FACTA UNIVERSITATIS (NIŠ)
SER.: ELEC. ENERG. vol. 16, 145-154

A Simplified Linearized Dinamic Model for Voltage Collapse Assessment in Power Systems

Dragan Tasić and Miodrag Stojanović

Abstract: A simplified linearized dynamic model for fast assessment of voltage collapse is developed in this paper. The simplification was made under assumption that the voltages at generator nodes are constant, which means that only power changes at load nodes are considered in analysis. Dimensions of the state matrix, which eigenvalues are used for voltage collapse assessment, decrease under this assumption. Appropriate transformations of linearized state matrix prove that knowledge of values of the dynamical load change time constants is not required for this model.

Keywords: Voltage collapse, assessment, load, state matrix.

1 Introduction

Analysis of faults caused by voltage collapse, and its consequences, pointed out this phenomenon is very complex and influenced by many factors. Because of complexity, for a long time this phenomenon occupies interest of researchers, what verify a number of published papers. Different approaches are used in voltage collapse researches, depending on: emphasized factors, used models of components, and introduced simplifications. In general, all approaches for analysis of voltage collapse and voltage (in)stability can be classified into two basic groups: static and dynamic.

Linearized models are often used in both static [1, 2, 5, 6] and dynamic [1,2,7-10] approaches. Eigenvalues of related linearized matrix are then used for voltage collapse presence indication.

Manuscript received 24 December 2002. An earlier version of this paper was presented at the 37rd International Conference on Information, Communication and Energy Systems and Technologies, ICEST 2002, October 1-4, 2002, Niš, Serbia.

The authors are with Faculty of Electronic Engineering, University of Niš, Beogradska 14, 18000 Niš (e-mail: dtasic@elfak.ni.ac.yu).

A simplified linearized dynamic model for voltage collapse assessment is formed in this paper. This model is obtained under assumption that voltages at generator nodes are constant. It means that only power changes at load nodes can be considered in analysis. Starting from this model, the voltage collapse is assessed by the eigenvalues of linearized state matrix. Sizes of this matrix are equal to the total number of load nodes.

2 Dynamic of power change at load nodes

The system under consideration has m generator and n load nodes. Dynamic of load changes at the i-th node can be expressed by following two differential equations [8, 10]

$$\frac{dP_{Li}}{dt} = -\frac{1}{T_i}(P_{Li} - f_{Pi}(V_i)) \ , \tag{1}$$

$$\frac{dQ_{Li}}{dt} = -\frac{1}{T_i}(Q_{Li} - f_{Qi}(V_i)) \ , \tag{2}$$

where P_{Li} is active load at i-th node, Q_{Li} reactive load at i-th node, T_i dynamical load change time constant at i-th node, $f_{Pi}(V_i)$ dependence of the active load at i-th node as a function of the voltage V_i, and $f_{Qi}(V_i)$ dependence of the reactive load at i-th node as a function of the voltage V_i.

Time constant T_i depends on the load structure. Major factor that influences on this value is time constant of asynchronous machine, which represents loads in proposed model. Value of T_i also depends on time constant of tap changing transformer regulator, if it is presented at load node. Determination of the time constant T_i is very complex problem for each particular case. Proposed approach does not require knowledge of time constants values T_i , which can be considered as an advantage.

Functional relations of f_{Pi} and f_{Qi} are static voltage characteristics of load at i-th node. In previous papers $[1-4]$, different methods for modelling of static voltage characteristics are presented. In this paper, following functional relations are used

$$f_{Pi}(V_i) = P_{Li}^o \left(\frac{V_i}{V_0}\right)^{k_{pvi}} \ , \tag{3}$$

$$f_{Qi}(V_i) = Q_{Li}^o \left(\frac{V_i}{V_0}\right)^{k_{qvi}} \ , \tag{4}$$

where k_{pvi} and k_{qvi} are voltage selfregulation coefficients of the active and reactive load at i-th node, P^o_{Li} is active and Q^o_{Li} reactive loads at i-th node that correspond to voltage V_0.

