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Variational Solution of the Steady Magnetic Field Distribution in Ferromagnetic Conductor

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Abstract: A distribution of the steady magnetic filed in a very large ferromagnetic conductor of arbitrary, but known cross-section is determined using variational approach with Ritz's method for approximate solving of the problem. Then an axial component of the magnetic vector potential is presented as finite functional series, automatically satisfying boundary condition that the normal component of the magnetic induction on the conductor's surface vanishes. The convergence and the accuracy of the proposed method are investigated. Several results for maps of the force lines and of the inductivity per unit conductor length are compared with corresponding numerical results obtained by Finite Element Method (program package FEMM) and excellent agreement s observed.

Keywords: Steady mgnetic field, magnetic vector potential, ferromagnetic conductor, Ritz's method, Finite Element Method.

1 Introduction

The aim of this paper is to determine a steady magnetic field distribution in a very large ferromagnetic conductor having known, but arbitrary shaped cross-section. For this purpose the standard variational principle is used [1, 2]. The axial component of the magnetic vector potential is presented as finite functional series, automatically satisfying boundary condition that the normal component of the magnetic induction on the conductor's surface

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vanishes [3, 4]. This boundary condition is automatically realized if an intensity of the magnetic vector potential vanishes on the contour defining the conductor's cross-section. The unknown coefficients in the magnetic vector potential approximation are determined numerically, using Ritz's variational method. Then all integrals appearing in the variational procedure are determined in closed form, so the influence of possible ill-conditioned linear equations system on the solution accuracy is minimized. The convergence and the accuracy of the obtained results are investigated. The obtained results for maps of the force lines, for intensity of the magnetic induction components and of the inductivity per unit conductor length are compared with corresponding values determined by program package FEMM [5], based on the Finite Element Method technique and very good agreement is concluded. Several numerical results for maps of the force lines and of the values of the inductivity per unit conductor length for ferromagnetic conductor of elliptical, rectangular, triangular, trapezoidal and semi-circular cross-sections are presented in this paper.

2 Short Theoretical Approach

A very large ferromagnetic conductor with steady current I and of known, but arbitrary shaped cross-section, as in Figure 1, is considered. It is supposed that the conductor's material is homogeneous and linear of very large magnetic permittivitty, μ, so the magnetic field exists only in the conductor's interior, with the zero value of the normal component of the magnetic induction on the conductor's surface. As it is known, the magnetic field satisfies the following Maxwell's equations [2]

$$\mathrm{rot}\vec{H} = J, \tag{1}$$

end

$$\mathrm{div}\vec{B} = 0, \tag{2}$$

including the constitutive relation

$$\vec{B} = \mu\vec{H}, \tag{3}$$

where

$$\vec{J} = J\hat{z}, \tag{4}$$

denotes the steady current density in the conductor and $\hat{z}$ is the unit vector collinear to the conductor's axis.

After introducing the magnetic vector potential $\vec{A} = A\hat{z}$, so it is

$$\vec{B} = \mathrm{rot}\vec{A} \quad \text{and} \quad \mathrm{div}\vec{A} = 0, \tag{5}$$

it is obtained

$$\Delta A = -\mu J. \tag{6}$$

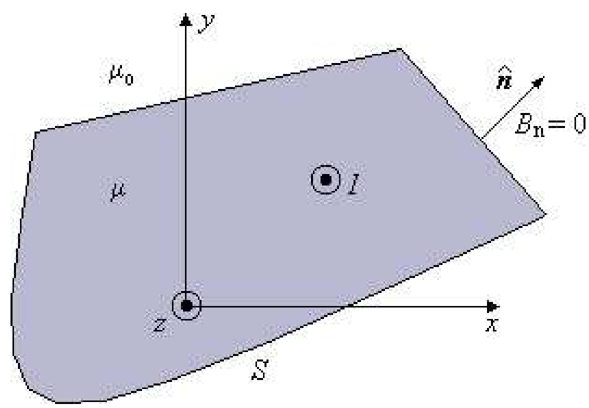

Fig. 1. The conductor's cross-section.

