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Adaptive Equalizer With Zero-Noise Constrained LMS Algorithm

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Abstract: This paper introduces a type of zero-noise constrained LMS (ZN-CLMS) algorithm in an adaptive equalizer. It is a particular case of mismatched noise constrained LMS (NCLMS) algorithm. It is also a variant of variable step-size LMS algorithm, where the step-size rule arises naturally from the constraints. We will show here that the adaptive equalizer based on the ZN-CLMS algorithm has favorable performance. Computer simulation results are provided to support the proposed implementation of the ZNCLMS.

Keywords: Adaptive Equalizer, LMS adaptive algorithm, zero noise-constrained LMS.

1 Introduction

Adaptive equalizer has been widely used in data communication systems to combat intersymbol interference [4]. An additional scenario of some practical interest, particularly in the mobile communication environment, is the operation of the adaptive equalizer in the presence of an interferer.

The most frequently used structure of equalizer is a transversal adaptive filter with appropriate algorithm. The computationally efficient least mean square (LMS) adaptive algorithm [1, 2, 3, 4, 5, 6] is often used in the equalizer implementation. However, it does not always converge in an acceptable manner. The step-size in the LMS algorithm controls the convergence rate of the adaptive filter coefficients, and also determines the steady state performance. Since the convergence time is inversely proportional to

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the step-size, a large step-size is often selected for fast convergence. However, this selection results in deterioration of the steady state performance (increased misalignement) [1, 2, 3].

There are numerous adaptive algorithms, [1, 2, 3, 7], derived from the conventional LMS algorithm. The objective of the alternative LMS-based algorithms is to reduce convergence time or misalignment, without substantial increase in computational complexity. Variable step-size LMS algorithms (VS LMS) are a popular modification of LMS that vary the step-size to better tradeoff the conflicting requirements of large step-size to maximize convergence rate and small step-size to minimize misadjustment. To design a VS LMS algorithm, intuitively one should use a larger ste-size when the estimate is far from the optimum and a smaller step-size as it approaches the optimum.

Noise-Constrained LMS (NCLMS) adaptive algorithm, proposed in [7], is a type of variable step-size LMS algorithm where the step-size rule arises naturally from the noise variance constraint. NCLMS exploits knowledge of channel noise variance for identification and tracking of FIR channels. A Robbins-Monro algorithm is used to minimize a mean square error criterion subject to a noise variance constraint [7]. For the constant channel case, the constraint does not alter the unconstrained Wiener solution but can positively affect the trajectory of the adaptive algorithm. Moreover, note that in many coomunication application (as the channel equalizer), additive white Gaussian noise is an appropriate model for thermal and possible interference noise, and significantly, the noise power can be accurately estimated prior to channel tap estimation.

The ZNCLMS is similar to the NCLMS. Naimely, if one can not estimate the noise variance, than it is replaced by zero, thus producing a zero-noise constrained LMS algorithm (ZNCLMS). That is a particular case of mismatched NCLMS algorithm.

This paper is organized as follows. The adaptive channel equalizer is summarized in Section II. In section III, NCLMS and ZNCLMS algorithms are presented. Simulation results are presented in Section IV, comparing ZNCLMS and LMS algorithm in adaptive equalizer.

2 Adaptive Equalizer

The classic model for a linear channel with an adaptive equalizer is shown in Figure 1. Observe that channel equalizer or inverse filtering consists of estimating a transfer function to compensate for the linear distortion caused by the channel. From another point of view, the objective is to force a prescribed dynamic behavior for the cascade of the channel (unknown system) and the adaptive filter, determined by the input signal. The first interpretation is more appropriate in communications, where the information is transmitted through dispersive channels. The second one is appropriate for control applications, where the inverse filtering scheme generates control signals to be used in the unknown system.

Let us define the signal vector at the equalizer input $\overline{X}(k) = [x(k)\, x(k-1)\, x(k-2) \ldots x(k-N+1)]^T$ and the vector of weighting coefficients $\overline{W}(k) = [W_0(k)\, W_1(k)\, W_2(k) \ldots W_{N-1}(k)]^T$ of the adaptive filter at an instant k, [1, 2, 3, 4, 5, 6, 7]. Moreover, the signal samples at the equalizer input are of the form:

$$x(k) = \sum_j h(j)a(k-j) + n(k), \tag{1}$$

where $a(k)$ denotes the *k-th* data sample, $n(k)$ - the additive noise with the variance σ_n^2, and $h(j)$ - the channel impulse response. The data samples take on values of ± 1 ($a(k) = \pm 1$) only, and the noise is assumed to be indenpendent and identically distributed. The equalizer output at the *k-th* iteration instant is:

$$y(k) = \overline{W}^T(k)\overline{X}(k). \tag{2}$$

The output y(k) is used in estimating the transmitted data symbol $a(k-Ko)$, with Ko denoting the delay. The *k-th* output error sample is:

$$e(k) = y(k) - a(k-Ko). \tag{3}$$

The weighting coefficients in the LMS algorithm are obtained from the following expression:

$$\overline{W}(k+1) = \overline{W}(k) + \mu\; e(k)\overline{X}(k), \tag{4}$$

where μ is the algorithm step-size.

