FACTA UNIVERSITATIS (NIŠ)
SER.: ELEC. ENERG. vol. 15, No. 3, dec. 2002, 399-408

Starting Generator Driving Torque Modelling in the Studies of Synchronous Starting of Synchronous Machines

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Abstract: The paper deals with significance of the starting generator (SG) driving torque modelling in the studies of synchronous starting of synchronous machines (SSSM). The detailed mathematical model of SSSM in reversible pumped storage plant (RPSP) "Bajina Bašta" is derived. Because it is very difficult to include the detailed model of the SG turbine in the system model, function of SG driving torque is modelled approximately. The most frequent case in SSSM studies is its approximation with a time linear function, until it reaches the value that should provide rotating of both machine rotors with synchronous velocities, which is also used in the paper. Afterwards, three different cases are considered: constant, linear and polynomial approximation in terms of SG rotor velocity. Numerical results obtained by applying developed models to a particular case of synchronous starting are compared with the corresponding experimental results. In this way, it is shown that the application of the most frequently used SG driving torque modelling leads to the erroneous results. The advantages of the polynomial approximation suggested in the paper and its validity is demonstrated.

Keywords: Synchronous starting, synchronous machines, "back-to-back start, torque modelling.

1 Introduction

Synchronous starting of synchronous machines (SSSM) in reversible pumped storage plants is very complex process affected by large number of different

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Manuscript received March 23, 2002.

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parameters. For this reason, in SSSM modelling, it is very difficult to develop a detailed mathematical model involving all the influential effects. Thus, many simplified approaches to the SSSM modelling and analysis, concerning the way of modelling of synchronous machines, SG driving torque, motor/generator (MG) load torque, machine interconnection, etc., are developed in literature ([1]-[7]).

Regarding the fact that the function of SG driving torque actually depends on large number of parameters, it is practically impossible to take care of all of them, unless the detailed model of the SG turbine is included in the system model. Since the mathematical modelling becomes extremely difficult in this way, in SSSM studies this function is usually modelled approximately. The most frequent case is its approximation with a time linear function, until it reaches the value that should provide rotating of both machine rotors with synchronous velocities. Afterwards, it is considered constant. Fewer authors make a step further taking into account the turbine selfregulation coefficients ([5]), which actually represent the SG driving torque as a linear function of SG rotor velocity, and which is much closer to the real situation. Besides two mentioned approaches, the authors suggested a new one in [6]: the polynomial approximation of SG driving torque in terms of SG rotor velocity. In this paper, all of the three mentioned approaches are analysed.

In order to show the differences between the introduced approaches and to point out the significance of SG driving torque modelling, detailed mathematical model of the SSSM in RPSP “Bajina Bašta” is derived.

2 Mathematical Model of Synchronous Starting in RPSP ”Bajina Bašta”

Mathematical model of synchronous starting in RPSP ”Bajina Bašta” is derived using one-line electrical scheme of synchronous starting ([7]), which is reduced to an equivalent one, shown in the Fig. 1.

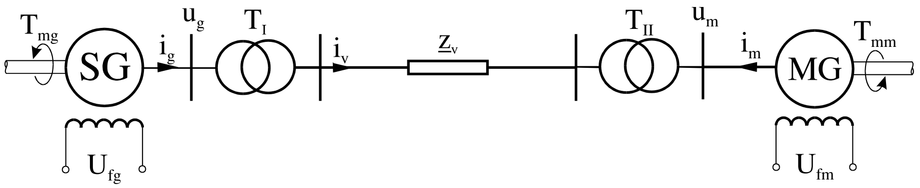


Fig. 1. Simplified electrical scheme of the SSSM.

This scheme represents the most general case of synchronous starting where SG and MG are machines with different rated parameters, so they can not be connected directly. Therefore, transformers T_I and T_{II} are used.