In order to apply the variational approach [1, 2] for obtaining the solution of the present problem, the following functional is formed

$$F = \int_S \Big(\frac{B^2}{2\mu} - JA\Big)\mathrm{d}S, \tag{7}$$

which extremes coincide with the presented Maxwell's equations.

If the Ritz's method for approximate solving of the present problem is used, the axial component of the magnetic vector potential as finite functional series is supposed as

$$A = \sum_{n=1}^{N} A_n f_n(x, y), \tag{8}$$

where A_n are unknown coefficients and $f_n(x, y)$ are the basic functions, so selected that the boundary condition that the normal component of the magnetic induction on the conductor surface vanishes. If the components of the magnetic induction are

$$B_x = \frac{\partial A}{\partial y}, \quad B_y = -\frac{\partial A}{\partial x} \quad \text{and} \quad B_z = 0, \tag{9}$$

the boundary condition can be written as $\vec{B}\hat{n}$ on the conductor's surface S, where $\hat{n}$ denotes the outward unit normal vector on the conductor's surface.

Substituting (8) in (7) and performing the Ritz's procedure, the following system of linear equations can be obtained,

$$\frac{\partial F}{\partial A_n} = 0, \text{ for } n = 1, 2, \ldots, N. \tag{10}$$

After solving this system, unknown coefficients in the magnetic vector potential approximation are determined and the necessary calculation can be realized in the standard way. The equation of the force lines can be presented as

$$A = C^{te}, \tag{11}$$

so

$$A = 0, \tag{12}$$

on the conductor surface can be used, because the contour of the conductor's cross-section coincides with the force line. The inductivity per unit conductor length can be expressed as

$$L' = \frac{1}{\mu I^2} \int_S B^2 \mathrm{d}S. \tag{13}$$

3 Examples

Example I

In the case of the ferromagnetic conductor of elliptical cross-section, asin Figure 2, the axial component of the magnetic vector potential is approximated with finite functional series of the following form

$$A = \sum_{n=1}^{N} A_n \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 \right)^n, \tag{14}$$

so the boundary condition $\vec{B}\hat{n} = 0$, or $A = 0$ on the conductor's surface is automatically satisfied. All integrals appearing in the variational solution of this problem can be determined in closed form (Appendix 1).

The equation of the force lines $A = C^{te}$ gives elliptical curves, $0 \leq x^2/a^2 + y^2/b^2 = C^{te} \leq 1$ (See Figure 3). The value of the magnetic vector potential is zero on the conductor's surface.

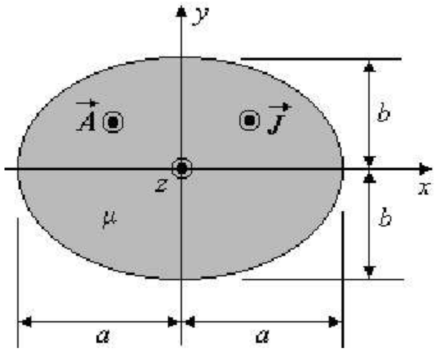

Fig. 2. Ferromagnetic conductor of elliptical cross-section.

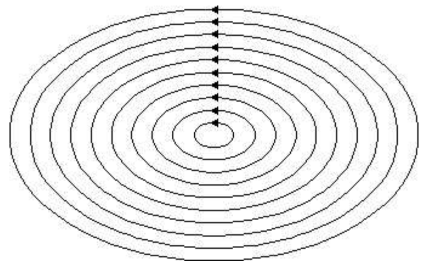

Fig. 3. Force lines of the conductor of elliptical cross-section.

In the case of elliptical conductor the exact solution exists, for $N = 1$ in the expression (14),

$$A = -\frac{\mu J a^2 b^2}{a^2 + b^2}\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right), \quad \text{for} \quad J = \frac{I}{ab\pi}, \tag{15}$$

where I is the total steady current in the conductor [2].

The exact value of the inductivity per unit conductor length is

$$L' = \frac{\mu}{4\pi}\frac{ab}{a^2 + b^2}. \tag{16}$$

As it is presented in Table 1, the approximate values of the inductivity per unit conductor length agree very well with the exact results.

Table 1. Inductivity per unit length of the elliptical conductor, $8\pi L'/\mu$, for different number of terms in (14) and different ratio a/b.