3 Linearized dynamic model

Analysis of the voltage collapse appearance, as usually, starts from known initial conditions, i.e. from initial values of the voltage phasors at all nodes. Moreover, constant magnitudes of voltages at the generator nodes are assumed, while magnitudes of the voltages at load nodes are treated as corresponding functions of the active and reactive loads. Small changes in voltages $\Delta\mathbf{V}$, then can be expressed as

$$\Delta\mathbf{V} = \left.\frac{\partial\mathbf{V}}{\partial\mathbf{P_L}}\right|_0 \Delta\mathbf{P_L} + \left.\frac{\partial\mathbf{V}}{\partial\mathbf{Q_L}}\right|_0 \Delta\mathbf{Q_L} \ , \tag{5}$$

where

$$\begin{aligned} \Delta\mathbf{V} &= [\Delta V_1\ \Delta V_2\ \dots\ \Delta V_n]^T \ , \\ \Delta\mathbf{P} &= [\Delta P_{L1}\ \Delta P_{L2}\ \dots\ \Delta P_{Ln}]^T \ , \\ \Delta\mathbf{Q} &= [\Delta Q_{L1}\ \Delta Q_{L2}\ \dots\ \Delta Q_{Ln}]^T \ , \end{aligned}$$

$$\left.\frac{\partial\mathbf{V}}{\partial\mathbf{P_L}}\right|_0 = \begin{bmatrix} \frac{\partial V_1}{\partial P_{L1}} & \cdots & \frac{\partial V_1}{\partial P_{Ln}} \\ \vdots & & \vdots \\ \frac{\partial V_n}{\partial P_{L1}} & \cdots & \frac{\partial V_n}{\partial P_{Ln}} \end{bmatrix}_0 , \qquad \left.\frac{\partial\mathbf{V}}{\partial\mathbf{Q_L}}\right|_0 = \begin{bmatrix} \frac{\partial V_1}{\partial Q_{L1}} & \cdots & \frac{\partial V_1}{\partial Q_{Ln}} \\ \vdots & & \vdots \\ \frac{\partial V_n}{\partial Q_{L1}} & \cdots & \frac{\partial V_n}{\partial Q_{Ln}} \end{bmatrix}_0 .$$

Subscripts "0" in these equations denote that partial derivatives are calculated for steady state before changes appear.

Linearizing the functions $f_{Pi}(V_i)$ and $f_{Qi}(V_i)$ around the analyzed initial state and respecting equation (5), equations (3) and (4) became

$$f_{Pi}(V_i) = P_{Li}(0) + \left(\frac{\partial f_{Pi}}{\partial V_i}\frac{\partial V_i}{\partial P_L}\right)_0 \Delta\mathbf{P_L} + \left(\frac{\partial f_{Pi}}{\partial V_i}\frac{\partial V_i}{\partial Q_L}\right)_0 \Delta\mathbf{Q_L} \ , \tag{6}$$

$$f_{Qi}(V_i) = Q_{Li}(0) + \left(\frac{\partial f_{Qi}}{\partial V_i}\frac{\partial V_i}{\partial P_L}\right)_0 \Delta\mathbf{P_L} + \left(\frac{\partial f_{Qi}}{\partial V_i}\frac{\partial V_i}{\partial Q_L}\right)_0 \Delta\mathbf{Q_L} \ , \tag{7}$$

where $P_{Li}(0)$ is initial steady-state active load, $Q_{Li}(0)$ is initial steady-state reactive load at the i-th node, and

$$\left.\frac{\partial V_i}{\partial P_L}\right|_0 = \left[\begin{array}{cccc} \frac{\partial V_i}{\partial P_{L1}} & \frac{\partial V_i}{\partial P_{L2}} & \cdots & \frac{\partial V_i}{\partial P_{Ln}} \end{array}\right]_0 \ ,$$
$$\left.\frac{\partial V_i}{\partial Q_L}\right|_0 = \left[\begin{array}{cccc} \frac{\partial V_i}{\partial Q_{L1}} & \frac{\partial V_i}{\partial Q_{L2}} & \cdots & \frac{\partial V_i}{\partial Q_{Ln}} \end{array}\right]_0 \ .$$