The output mean square error (MSE) is:

$$\begin{aligned} \varepsilon(k) &= E(e^2(k)) = \overline{W}^T(k)R\overline{W}(k) + E(a^2(k)) \\ &\quad -2\overline{W}^T(k)E(\bar{X}(k)a(k-Ko)), \end{aligned} \tag{5}$$

with $R = E(\bar{X}(k)\bar{X}^T(k))$. The average output MSE after *k-th* iteration can be expresed as:

$$\varepsilon_{avr}(k) = \varepsilon_{MIN} + E(\overline{V}^T(k)R\bar{V}(k)), \tag{6}$$

where ε_{MIN} is the MSE minimum ((5), for optimal weighting coefficients vector $\overline{W}^*(k)$, i.e Wiener vector) and $\overline{V}(k) = \overline{W}(k) - \overline{W}^*(k)$ is the weighting coefficient error vector. In the steady state, the MSE above ε_{MIN} is known as the excess MSE.

As shown in [1, 2, 3, 9], the excess MSE for LMS algorithm is given by:

$$\varepsilon_e = \frac{1}{2}\mu\sigma_n^2 tr(R). \tag{7}$$

It may be observed from (7) that the excess MSE is due to the gradient noise and is proportional to the step-size. The step-size must be selected to balance the conflicting goals of the fast convergence (large step-size) and small stedy-state error, i.e. small excess MSE (small step-size).

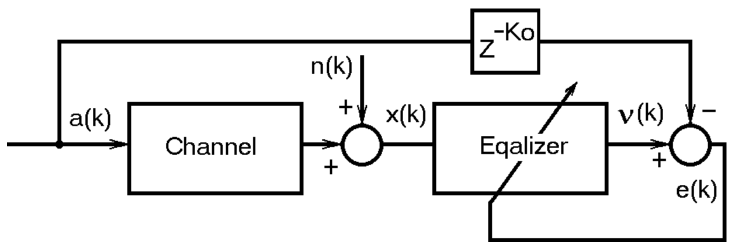

Fig. 1. Adaptive channel equalizer.

3 Zero-Noise Constrained LMS Algorithm (ZNCLMS)

Note that although the optimal weighting coefficients vector $\overline{W}^*(k)$ does not depend on the additive noise variance σ_n^2, this does not mean that a (partially) adaptive algorithm for estimating the optimum weighting vector cannot exploit knowledge of σ_n^2. In particular, away from the optimum, the knowledge of σ_n^2 might be useful in selecting search directions and/or step-size in an adaptive algorithm.

In [7], the problem of constrained MSE minimization, incorporating knowledge of σ_n^2, is analyzed in detail. Updates of the NCLMS algorithm weighting coefficients are obtained from following expressions:

$$\mu(k) = \mu(1 + \gamma\lambda(k)), \tag{8}$$

$$\overline{W}(k+1) = \overline{W}(k) + \mu(k)e(k)\overline{X}(k), \tag{9}$$

$$\lambda(k+1) = \lambda(k) + \alpha(\frac{1}{2}(e^2(k) - \sigma_n^2) - \lambda(k)). \tag{10}$$

where $\alpha, \gamma, \mu > 0$. If σ_n^2 in (10) is replaced by 0, then it reduces to:

$$\lambda(k+1) = (1-\alpha)\lambda(k) + \frac{\alpha}{2}(e^2(k)). \tag{11}$$

Now, relations (8), (9) and (11) represent the weighting coefficinets update in the so called zero-noise constrained LMS (ZNCLMS) algorithm. It is similar to the Kwong variable step-size algorithm, [8]. However, the ZNCLMS is not exactly the same as the VSLMS in [8], because the relation for step-size calculation is not the same. Moreover, the ZNCLMS has three (not two, as the VS LMS in [8]) independent parameters. This parameterization admits an analysis that explicitly shows how the convergence rate of ZNCLMS can be increased over LMS, without increasing misadjustment.