$\frac{a}{b}$	$N = 2\ldots5$	$N = 1$
1	1.000000	1.000000
2	0.800000	0.800000
3	0.600000	0.600000
4	0.470588	0.470599
5	0.384615	0.384615
6	0.324324	0.324324
7	0.280000	0.280000
8	0.246154	0.246154
9	0.219512	0.219512
10	0.198020	0.198020

The agreement with the Finite Element Method (FEMM [5]) is excellent, too (Figure 4).

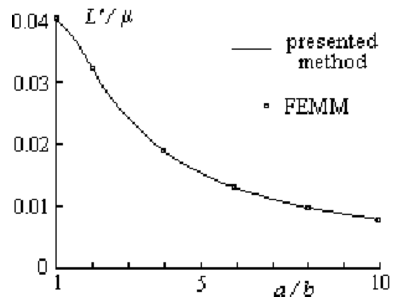

Fig. 4. Inductivity per unit length of the elliptical conductor versus ratio a/b.

Example II

For the ferromagnetic conductor of rectangular cross-section, as in Figure 5, the axial component of the magnetic vector potential is approximated with two-dimensional functional series of the following form

$$A = \sum_{n=1}^{N} \sum_{m=1}^{M} A_{nm} \left(\frac{x^2 - a^2}{a^2} \right)^n \left(\frac{y^2 - b^2}{b^2} \right)^m, \tag{17}$$

so the boundary condition $\vec{B}\hat{n} = 0$, or $A = 0$, on the conductor's surface is automatically satisfied. All integrals appearing in the variational solution of this problem can be determined in closed form (Appendix 2).

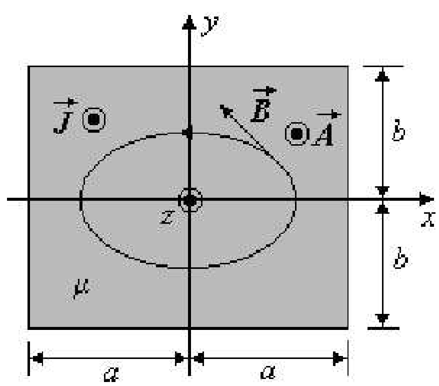

Fig. 5. Ferromagnetic conductor of rectangular cross-section.

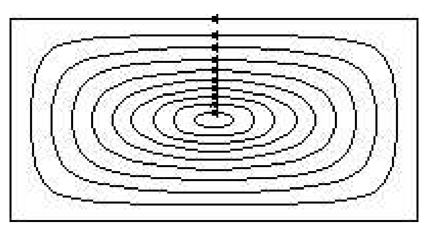

Fig. 6. Force lines of the conductor of rectangular cross-section.

The force lines are presented in Figure 6. The convergence of the maximal value of the magnetic induction, B_{max} (If $a > b$, the maximal value of the magnetic induction is on the conductor surface, at the points $x = 0,\ y = \pm b$),

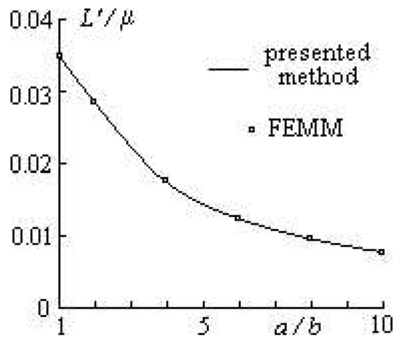

Fig. 7. Inductivity per unit length of the rectangular conductor versus ratio a/b.

Table 2. Convergence of the maximal value of the magnetic induction, B_{max}, and of the inductivity per unit length of the rectangular conductor, L', for different ratio a/b.

