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Ridges Extraction Method for a Polynimial Frequency Modulated Signal Covered with a Zero Mean Gaussian Noice

Cornelia E. Gordan and Romulus Reiz

Abstract: In this paper is presented an original method, developed by the authors, in order to extract the ridges of the polynomial frequency modulated signals, covered by a low-pass filtered Gaussian noise, using Short-Time Fourier, Gabor and Wigner-Ville time-frequency representations and mathematical morphology elements. In the paper are also presented practical results obtained for generated polynomial modulated chirps.

Keywords: Ridges extraction, time-frequency representations, mathematical morphology elements.

1 Introduction

The time-frequency representations as the Short-Time Fourier, Gabor, Wigner-Ville, Choi-Williams and "wavelet" transforms of a signal contain very important information concerning the regions from the time-frequency plane where the signal's energy is maximum. It was demonstrated that the ridges of the module of any time-frequency representation correspond to the maximum values of the signal's energy, which form the skelet of the analysed transform [3], [4]. These maxims are localised around the instantaneous frequency (IF) of the signal, which means that the detection of the ridges offers the possibility to estimate the IF and to reconstruct the original signal.

The importance of the IF concept stems from the fact that in many applications the signal analyst is confronted with the task of processing signals

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The authors are with the University of Oradea, Department of Applied Electronics, 5 Armatei Romane Str., 3700, Oradea, Romania (e-mail: cgordan@uoradea.ro)

whose spectral characteristics (in particular the frequency of the spectral peaks) are varying with time. These signals are often referred to as "non-stationary". For these signals, the IF is an important characteristic, because it is a time-varying parameter that defines the location of the signal's spectral peak as it varies with time.

In this paper is presented the theoretical basis of the proposed method, namely for the problem of ridges extraction of a polynomial frequency modulated signal covered by a low- pass filtered Gaussian noise, from its Short-Time Fourier, Gabor and Wigner-Ville time- frequency representations.

2 The Analysing Method

The polynomial frequency-modulated signal, covered by a low-pass filtered Gaussian noise, is generated with the MATLAB program. Then it is processed using the "Joint Time-Frequency Analysis" Program developed by National Instruments. This way can be obtained the Short Time Fourier, Gabor and Wigner-Ville time-frequency representations of the analysed signal.

In order to obtain the time-frequency localisation of the analysed signal is used a morphological algorithm, which eliminates the interference terms and gives a concrete, thin line approximation of the time-frequency dependence (ridges). This algorithm is based on the following assumptions

- The time-frequency dependence is represented as a grey level image;
- The highest grey level gives the best time-frequency localisation;
- Only the major component is important because the other components represent the interference terms;
- The best time-frequency approximation is the "Water-Shed" of the major component from the image.

This morphological algorithm can extract efficiently the time-frequency dependence from a time-frequency signature. Also, the interference terms will be filtered out. The advantage of the presented algorithm comparing with the morphological "Water-Shed" is that the interference terms do not have a bad influence on the time-frequency localisation.

After the image segmentation and the filtering of the interference terms can be notified that the described algorithm seems to be the SKIZ (Skeleton by Influence Zone) algorithm. The main difference is the implementation of the algorithm. While the SKIZ algorithm use some increasing circles to find the central point that describes the SKIZ line, the algorithm proposed here use repeated conditional erosion. Theoretically both algorithms should provide the same result but in practice, when are used discrete images (pixel images), it becomes obvious that the implementation of a given circle will not be accurate because of the image pixel representation. So, for small width surfaces the SKIZ algorithm can provide some zigzag errors. The proposed algorithm will not suffer from this symptom. Then every representation mentioned above is processed using the "Cyclope" program, which eliminates the interference terms and localises the maximum values. Finally, by the help of an original program, developed by the authors, is obtained the curve corresponding to the detected maximum values of the analysed curve, having the width of a pixel, and which, in fact, represents the ridges of the signal.

In the end the parameters corresponding to the curve are processed with another original program that calculates the coefficients of the polynomial, which interpolates the given curve. The program develops a comparison between the analysed and the interpolated curves, and makes possible the calculus of the errors of the interpolation polynomial coefficients. These errors are calculated after a comparison between the interpolation coefficients and their initial values used to generate the signal.