In spite of relatively simple appearance of the circuit in Fig. 1, it is difficult to derive the mathematical model suitable for numerical integration. A procedure of model deriving is complicated ([6]), so it will not be presented here. Due to simplicity, only a procedure of modelling of characteristical elements will be briefly presented in this paper.

Both of the synchronous machines (SG and MG) are represented by the well-known mathematical model:

$$
\begin{aligned}
p\Psi_{di} &= -\,u_{di} - r_{ai}i_{di} - \omega_i\Psi_{qi}, && (1)\\
p\Psi_{fi} &= U_{fi} - r_{fi}i_{fi}, && (2)\\
p\Psi_{Di} &= -\,r_{Di}i_{Di}, && (3)\\
p\Psi_{qi} &= -\,u_{qi} - r_{ai}i_{qi} + \omega_i\Psi_{di}, && (4)\\
p\Psi_{Qi} &= -\,r_{Qi}i_{Qi}, && (5)\\
p\omega_i &= \frac{T_{mi} - T_{ei}}{J_i}, && (6)\\
p\gamma_i &= \omega_i, && (7)\\
T_{ei} &= \Psi_{di}i_{qi} - \Psi_{qi}i_{di}, && (8)
\end{aligned}
$$

$$
\begin{bmatrix}\Psi_{di}\\ \Psi_{fi}\\ \Psi_{Di}\end{bmatrix} = \begin{bmatrix} x_{\gamma i} + x_{adi} & x_{adi} & x_{adi}\\ x_{adi} & x_{fi} + x_{adi} & x_{adi}\\ x_{adi} & x_{adi} & x_{Di} + x_{adi}\end{bmatrix} \begin{bmatrix} i_{di}\\ i_{fi}\\ i_{Di}\end{bmatrix}, \tag{9}
$$

$$
\begin{bmatrix}\Psi_{qi}\\ \Psi_{Qi}\end{bmatrix} = \begin{bmatrix} x_{\gamma i} + x_{aqi} & x_{aqi}\\ x_{aqi} & x_{Qi} + x_{aqi}\end{bmatrix} \begin{bmatrix} i_{qi}\\ i_{Qi}\end{bmatrix}. \tag{10}
$$

In the previous expressions $p = d/dt$ is the differentiating operator, and the suffix "i" ($i = g, m$) is used to designate variables and parameters relating to the SG and MG, respectively. All equations are normalized, parameters are per unit, and voltage and flux linkage equations are expressed in terms of reactances.

Machine interconnection is modelled by representing transformers T_I and T_{II} by their short circuit impedances $\underline{z}_{kI}$ and $\underline{z}_{kII}$, and connected line by impedance $\underline{Z}_V$ (Figure 2). Clearly, all parameters and variables shown in Figure 2 are referred to the same voltage level.

Usage of same more complex transformer models (e.g. "T"-equivalent scheme) only makes the system model more complicated without affecting

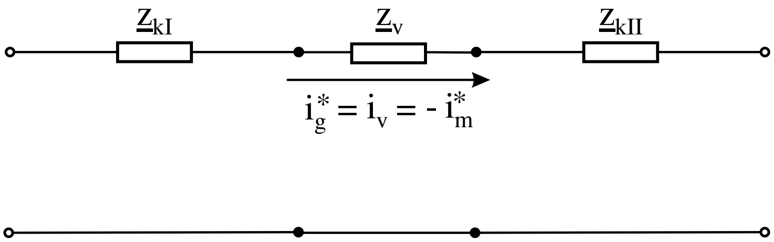


Fig. 2. Electrical scheme of SG and MG interconnection.