$N = M$	$\frac{aB_{max}}{\mu I}$	$\frac{L'}{\mu}$
1	0.15625	0.034722
2	0.16797	0.035139
3	0.16943	0.035144
4	0.16856	0.035144
5	0.16897	0.035144
6	0.16874	0.035144

and of the inductivity per unit length of the rectangular conductor, L', for different ratio a/b, with the number of the terms in the approximation (17) is presented in the Table 2. The agreement between presented results and the values obtained by Finite Element Method is excellent, as it is showed

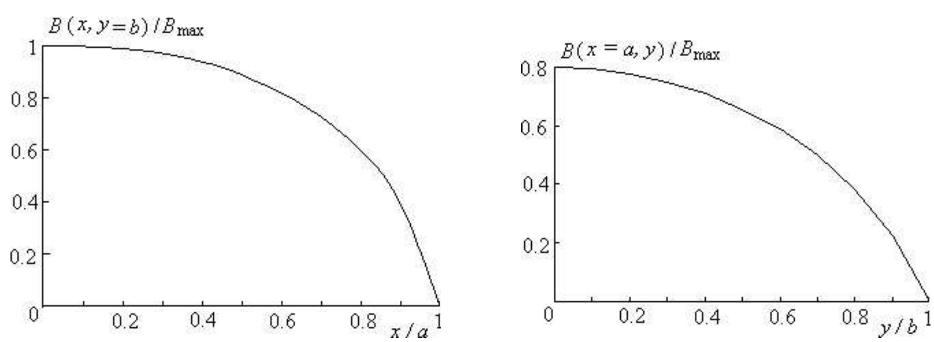

Fig. 8. Distribution of the magnetic induction on the surface of the rectangular conductor, for $a = 2b$

in Figure 7. The distribution of the magnetic induction on the surface of the rectangular conductor is presented in the Figure 8, when $a = 2b$.

Example III

For the ferromagnetic conductor of triangular cross-section, as in Figure 9, the axial component of the magnetic vector potential is approximated with two-dimensional functional series of the following form

$$A = \sum_{n=1}^{N} \sum_{m=1}^{M} A_{nm} \Big(\frac{x}{h}\Big)^n \Big[\Big(\frac{x}{h} - 1\Big)^2 - \Big(\frac{y}{a}\Big)^2\Big]^m, \tag{18}$$

so the boundary condition $\vec{B}\hat{n} = 0$, or $A = 0$, on the conductor's surface is automatically satisfied. All integrals appearing in the variational solution of this problem can be determined in closed form (Appendix 3).

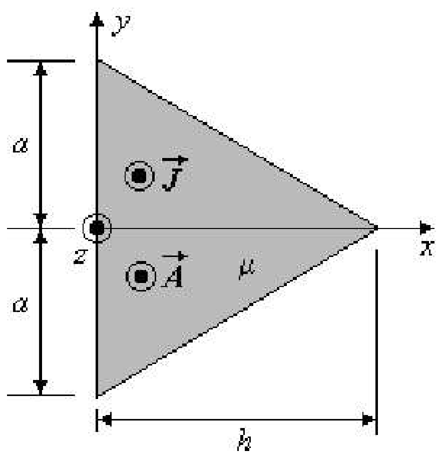

Fig. 9. Ferromagnetic conductor of triangular cross-section.

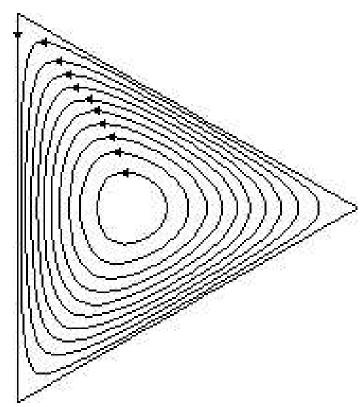

Fig. 10. Force lines of the conductor of triangular cross-section.

If $h = a\sqrt{3}/2$ the expression (18) gives the exact solution for $N = M = 1$, when the cross-section is equilateral triangle.

The inductivity per unit length of the triangular conductor, L', for different ratio a/h is compared with corresponding results obtained with the program package FEMM and very good agreement is obtained, Figure 11. The distribution of the magnetic induction on the surface of the triangular conductor is presented in the Figure 12, for $h = a\sqrt{3}/2$. Then the maximal value of the magnetic induction, B_{max}, is in the middle point of the sides of the triangular cross-section.