Power increments at i-th node can be expressed by following equations

$$\Delta P_{Li} = P_{Li} - P_{Li}(0) \ , \tag{8}$$

$$\Delta Q_{Li} = Q_{Li} - Q_{Li}(0) \ . \tag{9}$$

System of linearized differential equations is obtained if (6-9) are sequentially substituted in (1) and (2). For the case of n load nodes network, following system of linearized differential equation can be written in matrix form

$$\frac{d}{dt}\left[\begin{array}{c} \Delta\mathbf{P_L} \\ \Delta\mathbf{Q_L} \end{array}\right] = \left[\begin{array}{cc} -\mathbf{T}^{-1}\left(\mathbf{I} - \left(\frac{\partial \mathbf{f_P}}{\partial \mathbf{V}}\frac{\partial \mathbf{V}}{\partial \mathbf{P_L}}\right)_0\right) & \mathbf{T}^{-1}\left(\frac{\partial \mathbf{f_P}}{\partial \mathbf{V}}\frac{\partial \mathbf{V}}{\partial \mathbf{Q_L}}\right)_0 \\ \mathbf{T}^{-1}\left(\frac{\partial \mathbf{f_Q}}{\partial \mathbf{V}}\frac{\partial \mathbf{V}}{\partial \mathbf{P_L}}\right)_0 & -\mathbf{T}^{-1}\left(\mathbf{I} - \left(\frac{\partial \mathbf{f_Q}}{\partial \mathbf{V}}\frac{\partial \mathbf{V}}{\partial \mathbf{Q_L}}\right)_0\right) \end{array}\right]\left[\begin{array}{c} \Delta\mathbf{P_L} \\ \Delta\mathbf{Q_L} \end{array}\right], \tag{10}$$

where $\mathbf{I}$ is unit $n \times n$ matrix, $\Delta\mathbf{P_L} = [\Delta P_{L1}\ \Delta P_{L2}\ \ldots\ \Delta P_{Ln}]^T$,

$$\Delta\mathbf{Q_L} = [\Delta Q_{L1}\ \Delta Q_{L2}\ \ldots\ \Delta Q_{Ln}]^T,\ \left.\frac{\partial \mathbf{f_P}}{\partial \mathbf{V}}\right|_0 = \left[\begin{array}{ccc} \frac{\partial f_{P1}}{\partial V_1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \frac{\partial f_{Pn}}{\partial V_n} \end{array}\right]_0 ,$$
$$\left.\frac{\partial \mathbf{f_Q}}{\partial \mathbf{V}}\right|_0 = \left[\begin{array}{ccc} \frac{\partial f_{Q1}}{\partial V_1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \frac{\partial f_{Qn}}{\partial V_n} \end{array}\right]_0 ,\ \mathbf{T} = \left[\begin{array}{ccc} T_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & T_n \end{array}\right]_0 .$$

In mentioned mathematical model (10), appearance of voltage collapse is assessed basing on state matrix eigenvalues. This practically means that the answer results from the solution of the following algebraic equation

$$\left|\begin{array}{cc} -\mathbf{T}^{-1}\left(\mathbf{I} - \left(\frac{\partial \mathbf{f_P}}{\partial \mathbf{V}}\frac{\partial \mathbf{V}}{\partial \mathbf{P_L}}\right)_0\right) - \lambda\mathbf{I} & \mathbf{T}^{-1}\left(\frac{\partial \mathbf{f_P}}{\partial \mathbf{V}}\frac{\partial \mathbf{V}}{\partial \mathbf{Q_L}}\right)_0 \\ \mathbf{T}^{-1}\left(\frac{\partial \mathbf{f_Q}}{\partial \mathbf{V}}\frac{\partial \mathbf{V}}{\partial \mathbf{P_L}}\right)_0 & -\mathbf{T}^{-1}\left(\mathbf{I} - \left(\frac{\partial \mathbf{f_Q}}{\partial \mathbf{V}}\frac{\partial \mathbf{V}}{\partial \mathbf{Q_L}}\right)_0\right) - \lambda\mathbf{I} \end{array}\right| = 0 \ . \tag{11}$$