Based on convergence analysis and properties of the NCLMS algorithm, its excess MSE in the steady state is approximately given by [7]:

$$\varepsilon_e = \frac{1}{2}\mu tr(R)\sigma_n^2\Big[1 + \frac{\gamma^2\alpha\sigma_n^4}{2(2-\alpha)}\Big], \tag{12}$$

while, for the ZNCLMS it is given by:

$$\varepsilon_e = \frac{1}{2}\mu tr(R)\sigma_n^2\left[1 + \frac{\gamma\sigma_n^2}{2} + \frac{\gamma^2\alpha\sigma_n^4}{2(2-\alpha)(1+\frac{\gamma\sigma_n^2}{2})}\right]. \tag{13}$$

It may be observed from (7), (12) and (13) that the excess MSE's for NCLMS and ZNCLMS are negligibly larger then the excess MSE for LMS algorithm. To compare the algorithms, the step-size for the LMS is chosen in order to obtain a specified misadjustment, and then the step-sizes μ_{NCLMS} and μ_{ZNCLMS}, for NCLMS and ZNCLMS, respectively, are chosen so as to make the misadjustment for each considered algorithm equal. This is achieved by the appropriate choice of the parameters α and γ, for NCLMS and ZNCLMS, respectively. The performance advantage of the NCLMS and ZNSLMS over LMS algorithm is obtained by suitable choice of the parameters α, γ, [7].

Comparison of largest time constants for mean weighting coefficients convergence (which is a simple measure of inverse convergence rate) of LMS, NCLMS and ZNCLMS algorithms, [7], shows that the NCLMS and ZNCLMS have higher learning rate than the LMS algorithm.

Moreover, NCLMS can exploit knowledge of the noise variance to increase the learning rate over ZNCLMS, which does not use the noise variance.

4 Simulation Results

Here we consider the case when the noise variance is not known and/or it may not be estimated. This is an appropriate case for using the adaptive equalizer with the ZNCLMS algorithm. The performance of the algorithm is compared with standard LMS algorithms with different step-sizes.

In the presented simulations, the signaling is binary ($a(k) = \pm 1$) and the sampled channel impulse response is raised-cosine pulse defined by:

$$h(i) = \begin{cases} \frac{1}{2}(1 + \cos(\frac{2\pi(i-1)}{z}), & 1 \leq i \leq 3 \\ 0, & \text{otherwise} \end{cases}, \tag{14}$$

where z is set equal to 3.0 or 3.3, corresponding to the input autocorrelation matrix eigenvalue ratio (spread) of 8 or 21, respectively. The additive noise $n(k)$ is a white Gaussian one, uncorrealted with the input signal, and with zero mean and variance $\sigma_n^2. = 10^{-3}$. Here we present the resultas from two examples, the first for $z = 3.0$ and the second for $z = 3.3$. Figures 2 and 3 show the MSE characteristics for each considered algorithm, for $z = 3.0$ and $z = 3.3$, respectively. Results are obtained by averaging over 200 independent runs.

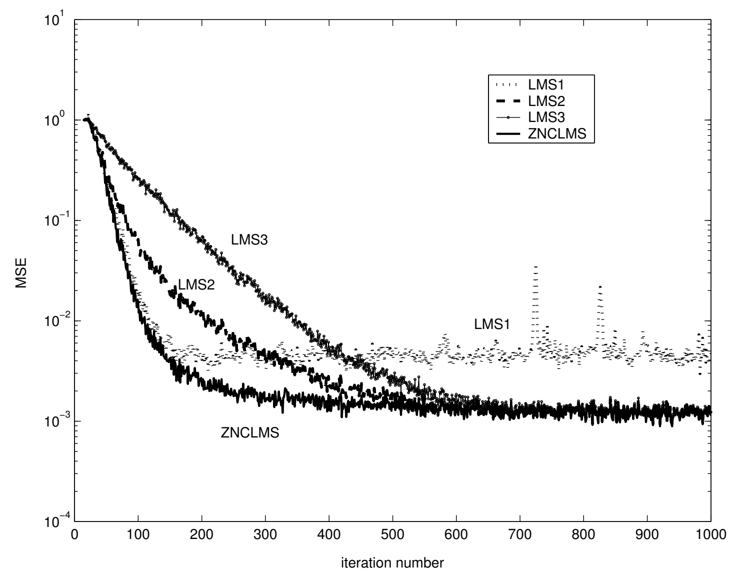

Fig. 2. Comparison of MSE for considered algorithms.

In both examples the adaptive filter has 15 weighting coefficients and the delay parameter was $Ko = 7$. For both of these examples, the ZNCLMS has the following parameter values: $\alpha = 0.02$, $\gamma = 120$ and $\mu = 0.0029$. These parameters are obtained by simulation and according to the suggestions from [7]. For comparison with the ZNCLMS, we use three LMS algorithms with different step-sizes: $\mu_L = 0.08$ (LMS1), $\mu_L/4$ (LMS2) and $\mu_L/8$ (LMS3).