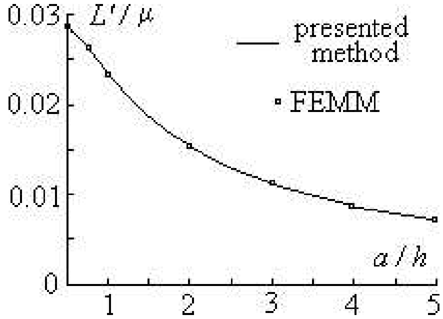

Fig. 11. Inductivity per unit length of the triangular conductor versus ratio a/h.

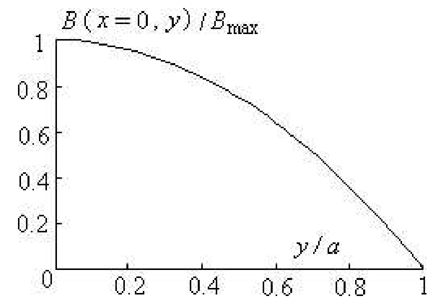

Fig. 12. Distribution of the magnetic induction on the surface of the triangular conductor, for $h = \sqrt{3}/2$.

Example IV

For the ferromagnetic conductor of trapezoidal cross-section, as in Figure 13, the axial component of the magnetic vector potential is approximated with three-dimensional functional series of the following form

$$A = \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{k=1}^{K} A_{mnk}\left(\frac{x}{h}\right)^m\left(\frac{x}{h}-1\right)^n\left\{\frac{y^2}{a^2}-\left[\left(\frac{x}{h}\right)\left(\frac{b}{a}-1\right)+1\right]^2\right\}^k, \quad (19)$$

so the boundary condition $\vec{B}\hat{n} = 0$, or $A = 0$, on the conductor's surface is automatically satisfied. All integrals appearing in the variational solution of this problem can be determined in closed form (Appendix 4).

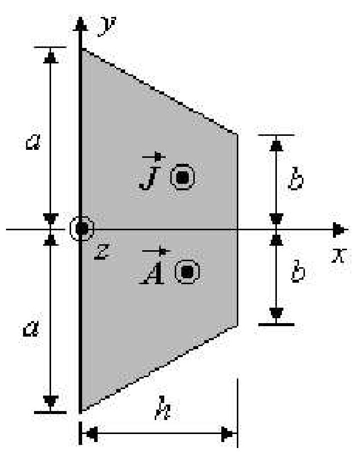

Fig. 13. Ferromagnetic conductor of trapezoidal cross-section.

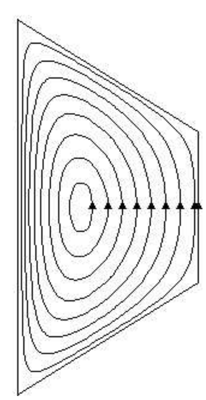

Fig. 14. Force lines of the conductor of trapezoidal cross-section.

The convergence of the results of the inductivity per unit length of the trapezoidal conductor, L', for different terms in the approximation (19) and for $a = 2b = h$, is presented in the Table 3. The force lines in the conductor from Figure 13 are presented in Figure 14.

Table 3. Inductivity per unit length of the trapezoidal conductor, L'/μ, for different number of terms in (19) and $a = 2b = h$. The value are determined using FEMM is $L'/\mu = 0.0305497$.

M	N	K	$\frac{L'}{\mu}$
1	1	1	0.024198
1	1	2	0.027013
1	2	1	0.028902
2	1	2	0.029814
2	1	3	0.030335
2	2	3	0.030448
3	2	3	0.030511
3	3	3	0.030531
4	4	4	0.030553
5	5	5	0.030557

Example V

For the ferromagnetic conductor of semi-circular cross-section, as in Figure 15, the axial component of the magnetic vector potential is approximated with two-dimensional functional series of the following form

$$A = \sum_{n=1}^{N} \sum_{k=1}^{K} A_{nk} \left(\frac{x}{a}\right)^{n\alpha} \left(\frac{x^2}{a^2} + \frac{y^2}{a^2} - 1\right)^{k\beta}, \tag{20}$$

where α and β are greater than one. So the boundary condition $\vec{B}\hat{n} = 0$, or $A = 0$, on the conductor's surface is automatically satisfied. All integrals appearing in the variational solution of this problem can be determined in closed form (Appendix 5).