Using elementary transformations, the system of equations (11) can be reduced to the following, simple form

$$\begin{vmatrix} -\mathbf{T}^{-1}-\lambda\mathbf{I} & \mathbf{0} \\ \mathbf{T}^{-1}\left(\frac{\partial \mathbf{f_Q}}{\partial \mathbf{V}}\frac{\partial \mathbf{V}}{\partial \mathbf{P_L}}\right)_0 & -\mathbf{T}^{-1}\left(\begin{array}{l}\mathbf{I}-\left(\frac{\partial \mathbf{f_Q}}{\partial \mathbf{V}}\frac{\partial \mathbf{V}}{\partial \mathbf{Q_L}}\right)_0 - \\ \quad -\left(\frac{\partial \mathbf{f_Q}}{\partial \mathbf{V}}\frac{\partial \mathbf{V}}{\partial \mathbf{P_L}}\right)_0\left(\frac{\partial \mathbf{f_Q}}{\partial \mathbf{V}}\right)_0^{-1}\left.\frac{\partial \mathbf{f_P}}{\partial \mathbf{V}}\right|_0\end{array}\right)-\lambda\mathbf{I}\end{vmatrix}=0. \tag{12}$$

From equation (12) it is obvious that n eigenvalues are always real and negative $(-1/T_1,\ldots,-1/T_n)$. For the voltage collapse assessment purpose, only sign of appropriate eigenvalues are needed, i.e. quantification of the first n eigenvalues is not necessary. Then, problem is reduced to the determination of n eigenvalues

$$\mathbf{A}=-\mathbf{T}^{-1}\left(\mathbf{I}-\left(\frac{\partial \mathbf{f_Q}}{\partial \mathbf{V}}\frac{\partial \mathbf{V}}{\partial \mathbf{Q_L}}\right)_0-\left(\frac{\partial \mathbf{f_Q}}{\partial \mathbf{V}}\frac{\partial \mathbf{V}}{\partial \mathbf{P_L}}\right)_0\left(\frac{\partial \mathbf{f_Q}}{\partial \mathbf{V}}\right)_0^{-1}\left.\frac{\partial \mathbf{f_P}}{\partial \mathbf{V}}\right|_0\right). \tag{13}$$

Regarding that $\left.\frac{\partial \mathbf{f_Q}}{\partial \mathbf{V}}\right|_0$ (i.e. $\left(\frac{\partial \mathbf{f_Q}}{\partial \mathbf{V}}\right)_0^{-1}$) is diagonal, after some elementary transformations, the matrix $\mathbf{A}$ can be written in the following form

$$\mathbf{A}=-\mathbf{T}^{-1}\left.\frac{\partial \mathbf{V}}{\partial \mathbf{Q_L}}\right|_0\left(\left(\frac{\partial \mathbf{V}}{\partial \mathbf{Q_L}}\right)_0^{-1}-\left.\frac{\partial \mathbf{f_Q}}{\partial \mathbf{V}}\right|_0-\left(\frac{\partial \mathbf{V}}{\partial \mathbf{Q_L}}\right)_0^{-1}\left(\frac{\partial \mathbf{V}}{\partial \mathbf{P_L}}\right)_0\left.\frac{\partial \mathbf{f_P}}{\partial \mathbf{V}}\right|_0\right). \tag{14}$$

