$m, parallel to the ground surface, on the height of 2m. In the plane γ, for $z = 10$m, on the height 4m from the ground surface, are the points: F(–6m,4.5m,10m), G(–4m,3m,10m), H(–4m,8m,10m), I(–10m,3m,10m) and J(–10m,8m,10m). The results for electric field components are presented in Figs. 5 – 10, for the points: A (Fig. 5), B (Fig. 6), C (Fig. 7), D (Fig. 8), E (Fig. 9) and F (Fig. 10). First node is the origin of the Descartes' co-ordinate system with segment (1) as z-axis. These graphics present pulse character of electric field inside parallelepiped structure, but maximum values are different in different points. These values enable estimation of electric field values for the purpose of equipment protection against EM field influence inside objects. $N = 8192$ points for FFT prolongs the calculation time, but much smaller number than this is also enough for obtaining results. Using results for $N/10$ points and adding nulls for the rest of the points, because the expected solution for field components in each point has to be also of pulse shape, results for the field can be obtained using IFFT. It is also possible that the number of the points of interest for obtaining solution can be even smaller and this should be investigated, because it decreases the time for calculations, but that is also related to desired accuracy.

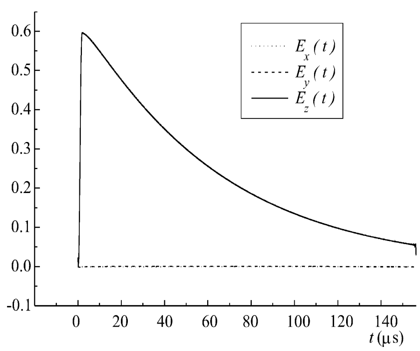

Fig. 5. Electric field components in the point A(–6m,4.5m,8m) versus time for the case(a).

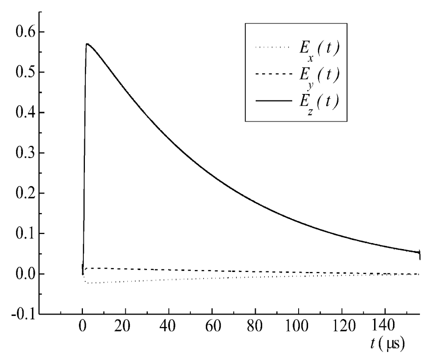

Fig. 6. Electric field components in the point B(–4m,3m,8m) versus time for the case(a).

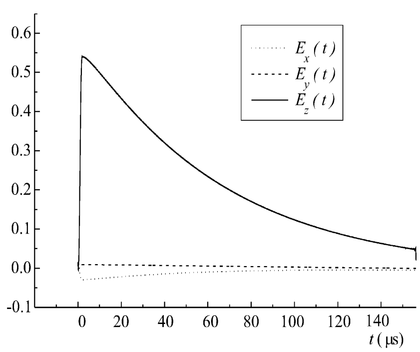

Fig. 7. Electric field components in the point C(–4m,8m,8m) versus time for the case(a).

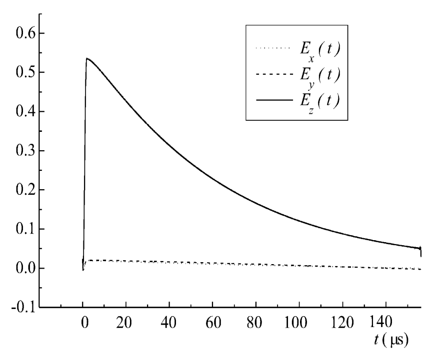

Fig. 8. Electric field components in the point D(–10m,3m,8m) versus time for the case(a).