The convergence of the results of the inductivity per unit length of the semi-circular conductor, L', for different terms in the approximation (20) and for $\alpha = 2\beta$, is presented in the Table 4. The force lines in the conductor from Figure 15 are presented in Figure 16.

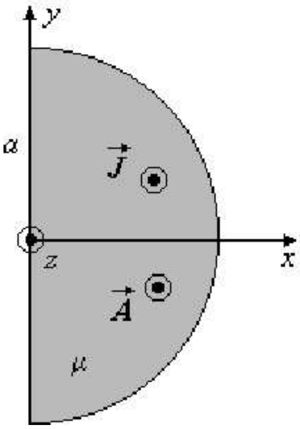

Fig. 15. Ferromagnetic conductor of semi-circular cross-section.

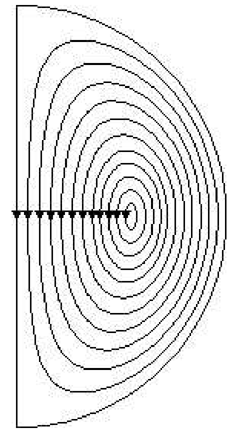

Fig. 16. Force lines of the conductor of semi-circular cross-section.

Table 4. Inductivity per unit length of the semi-circular conductor, L'/μ, for different number of terms in (20). The value are determined using FEMM is $L'/\mu = 0.029452$.

$N = K$	$\frac{L'}{\mu}$
1	0.0224664
2	0.0253785
3	0.0268818
4	0.0276212
5	0.0279122
6	0.0284405
8	0.0288588
10	0.0290261
15	0.0293358
20	0.0293394

4 Conclusion

The paper shows a variational approach for approximate determination of the magnetic field distribution in a very large ferromagnetic conductor of known, but arbitrary shaped cross-section and with steady current. The axial component of the magnetic vector potential is presented in the form of finite functional series of known convenient basis functions, so the boundary condition on the conductor's surface that normal component of the magnetic induction vanishes is automatically satisfied. If the polynomial approximation of magnetic vector potential function is adopted, all integrals obtained in variational solution of the presented problem can be determined in closed form, so the influence of possible ill-conditioned linear equations system on the solution accuracy is minimized. The obtained results are compared with corresponding exact and numerical values determined using Finite Element Method (FEMM) and excellent agreement is concluded.

Appendix

Appendix 1

All integrals appearing in the variational solution of the conductor having elliptical cross-section can be determined as

$$\begin{aligned} I_p &= \int\limits_{x=0}^{a} \int\limits_{y=0}^{b\sqrt{1-\frac{x^2}{a^2}}} \left(\frac{y}{b}\right)^2 \left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)^p \frac{\mathrm{d}x}{a}\frac{\mathrm{d}y}{b} \\ &= \sum_{r=0}^{p} \binom{p}{r} (-1)^{p-r} \frac{\pi}{2r+3} \frac{(2p+3)!!}{2(2p+4)!!}, \end{aligned} \tag{21}$$

and

$$\begin{aligned} L_p &= \int\limits_{x=0}^{a} \int\limits_{y=0}^{b\sqrt{1-\frac{x^2}{a^2}}} \left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)^p \frac{\mathrm{d}x}{a}\frac{\mathrm{d}y}{b} \\ &= \sum_{r=0}^{p} \binom{p}{r} (-1)^{p-r} \frac{\pi}{2r+1} \frac{(2p+1)!!}{2(2p+2)!!}. \end{aligned} \tag{22}$$

Appendix 2

All integrals appearing in the variational solution of the conductor having rectangular cross-section can be determined as

$$I_p = \int\limits_{x=0}^{a} \frac{x^2}{a^2} \left(\frac{x^2-a^2}{a^2}\right)^p \frac{\mathrm{d}x}{a} = (-1)^p \frac{(2p)!!}{(2p+3)!!}, \tag{23}$$

and

$$L_p = \int\limits_{x=0}^{a} \left(\frac{x^2-a^2}{a^2}\right)^p \frac{\mathrm{d}x}{a} = (-1)^p \frac{(2p)!!}{(2p+3)!!}. \tag{24}$$