the accuracy of obtained numerical results. According to adopted convention for currents and torques, the MG load torque is represented by the relation:

$$T_m = -(T_{m0} + c_0\omega_m^2). \tag{11}$$

Regarding the basic aim of this study, influence of representing of SG driving torque by different functions to obtained numerical results is analysed in the paper. In this sense, three different functions are considered. First of all, SG driving torque is approximated with time linear function, until it reaches the value (T_{mu}) that should provide rotating of both machine rotors with synchronous speeds. Afterwards, it is considered constant, being the most frequently used modelling way of SG driving torque function in literature ([1]-[3]):

$$T_{mg} = \begin{cases} \dfrac{T_{mu}}{t_1}t & \text{for} \quad t < t_1 \\ T_{mu} & \text{for} \quad t \geq t_1 \end{cases}. \tag{12}$$

Characteristical point (T_{mu}, t_1) of the curve (12) relates to the point when guide vane opening of SG turbine is set to a constant value. In practice, before this moment, guide vane opening speed is commonly time linear function.

Dealing with the described model of SG driving torque, the authors noticed that significant differences appeared between numerical and experimental results. Only a few numerical experiments were enough to conclude that the basic reason for this was the way of modelling of SG driving torque, and that real function does not have the form (12). Analyses conducted in [6] show that after reaching the value T_{mu} SG driving torque decays as the SG rotor accelerates, even though the guide vane opening remains constant.

In order to make identification of real SG driving torque function the authors introduced its polynomial approximation.

By using the first order polynomial for representing a SG driving torque, it can be expressed in the following way:

$$T_{mg} = \begin{cases} \dfrac{T_{mu}}{t_1} t & \text{for} \quad t < t_1 \\ T_{mu} - c(\omega_g - \omega_{pr}) & \text{for} \quad t \geq t_1 \end{cases} . \tag{13}$$

where c represents turbine speed selfregulation coefficient ([5]), and ω_{pr} is SG rotor speed in the moment when guide vane opening reaches the constant value.

Introducing the second order polynomial, the next function is obtained:

$$T_{mg} = \begin{cases} \dfrac{T_{mu}}{t_1} t & \text{for} \quad t < t_1 \\ T_{mu} - c(\omega_g - \omega_{pr}) - c_1(\omega_g - \omega_{pr})^2 \ldots & \text{for} \quad t \geq t_1 \end{cases} . \tag{14}$$

In all the previously stated relations ((12)-(14)), function of SG driving torque is represented by the same expression for $t < t_1$. Strictly speaking, this presumption is not correct, and there are lot of reasons for new polynomial approximation introducing. But, it will be shown in this paper that it is not necessary.

It is significant to point out the fact that with all three presented functions, procedure of system model deriving is practically the same, which results in the same values of time needed for numerical simulations.

3 Parameters of Synchronous Starting in RPSP "Bajina Bašta"

Formed models are applied on the case of test synchronous starting in RPSP "Bajina Bašta" done in December 20th, 1981 with an aim to perform its computer simulation. Recordings of characteristic variables and values of numerous parameters were available ([8]). Values of some unknown parameters had to be estimated. Values of all parameters needed for numerical simulation are presented in the following part.

Synchronous machines parameters used in simulations are shown in Table 1. All resistances and reactances are expressed as values per unit on the own machine base.

Table 1. Parameters of machines in RPSP "B. Bašta"

Parameter	SG	MG
S_n [MVA]	100	310
U_n [kV]	15.65	11
I_n [kA]	3.68914	16.2708
n_n [rpm]	136.3636	428.57143
r_a [p.u.]	0.00355214	0.0015756
r_f [p.u.]	0.00047914	0.00040048
r_D [p.u.]	0.014296	0.0154329
r_Q [p.u.]	0.0118055	0.0148583
x_γ [p.u.]	0.14	0.1
x_{ad} [p.u.]	0.756	1.08
$x_{f\gamma}$ [p.u.]	0.202953	0.230562
$x_{D\gamma}$ [p.u.]	0.096	0.0876923
x_{aq} [p.u.]	0.44	0.68
$x_{Q\gamma}$ [p.u.]	0.0977778	0.0906667
J [p.u.]	2576.105976	3264.1147

Total value of connected line impedance, used in numerical simulations, is taken from [7]: $\underline{z}_{vu} = \underline{z}_{kI} + \underline{z}_v + \underline{z}_{kII} = (0.01981 + j0.5079)$ [p.u.].