3 Practical Results

The If variation law of a four-degree polynomial frequency modulated signal is also a polynomial described by the relation

$$f(t) = at^3 + bt^2 + ct + d \tag{1}$$

Using the following definition

$$f_i(t) = \frac{1}{2\pi}\frac{\mathrm{d}f(t)}{\mathrm{d}t} \tag{2}$$

can be easily obtained the phase of the frequency-modulated signal, which is defined by a four-degree polynomial

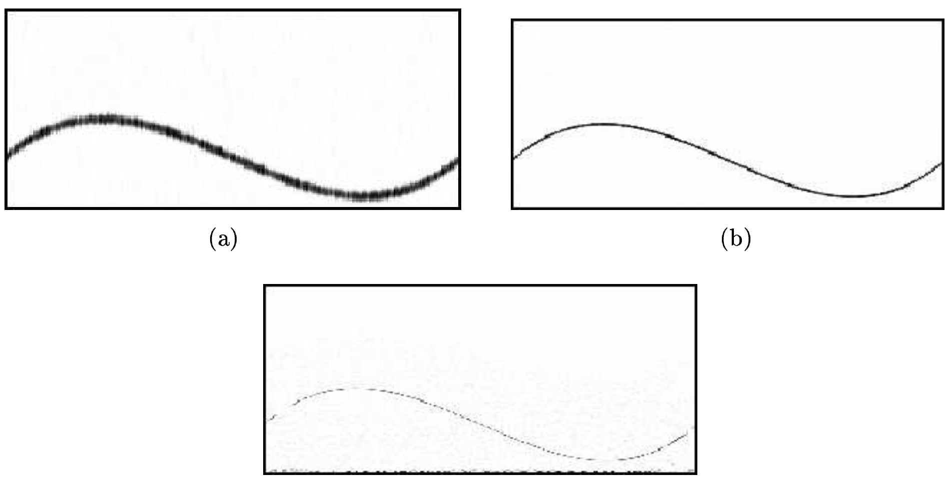
$$s(t) = \cos\left[\frac{2\pi}{4}at^4 + \frac{2\pi}{3}bt^3 + \frac{2\pi}{2}ct^2 + 2\pi\mathrm{d}t\right] \tag{3}$$

The parameters included in the above relation were chosen as follows: $a = 500/27$, $b = -500/3$, $c = 500 \times 2/3$, $d = 500/2$. In these conditions the signal is 6 seconds long, the frequency band is [50,500] Hz and the sampling frequency is equal to 2 kHz. This signal was generated using the MATLAB program.

In this paper the instantaneous frequency of the signal defined with relation (3) is estimated using the Short-Time Fourier, Gabor and Wigner-Ville, time-frequency transforms for the case when the signal is covered with a low-pass filtered Gaussian noise. The following relation defines the transfer function of this filter

$$H(z) = \frac{1 - A_1^2 - A_2^2}{1 - A_1 z^{-1} - A_2 z^{-2}} \tag{4}$$

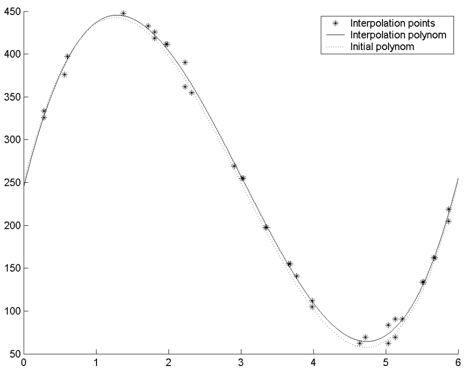
First of all, using the "Joint Time-Frequency Analysis" program are obtained Short-Time Fourier, Gabor and Wigner-Ville, representations of the signal covered by noise, for different values of the noise variance σ^2: 0.01, 0.04, 0.09, 0.16 and 0.36. These representations characterised by σ^2=0.04 are presented in Figure 1: (a) Short-Time Fourier, (b) Gabor, (c) Wigner-Ville.



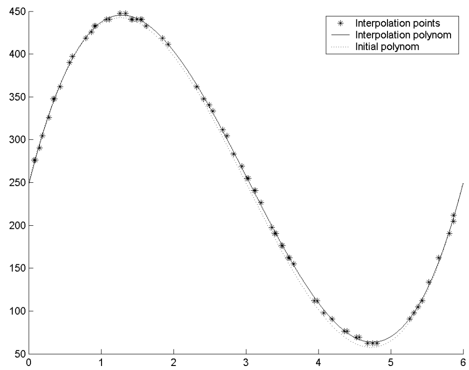
(a) (b) (c)

Fig. 1. Time-frequency representations of $s(t)$ defined by relation (3) when $\sigma = 0.4$: (a) Short-Time Fourier, (b) Gabor and (c) Wigner-Ville.