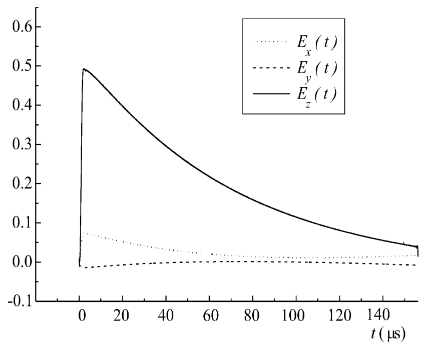

Fig. 9. Electric field components in the point E(–10m,8m,8m) versus time for the case(a).

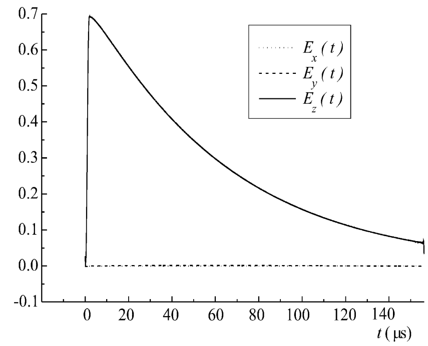

Fig. 10. Electric field components in the point F(–6m,4.5m,10m) versus time for the case(a).

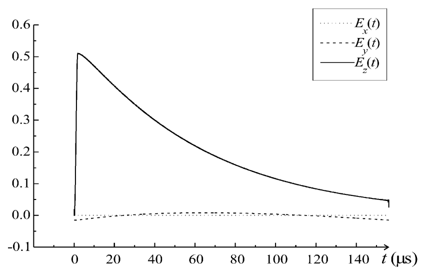

Fig. 11. Electric field components in the point A(–6m, 4.5m, 8m) versus time for the case (b).

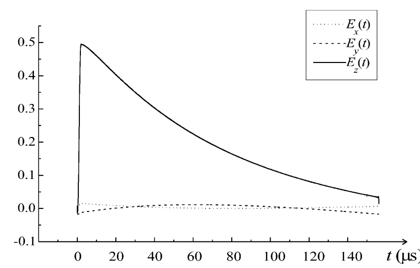

Fig. 12. Electric field components in the point B(–4m, 3m, 8m) versus time for the case (b).

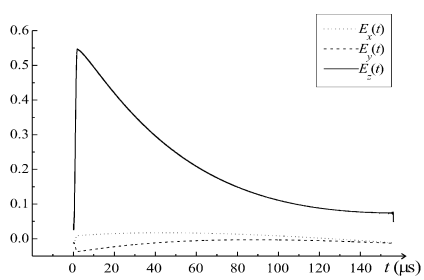

Fig. 13. Electric field components in the point C(–4m, 8m, 8m) versus time for the case (b).

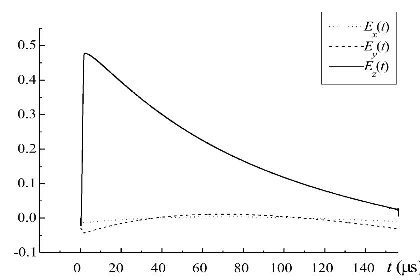

Fig. 14. Electric field components in the point D(–10m, 3m, 8m) versus time for the case (b).

Values of $\Re\{\underline{Y}(f)\}$, $\Im\{\underline{Y}(f)\}$ and $|\underline{Y}(f)/\underline{Y}(0)|$ for different values of frequency f are presented in the Table I. The obtained results show that standard pulse has LF spectrum and that the main part of the power of lightning discharge pulse is in the frequency range $0-10$kHz. For conductive structure of parallelepiped shape, with basic dimensions $a = 9\text{m}$, $b = 12\text{m}$ and of height $c = 10\text{m}$, case (b), with radii of all conductive segments $a^{(i)} = 0.03\text{m}$, the results for electric field components are presented in Figs. 11 − 14. Figure 11 presents electric field components $E_x(t)$, $E_y(t)$ and $E_z(t)$ versus time (from 0 to 156.172μs) in the point A(−6m, 4.5m, 8m), Fig. 12 presents these components in the point B(−4m, 3m, 8m), Fig. 13 in the point C(−4m, 8m, 8m) and Fig. 14 in the point D(−10m, 3m, 8m). It can be noticed that for higher object, electric field in the same chosen points is of smaller values. This can be used for choosing more secure places for some equipment inside one conductive structure exposed to external pulse electric field, such as of lightning discharge.