The machine excitations were set to the values: $U_{fg} = 1$ [p.u.], $U_{fm} = 0.8$ [p.u.] in experiment.

For MG load torque parameters, the next values are adopted: $T_{m0} = 0.01$ [p.u.], $c_0 = 0.035$.

Identification of SG driving torque function can not be done because of the lack of parameters related to this function. To solve this problem the authors have used a great sensitivity of machine stator currents to SG driving torque changing.

In the simplest case (12), only a few numerical experiments were enough to estimate the range of parameters T_{mu} and t_1 which result in satisfactorily precise MG stator current shape. In this way, the following values are determined and used in simulations: $T_{mu} = 0.52$ [p.u.], $t_1 = 56.5$ [s]. In a similar way parameter identification of functions (13) and (14) are performed. Parameter values used in simulations are listed below

$$
\begin{aligned}
T_{mu} &= 0.52[p.u.], \quad t_1 = 56.5[s], \quad c = 0.31; \\
T_{mu} &= 0.52[p.u.], \quad t_1 = 56.5[s], \quad c = 0.28, \quad c_1 = 0.1.
\end{aligned}
$$

The fact that in the real case SG turbine guide vane starts to open $t_2 = 16.5s$ after the beginning of starting process is taken into account in all three cases.

4 Comparison Between Numerical and Experimental Results

In order to demonstrate the significance of SG driving torque modelling, numerical results obtained by applying of mathematical models previously described on the mentioned synchronous starting test are compared with the corresponding experimental ones.

Time dependent characteristics of MG rotor velocities in numerical simulations and experiment are shown in Figure 3. MG stator currents versus time are shown in Figure 4. (Certain deviations occurred in presented experimental curves because of low quality of experimental result recordings. Certainly, these deviations will not affect the following analysis validity.)

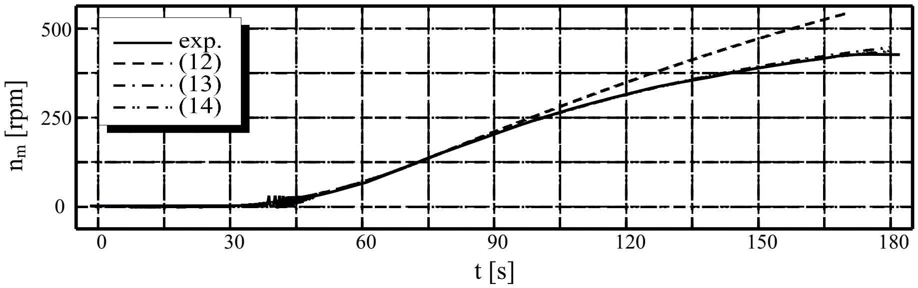
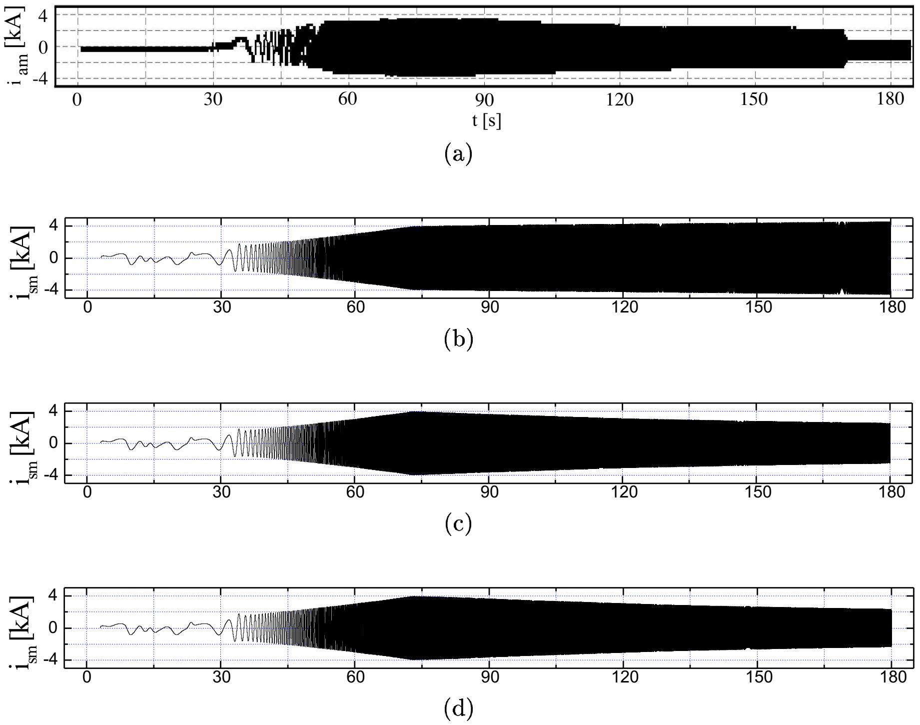