Starting from the expressions for $\mathbf{Q_L}$ and $\mathbf{P_L}$ matrices $\left.\frac{\partial \mathbf{V}}{\partial \mathbf{Q_L}}\right|_0$ and $\left.\frac{\partial \mathbf{V}}{\partial \mathbf{P_L}}\right|_0$ can be determined as functions of voltage magnitude $\mathbf{V}$ and angle $\mathbf{\Theta}$, i.e. $\mathbf{Q_L}=\mathbf{g_Q}(\mathbf{V},\mathbf{\Theta})$ and $\mathbf{P_L}=\mathbf{g_P}(\mathbf{V},\mathbf{\Theta})$

$$\left.\frac{\partial \mathbf{V}}{\partial \mathbf{Q_L}}\right|_0=\left(\left.\frac{\partial \mathbf{g_Q}}{\partial \mathbf{V}}\right|_0-\left.\frac{\partial \mathbf{g_Q}}{\partial \mathbf{\Theta}}\right|_0\left(\frac{\partial \mathbf{g_P}}{\partial \mathbf{\Theta}}\right)_0^{-1}\left.\frac{\partial \mathbf{g_P}}{\partial \mathbf{V}}\right|_0\right)^{-1}, \tag{15}$$

$$\left.\frac{\partial \mathbf{V}}{\partial \mathbf{P_L}}\right|_0=-\left(\left.\frac{\partial \mathbf{g_Q}}{\partial \mathbf{V}}\right|_0-\left.\frac{\partial \mathbf{g_Q}}{\partial \mathbf{\Theta}}\right|_0\left(\frac{\partial \mathbf{g_P}}{\partial \mathbf{\Theta}}\right)_0^{-1}\left.\frac{\partial \mathbf{g_P}}{\partial \mathbf{V}}\right|_0\right)^{-1}\left.\frac{\partial \mathbf{g_Q}}{\partial \mathbf{\Theta}}\right|_0\left(\frac{\partial \mathbf{g_P}}{\partial \mathbf{\Theta}}\right)_0^{-1}. \tag{16}$$

In (14), $\mathbf{T}$ is diagonal matrix and does not have effect on sign of eigenvalues. For this reason it can be neglected. Then, respecting (15) and (16), following matrix of order n can be used for voltage collapse assessment

$$\begin{aligned}\mathbf{PQV}=-&\left(\left.\frac{\partial \mathbf{g_Q}}{\partial \mathbf{V}}\right|_0-\left.\frac{\partial \mathbf{g_Q}}{\partial \mathbf{\Theta}}\right|_0\left(\frac{\partial \mathbf{g_P}}{\partial \mathbf{\Theta}}\right)_0^{-1}\left.\frac{\partial \mathbf{g_P}}{\partial \mathbf{V}}\right|_0\right)^{-1}\cdot\\ &\cdot\left(\left.\frac{\partial \mathbf{g_Q}}{\partial \mathbf{V}}\right|_0-\left.\frac{\partial \mathbf{f_Q}}{\partial \mathbf{V}}\right|_0+\left.\frac{\partial \mathbf{g_Q}}{\partial \mathbf{\Theta}}\right|_0\left(\frac{\partial \mathbf{g_P}}{\partial \mathbf{\Theta}}\right)_0^{-1}\left(\frac{\partial \mathbf{f_P}}{\partial \mathbf{V}}-\frac{\partial \mathbf{g_P}}{\partial \mathbf{V}}\right)_0\right).\end{aligned} \tag{17}$$

Elements of this matrix could be simply obtained on the basis of known parameters and state variables of the power system. Thus, a relatively simple mathematical model is formed. This model is very suitable for the analysis of the power system voltage (in)stability, because problem is reduced to determination of eigenvalues of a real matrix **PQV**, of relatively low order (equal to the number of load nodes).