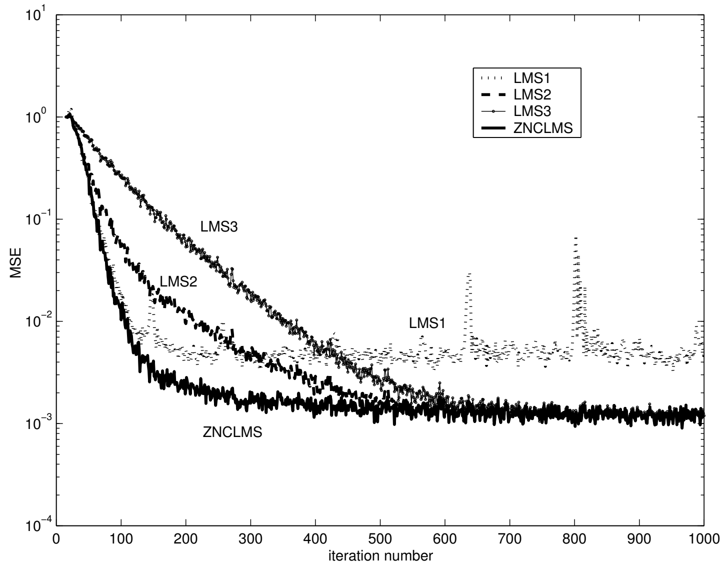

Fig. 3. Comparison of MSE for considered algorithms.

In order to clearly compare the obtained results, for each algorithm we calculated the average MSE ($MSE'_a = 1/P \sum_p [1/(IT) \sum_k e^2(k)]$, where $P = 200$ is the number of independent runs and $IT = 1000$ is number of iterations). The ZNCLMS resulted in MSE_a=0.0288, the LMS1 produced MSE_a=0.0341, the LMS2 gives MSE_a=0.0362, and the LMS3 resulted in MSE_a=0.0696; this is the first example (Figure 2). For the secend example (Figure 3), these values were: MSE_a=0.0291, MSE_a=0.0345, MSE_a=0.0365 and MSE_a=0.0673, for the ZNCLMS, LMS1, LMS2 and LMS3, respectively. Note that the ZNLMS algorithm has favorable performnace over the considered LMS algorithms in the transient phase, as well as in the steady state, as it may be observed from Figure 2 and Figure 3.

Figure 4 shows the averaged value of the step-size for the ZNCLMS algorithm, in the first example. Note that the advantage of the proposed algorithm stems from the appropriate changes of its step-size values.

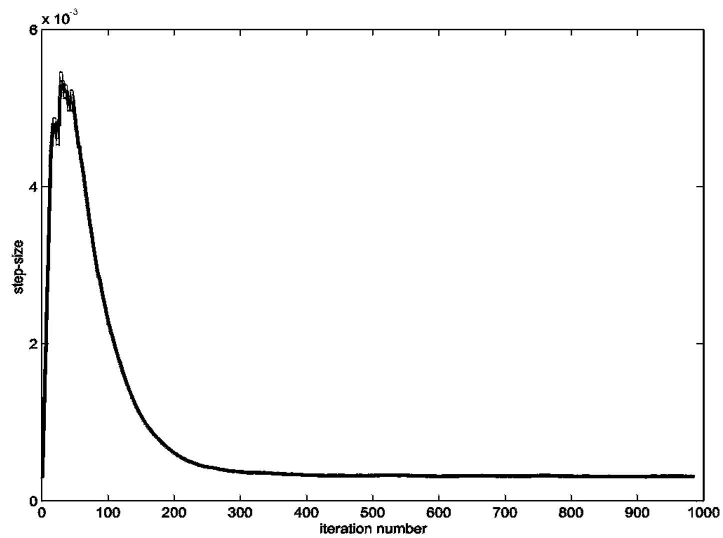

Fig. 4. Change of step size for the ZNCLMS algorithm.

5 Conclusions

A type of noise constrained LMS (NCLMS) algorithm in an adaptive equalizer, called zero noise constrained LMS (ZNCLMS), is introduced. This kind of algorithm is used because of its favourable properties in the case when the noise variance is not known and/or it may not be estimated. It is a variant of variable step-size LMS algorithm, where the step-size rule arises naturally from the constraints. We show that the adaptive equalizer based on the ZNCLMS algorithm have favorable performance over standard LMS algorithms in the considered application. Simulation results support the proposed implementation of ZNCLMS.

Acknowledgments

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