Appendix 3

All integrals appearing in the variational solution of the conductor having triangular cross-section can be determined as

$$I_{1pq} = \int_{x=0}^{h} \int_{y=0}^{\frac{a}{2}(1-\frac{x}{h})} \left(\frac{x}{h}\right)^p \left[\left(\frac{x}{h}-1\right)^2 - \left(\frac{2y}{a}\right)^2\right]^q \frac{\mathrm{d}x}{h} 2\frac{\mathrm{d}y}{a}$$
$$= \sum_{t=0}^{q} \binom{q}{t} (-1)^{2q-t} \frac{1}{2t+1} \sum_{s=0}^{2q+1} \binom{2q+1}{s} (-1)^s \frac{1}{p+s+1}, \tag{25}$$

$$I_{2pq} = \int_{x=0}^{h} \int_{y=0}^{\frac{a}{2}(1-\frac{x}{h})} \left(\frac{x}{h}\right)^p \left(\frac{x}{h}-1\right) \left[\left(\frac{x}{h}-1\right)^2 - \left(\frac{2y}{a}\right)^2\right]^q \frac{\mathrm{d}x}{h} 2\frac{\mathrm{d}y}{a}$$
$$= \sum_{t=0}^{q} \binom{q}{t} (-1)^{2q-t+1} \frac{1}{2t+1} \sum_{s=0}^{2q+2} \binom{2q+2}{s} (-1)^s \frac{1}{p+s+1}, \tag{26}$$

$$I_{3pq} = \int_{x=0}^{h} \int_{y=0}^{\frac{a}{2}(1-\frac{x}{h})} \left(\frac{x}{h}\right)^p \left(\frac{x}{h}-1\right)^2 \left[\left(\frac{x}{h}-1\right)^2 - \left(\frac{2y}{a}\right)^2\right]^q \frac{\mathrm{d}x}{h} 2\frac{\mathrm{d}y}{a}$$
$$= \sum_{t=0}^{q} \binom{q}{t} (-1)^{2q-t+2} \frac{1}{2t+1} \sum_{s=0}^{2q+3} \binom{2q+3}{s} (-1)^s \frac{1}{p+s+1}, \tag{27}$$

and

$$I_{4pq} = \int_{x=0}^{h} \int_{y=0}^{\frac{a}{2}(1-\frac{x}{h})} \left(\frac{x}{h}\right)^p \left(\frac{2y}{a}\right)^2 \left[\left(\frac{x}{h}-1\right)^2 - \left(\frac{2y}{a}\right)^2\right]^q \frac{\mathrm{d}x}{h} 2\frac{\mathrm{d}y}{a}$$
$$= \sum_{t=0}^{q} \binom{q}{t} (-1)^{2q-t} \frac{1}{2t+3} \sum_{s=0}^{2q+3} \binom{2q+3}{s} (-1)^s \frac{1}{p+s+1}. \tag{28}$$

Appendix 4

All integrals appearing in the variational solution of the conductor having trapezoidal cross-section can be determined as

$$I_{1pql} = \int\limits_{x=0}^{h} \int\limits_{y=0}^{a[\frac{x}{h}(\frac{b}{a}-1)+1]} \left(\frac{y}{a}\right)^2 \left(\frac{x}{h}\right)^p \left(\frac{x}{h}-1\right)^q \times \left\{ \left(\frac{y}{a}\right)^2 - \left[\frac{x}{h}\left(\frac{b}{a}-1\right)+1\right]^2 \right\}^l \frac{\mathrm{d}x}{h}\frac{\mathrm{d}y}{a}$$
$$= \sum_{t=0}^{q} \binom{q}{t} (-1)^{q+t} \sum_{s=0}^{l} \binom{l}{s} (-1)^{l+s} \frac{1}{2s+3} \sum_{r=0}^{2l+3} \binom{2l+3}{r} \frac{(\frac{b}{a}-1)^r}{p+t+r+1}, \tag{29}$$