4 Uniform movement of generator rotors

Voltage (in)stability is practically always performed for characteristic post dynamic quasi-states of the power system. The assumption that the change in power at load nodes coincides with the uniform synchronous generators movements is there completely justified. In practice, this means that synchronous machines participate in total acceleration power P_{ac} according to their inertia constants [9, 10], i.e.

$$\frac{P_{Ti} - P_i}{M_i} = \frac{P_{ac}}{M_s} \ , \tag{18}$$

where P_{Ti} is mechanical power, P_i injected active power, $M_i = (T_{ji} S_{ni}) / \omega_s$ inertia constant, T_{ji} inertia time constant, and S_{ni} nominal power of i-th synchronous machine. In the same equation ω_s is synchronous velocity of rotation, $M_s = \sum\limits_{i=1}^{m} M_i$ and m is number of synchronous machines.

If participation of M_i in M_s is noted as F_i ($F_i = M_i/M_s$; $\sum\limits_{i=1}^{m} F_i = 1$) and if m-th machine is reference one

$$\frac{P_{Ti} - P_i}{F_i} = \frac{P_{Tm} - P_m}{F_m} \ ; \qquad i = 1, \ldots, m-1 \ . \tag{19}$$

Thus, the condition of uniform movement of generator rotors reduces to the form

$$R_{Ti} = R_i \ ; \qquad i = 1, \ldots, m-1 \ , \tag{20}$$

$$R_{Ti} = F_m P_{Ti} - F_i P_{Tm} \qquad \text{and} \qquad R_i = F_m P_i - F_i P_m \ . \tag{21}$$

Changes in injected power at i-th generator node, as result of load power changes, can be expressed as

$$\Delta P_i = k_i \Delta \left(P_{loss} + \sum_{j=1}^{n} P_{Lj} \right) \ ; \qquad \sum_{i=1}^{m} k_i = 1 \ , \tag{22}$$

where k_i is incremental coefficient which corresponds to generator production at node i, and P_{loss} is total active power loses of system. Regarding to the two last expressions, for matrix **PQV** can be finally written

$$\mathbf{PQV} = -\mathbf{G}^{-1}\left[\left.\frac{\partial \mathbf{g_Q}}{\partial \mathbf{V}}\right|_0 - \left.\frac{\partial \mathbf{f_Q}}{\partial \mathbf{V}}\right|_0 + \left.\frac{\partial \mathbf{g_Q}}{\partial \mathbf{\Theta}}\right|_0 \begin{bmatrix} \frac{\partial \mathbf{g_P}}{\partial \mathbf{\Theta}} \\ \\ \frac{\partial \mathbf{g_R}}{\partial \mathbf{\Theta}} \end{bmatrix}_0^{-1} \begin{bmatrix} \frac{\partial \mathbf{f_P}}{\partial \mathbf{V}} - \frac{\partial \mathbf{g_P}}{\partial \mathbf{V}} \\ \\ \mathbf{F}\frac{\partial \mathbf{f_P}}{\partial \mathbf{V}} - \frac{\partial \mathbf{g_R}}{\partial \mathbf{V}} \end{bmatrix}_0\right], \tag{23}$$

where

$$\mathbf{G} = \left[\left.\frac{\partial \mathbf{g_Q}}{\partial \mathbf{V}}\right|_0 - \left.\frac{\partial \mathbf{g_Q}}{\partial \mathbf{\Theta}}\right|_0 \begin{bmatrix} \frac{\partial \mathbf{g_P}}{\partial \mathbf{\Theta}} \\ \\ \frac{\partial \mathbf{g_R}}{\partial \mathbf{\Theta}} \end{bmatrix}_0^{-1} \begin{bmatrix} \frac{\partial \mathbf{g_P}}{\partial \mathbf{V}} \\ \\ \frac{\partial \mathbf{g_R}}{\partial \mathbf{V}} \end{bmatrix}_0\right] . \tag{24}$$