Table 1. FFT for the standard pulse function according to expression (16).

f [Hz]	$\Re\{\underline{Y}(f)\}$	$\Im\{\underline{Y}(f)\}$	$\lvert\underline{Y}(f)/\underline{Y}(0)\rvert$
0.00000000E+00	0.62319951E−04	0.00000000E+00	0.10000000E+01
0.64031901E+04	0.68918465E−05	−0.21723406E−04	0.36570046E+00
0.12806380E+05	0.87132777E−06	−0.11785367E−04	0.18962681E+00
0.19209570E+05	−0.28394736E−06	−0.78672642E−05	0.12632209E+00
0.25612760E+05	−0.66103950E−06	−0.58416039E−05	0.94333944E−01
0.32015951E+05	−0.81895382E−06	−0.46101970E−05	0.75134389E−01
0.38419141E+05	−0.89429051E−06	−0.37820656E−05	0.62361373E−01
0.44822331E+05	−0.93241165E−06	−0.31856212E−05	0.53261805E−01
0.51225521E+05	−0.95154399E−06	−0.27342230E−05	0.46454894E−01
0.57628711E+05	−0.96003311E−06	−0.23795481E−05	0.41173230E−01
0.64031901E+05	−0.96208314E−06	−0.20925731E−05	0.36956747E−01
0.70435091E+05	−0.95997289E−06	−0.18548385E−05	0.33513086E−01
0.76838281E+05	−0.95501187E−06	−0.16540564E−05	0.30647662E−01
0.83241471E+05	−0.94798996E−06	−0.14817381E−05	0.28226003E−01
0.89644661E+05	−0.93940390E−06	−0.13318356E−05	0.26152226E−01
0.96047852E+05	−0.92957807E−06	−0.11999270E−05	0.24356143E−01
0.10245104E+06	−0.91873193E−06	−0.10827066E−05	0.22785199E−01
0.10885423E+06	−0.90701922E−06	−0.97765520E−06	0.21399274E−01
0.21770846E+06	−0.63342396E−06	−0.61643722E−07	0.10212082E−01
0.68514134E+06	0.45101933E−07	0.45967292E−07	0.10333541E−02
0.22475197E+07	0.49313361E−09	0.62586716E−08	0.10073931E−03
0.15860702E+08	0.81575669E−12	0.62365368E−09	0.10007296E−04
0.25030070E+08	−0.20139125E−12	0.62612237E−10	0.10046953E−05
0.26105806E+08	−0.14171106E−12	0.63502551E−11	0.10192300E−06
0.26214660E+08	−0.13772593E−12	0.66891888E−12	0.10958772E−07

7 Conclusion

Using simple approximation for the pulse excitation, program FAS [6] for FFT and program SPAN [1,2] for frequency domain, pulse response of receiving structure in external EM field is obtained, for example of a cage conductive structure. The example can represent one small building configuration with edge lightning protection conductors of given dimensions [8], but this procedure can be used for any conductive configuration consisting of arbitrary positioned linear segments in any external electromagnetic field. Results for the field in some points inside the object present pulse character of electric field function, but also show that some points inside the object are more protected from external field than the other. This can be used for protection of some important equipment and instruments against dangerous EM field values. The procedure that is used for calculating electric field in the case of a lightning discharge, can be used in cases of other pulse excitations with respectable energy dissipation, such as EM pulse of nuclear explosions (EMINE), and other important analyses of EM field influence on certain objects. Further research and improvement of SPAN program package should include imperfectly conducting ground [9] and more precise calculation of the integrals [10].

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