$$I_{2pql} = \int\limits_{x=0}^{h} \int\limits_{y=0}^{a[\frac{x}{h}(\frac{b}{a}-1)+1]} \left(\frac{x}{h}\right)^p \left(\frac{x}{h}-1\right)^q \left[\frac{x}{h}\left(\frac{b}{a}-1\right)+1\right]^2 \times \left\{ \left(\frac{y}{a}\right)^2 - \left[\frac{x}{h}\left(\frac{b}{a}-1\right)+1\right]^2 \right\}^l \frac{\mathrm{d}x}{h}\frac{\mathrm{d}y}{a}$$
$$= \sum_{t=0}^{q} \binom{q}{t} (-1)^{q+t} \sum_{s=0}^{l} \binom{l}{s} (-1)^{l+s} \frac{1}{2s+1} \sum_{r=0}^{2l+3} \binom{2l+3}{r} \frac{(\frac{b}{a}-1)^r}{p+t+r+1}, \tag{30}$$

$$I_{3pql} = \int\limits_{x=0}^{h} \int\limits_{y=0}^{a[\frac{x}{h}(\frac{b}{a}-1)+1]} \left(\frac{x}{h}\right)^p \left(\frac{x}{h}-1\right)^q \left[\frac{x}{h}\left(\frac{b}{a}-1\right)+1\right] \times \left\{ \left(\frac{y}{a}\right)^2 - \left[\frac{x}{h}\left(\frac{b}{a}-1\right)+1\right]^2 \right\}^l \frac{\mathrm{d}x}{h}\frac{\mathrm{d}y}{a}$$
$$= \sum_{t=0}^{q} \binom{q}{t} (-1)^{q+t} \sum_{s=0}^{l} \binom{l}{s} (-1)^{l+s} \frac{1}{2s+1} \sum_{r=0}^{2l+2} \binom{2l+2}{r} \frac{(\frac{b}{a}-1)^r}{p+t+r+1}, \tag{31}$$

and

$$I_{4pql} = \int\limits_{x=0}^{h} \int\limits_{y=0}^{a[\frac{x}{h}(\frac{b}{a}-1)+1]} \left(\frac{x}{h}\right)^p \left(\frac{x}{h} - 1\right)^q \times \left\{ \left(\frac{y}{a}\right)^2 - \left[\frac{x}{h}\left(\frac{b}{a} - 1\right) + 1\right]^2 \right\}^l \frac{\mathrm{d}x}{h} \frac{\mathrm{d}y}{a}$$
$$= \sum_{t=0}^{q} \binom{q}{t} (-1)^{q+t} \sum_{s=0}^{l} \binom{l}{s} (-1)^{l+s} \frac{1}{2s+1} \sum_{r=0}^{2l+1} \binom{2l+1}{r} \frac{(\frac{b}{a} - 1)^r}{p+t+r+1}. \quad (32)$$

Appendix 5

All integrals appearing in the variational solution of the conductor having semi-circular cross-section can be determined as

$$I_{pq} = \int\limits_{x=0}^{a} \int\limits_{y=0}^{a\sqrt{1-\frac{x^2}{a^2}}} \left(\frac{y}{a}\right)^2 \left(\frac{x}{a}\right)^q \left(\frac{x^2}{a^2} + \frac{y^2}{a^2} - 1\right)^p \frac{\mathrm{d}x}{a} \frac{\mathrm{d}y}{a}$$
$$= \sum_{t=0}^{q} \binom{q}{t} (-1)^{q-t} \frac{1}{2t+3} \frac{(\frac{p}{2} - \frac{1}{2})!(q + \frac{3}{2})!}{2(\frac{p}{2} + q + 2)!}, \quad (33)$$

and

$$L_{pq} = \int\limits_{x=0}^{a} \int\limits_{y=0}^{a\sqrt{1-\frac{x^2}{a^2}}} \left(\frac{x}{a}\right)^p \left(\frac{x^2}{a^2} + \frac{y^2}{a^2} - 1\right)^q \frac{\mathrm{d}x}{a} \frac{\mathrm{d}y}{a}$$
$$= \sum_{t=0}^{q} \binom{q}{t} (-1)^{q-t} \frac{1}{2t+1} \frac{(\frac{p}{2} - \frac{1}{2})!(q + \frac{1}{2})!}{2(\frac{p}{2} + q + 1)!}. \quad (34)$$

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