$\mathbf{g_R}$ is vector that shows conditions of uniform movement of rotors, $\mathbf{g_R}(\mathbf{V}, \mathbf{\Theta})$, and $\mathbf{F}$ is matrix with n identical columns

$$[F_m k_1 - F_1 k_m \; F_m k_2 - F_2 k_m \ldots F_m k_{m-1} - F_{m-1} k_m]^T \; . \tag{25}$$

5 Test example

Presented procedure is applied for voltage collapse assessment of a power system with 13 nodes that is shown in Fig. 1 [10]. Four of the nodes are generator ones. The data in per unit values of the system (generated powers, load powers and nodes voltages) are shown in Table 1. The voltages in the same table are calculated using Newton-Raphson method.

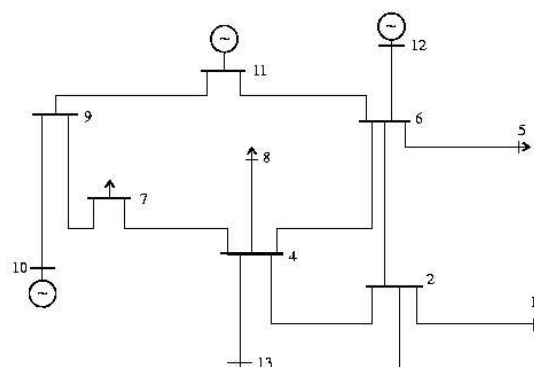

Fig. 1. Test system for voltage collapse assessment.

The corresponding incremental factors k_i are $k_1 = 0.2$, $k_2 = 0.28$, $k_3 = 0.22$, $k_4 = 0.3$, while factors F_i are: $F_1 = 0.15$, $F_2 = 0.3$, $F_3 = 0.25$, $F_4 = 0.3$. For this initial state of the test network, and for different values of voltage selfregulation coefficients (with assumption that they are the same for all load nodes), voltage collapse assessment is made.

Table 1. Generated powers, load powers and voltages for test system

Node	Generated power $P_G(p.u.)$	Load power $P_L(p.u.)$	Load power $Q_L(p.u.)$	Voltage magnitude $V(p.u.)$	Voltage phase angle $\theta(^\circ)$
1		3.5	2.0	0.826466	-24.4156
2		0.0	0.0	0.896634	-16.8234
3		2.12	1.1	0.859977	-23.9969
4		0.0	0.0	0.944272	-10.657
5		1.3	0.8	0.905786	-15.3282
6		0.0	0.0	0.987532	-5.5421
7		2.95	1.4	0.920813	-11.5256
8		1.1	0.7	0.907940	-18.702
9		0.9	0.5	1.027933	5.2696
10	2.7			1.05	10.2098
11	3.0			1.05	6.3277
12	3.2			1.05	0.3605
13	3.452			1.05	0.0

On the basis of results shown at the Tabes 1 and 2 following statement can be established: eigenvalues obtained if influence of synchronous generators are not considered are approximately equal to ones obtained when uniform movement of generator rotors is included. Thus, for purpose of voltage collapse fast assessment, it is suitable to use (17) for calculation matrix **PQV**, because of its simplicity comparing to (23).

In any of two mentioned cases, when $k_{pv} = k_{qv} = 1$ or $k_{pv} = k_{qv} = 0$, one of eigenvalues is positive what indicate possibility of voltage collapse appearing. Test example verifies the fact that loads with smallest voltage selfregulation coefficients of active and reactive power are critical, i.e. they have larger contribution to voltage collapse appearance.