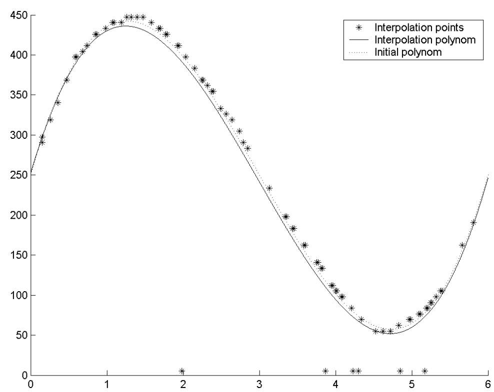
Using the detection algorithm, presented in this paper, were obtained the instantaneous frequency variation laws of the signal $s(t)$ for all three time-frequency representations and for every value of the noise variance σ^2. The curves estimated for $\sigma^2 = 0.16$ are presented in Figure 2: (a) Short-Time Fourier, (b) Gabor and (c) Wigner-Ville. For every detected curve



(a)



(b)



(c)

Fig. 2. IF curves of $s(t)$ when $\sigma = 0.4$ for the following representations: (a) Short-Time Fourier, (b) Gabor and (c) Wigner-Ville.

were also estimated the interpolation polynomials and were calculated the coefficients of these polynomials and the errors which characterise them.

In tables 1, 2, 3, 4 and 5 are presented the interpolation coefficients and their errors for all three time-frequency representations and for the five different values of the noise variance mentioned above.

Table 1. The interpolation coefficients and their errors for $\sigma = 0.1$.

Time – freq. represent.	a	b	c	d	er_a [%]	er_b [%]	er_c [%]	er_d [%]
STFT	18.36	−166.02	334.12	252.03	−0.847	−0.389	0.237	0.811
Gabor	18.38	−166.37	336.54	248.91	−0.752	−0.177	0.962	−0.437
Wign-Ville	18.72	−168.85	341.37	241.61	1.114	1.331	2.411	−3.354

Table 2. The interpolation coefficients and their errors for $\sigma = 0.2$.

Time – freq. represent.	a	b	c	d	er_a [%]	er_b [%]	er_c [%]	er_d [%]
STFT	18.34	−165.55	333.44	254.71	−0.973	−0.672	0.233	0.884
Gabor	18.37	−166.32	336.39	251.33	−0.778	−0.209	0.917	0.531
Wign-Ville	18.28	−166.49	342.71	241.38	−1.261	−0.103	2.813	−3.450

Table 3. The interpolation coefficients and their errors for $\sigma = 0.3$.

Time – freq. represent.	a	b	c	d	er_a [%]	er_b [%]	er_c [%]	er_d [%]
STFT	18.27	−163.98	324.40	253.43	−1.357	−1.614	−1.780	1.372
Gabor	18.56	−168.24	341.91	254.48	0.239	0.946	2.573	1.971
Wign-Ville	17.58	−158.45	319.75	246.77	−5.089	−4.930	−4.075	−1.291

Table 4. The interpolation coefficients and their errors for $\sigma = 0.4$.

Time – freq. represent.	a	b	c	d	er_a [%]	er_b [%]	er_c [%]	er_d [%]
STFT	18.80	−163.06	327.40	253.43	−1.524	−2.165	−2.118	1.948
Gabor	18.59	−169.87	339.74	254.93	0.361	1.910	0.917	1.970
Wign-Ville	18.07	−155.73	317.66	245.61	−7.82	−6.560	−4.701	−1.756

Table 5. The interpolation coefficients and their errors for $\sigma = 0.5$.

Time – freq. represent.	a	b	c	d	er_a [%]	er_b [%]	er_c [%]	er_d [%]
STFT	19.53	−159.02	320.10	257.20	4.497	−4.588	−3.966	2.881
Gabor	19.29	−160.21	342.19	257.87	4.152	−3.871	2.658	3.148
Wign-Ville	16.54	−149.62	311.29	236.12	−10.67	−10.23	−6.611	−5.549

4 Conclusions

In [5] the author has demonstrated that the wavelet time-frequency representation has the best behaviour in the process of instantaneous frequency estimation, in comparison with other time-frequency transforms, namely the Short-Time Frequency, Gabor, Wigner-Ville and Choi-Williams transforms. This demonstration was performed for the case of frequency modulated signals covered by a zero-mean Gaussian noise characterised by different values of the noise variance. It was shown that the IF detection was possible in very good conditions for noise variance values less or equal to 0.16. and in satisfactory conditions even for $\sigma^2 = 0.25$.

In this paper the same method is used for the case of the same family of signals but when the Gaussian noise was low-pass filtered and when were used only Short-Time Frequency, Gabor and Wigner-Ville time-frequency representations. Based on the results presented in tables 1, 2, 3, 4 and 5 it can be concluded that the Gabor transform has a good behaviour only for noise variances lower than 0.16.

Other authors whose papers are mentioned in the references published similar results. But their results were only qualitative, based on images, while in this paper the conclusions are based on qualitative results.

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