As we saw, generator nodes were modelled, in this approach, only as constant voltage sources. Because of that, when critical points are determined,

Table 2. Eigenvalues of the **PQV** matrix without consideration influence of synchronous generators

	$k_{pv} = k_{qv} = 2$	$k_{pv} = k_{qv} = 1$	$k_{pv} = k_{qv} = 0$
λ_1	-3.91786	0.004088	1.05967
λ_2	-1.42943	-0.85222	-0.69687
λ_3	-1.20511	-0.92942	-0.85522
λ_4	-1.30729	-0.89426	-0.78309
λ_5	-1.14506	-0.95008	-0.89760
λ_6	-1.00000	-1.00000	-1.00000
λ_7	-1.01299	-0.99553	-0.99083
λ_8	-1.00000	-1.00000	-1.00000
λ_9	-1.00000	-1.00000	-1.00000

Table 3. Eigenvalues of the **PQV** matrix when uniform movement of synchronous generators is considered

	$k_{pv} = k_{qv} = 2$	$k_{pv} = k_{qv} = 1$	$k_{pv} = k_{qv} = 0$
λ_1	-3.60740	0.0082376	1.15838
λ_2	-1.40995	-0.867497	-0.72057
λ_3	-1.29324	-0.90344	-0.79801
λ_4	-1.17375	-0.93441	-0.85790
λ_5	-1.12872	-0.94556	-0.89901
λ_6	-0.99958	-0.99091	-0.98719
λ_7	-0.99958	-1.00355	-1.00498
λ_8	-1.00008	-1.00000	-1.00004
λ_9	-0.99999	-1.00003	-1.00000

we should reconsider them using some exact method to test if voltage collapse appears.

6 Conclusion

The simplified linearized dynamic model for fast voltage collapse assessment is formed in this paper. For the case of n-load nodes network appropriate transformations prove that instead of $2n$, it is needed to determine only n eigenvalues. Additionally, knowledge of the values of load time constant is needless also, as it is shown in this paper. Presented linearized dynamical model, because of introduced assumptions, can be used only for fast assessment of voltage collapse. When critical states are identified, other method should be utilised in order to clearly determine whether voltage collapse appears.

References

[1] P. Kundur: *Power Systems Stability and Control*, New York, Mc Graw-Hill, 1993.
[2] C. W. Taylor: *Power Sustem Voltage Stability*, New York, Mc Graw-Hill, 1994.
[3] IEEE Task Force on Load Representation for Dynamic Performance: *Bibliography on Load Models for Power Flow and Dynamic Performance Simulation*, IEEE Trans., Power Systems, vol. 10, No. 1, pp. 523-538, 1995.
[4] Yu. E. Gurevich, L. E. Libova, E. A. Khachatryan: *Ustoichivost nagruzki elektricheskikh sistem*, Moskva, Energoizdat, 1981.
[5] P. W. Sauer, M. A. Pai: *Power Systems Steady-State Stability and the Load Flow Jacobian*, IEEE Trans., Power Systems, vol. 5, No. 4, pp.1374-1382, 1990.
[6] A. Tiranuchit, R. J. Thomas: *A Posturing Strategy, Against Voltage Instabilities in Electric Power Systems*, IEEE Trans. Power Systems, vol. 3, No. 1, pp. 87-93, 1988.
[7] M. Brucoli, F. Rossi, F. Torelli, M. Trovato: *A Generalized Approach to the Analysis of Voltage Stability in Electric Power Systems*, Electric Power Systems Research, No 9,pp. 49-62, 1985.
[8] S. Abe, Y. Fukunaga, A. Isoko, B. Kondo:*Power System Voltage Stability*, IEEE Trans., PAS-101, No. 10, pp. 3830-3838, 1982.
[9] D. Tasic, D. P. Popovic: *Jedna metoda za analizu naponske stabilnosti elektroenergetskih sistema*, Elektroprivreda, No. 7-8, p.p. 281-288, 1989. (in Serbian).
[10] D. Tasic, D.P. Popovic: *An Efficient Method for Voltage Stability Assessment of Power Systems*, AMSE, vol 36, No 3, pp. 33-44, 1991.
[11] D.J. Hill, I.A. Hiskens, D.H. Popovic: *Stability Analysis of Power System Loads With Recovery Dynamics*, Int. Journal of Electric Power and Energy Systems, vol. 16, No. 4, pp. 277-286, 1994.