Fig. 3. MG rotor velocity versus time in numerical simulations and experiment.

Obviously, results of numerical simulations significantly depend on way of SG driving torque modelling. In the simplest case, when SG driving torque is represented by the relation (12), differences between numerical and experimental results are great. Namely, one may notice that MG rotor velocity in simulations grows up more rapidly than in real situation, so great differences between these two curves appears after approximately 75 seconds from the beginning of starting process.

MG stator currents also differ from each other in this case. Unlike the magnitude of current obtained by numerical simulation being nearly constant after 75 seconds (in fact, slight magnitude increasing takes place), real magnitude decays as the machine rotor accelerates. Previously stated facts directly lead to the conclusion that presumption that SG driving torque remains constant after guide vane opening reaches constant value is not correct. It is obvious that this function real shape has to decay as the SG rotor accelerates. Consequently, relations either (13) or (14) have to be



(a)

(b)

(c)

(d)

Fig. 4. MG stator current versus time: (a) exp., (b) (12), (c) (13), (d) (14).

better SG driving torque approximations than relation (12).

Confirmation of this statement is given by the curves shown in Figures 3, 4(c), and 4(d), because these are very much close to the experimental ones. Deviation of the curve (13) given in Figure 3 from the experimental one, starts approximately 130 seconds after beginning of starting process. On the contrary, deviation of the curve (14) is not visible until 170th second, and than this is direct consequence of acting of SG turbine governor which is not included in the system model.

Since the modelling of SG driving torque is done by the same function in all three cases, waveshapes of stator currents given in the Figures 4(b), 4(c), and 4(d) are identical in the first 75 seconds of starting process. Comparing any of these curves with the experimental one (Figure 4(a)) one may conclude that a function of the form:

$$T_{mg} = c_2(t - t_2) - c_3(t - t_2)^2 - \dots \text{ for } t_2 < t < t_1, \tag{15}$$

will be more appropriate than the function applied. But, because of low quality of experimental result recordings, any serious analysis is impossible in this paper.

Finally, SG driving torque functions used in numerical simulations versus SG rotor velocity are shown in Figure 5. According to slight differences between the curves obtained using relations (13) and (14), conclusion that it is not necessary to deal with the polynomial approximation of SG driving torque can be drawn. But, according to the authors' experience, there are lot of cases of SSSM where benefits of polynomial approximation introducing are much more obvious. It should be pointed out in this stage that this characteristic depends on lot of parameters of SG turbine, of turbine governor and parameters of complete hydrosystem. For example, even though the water head is the only parameter differing in two starting processes significant differences between results obtained with linear and polynomial approaches can appear.

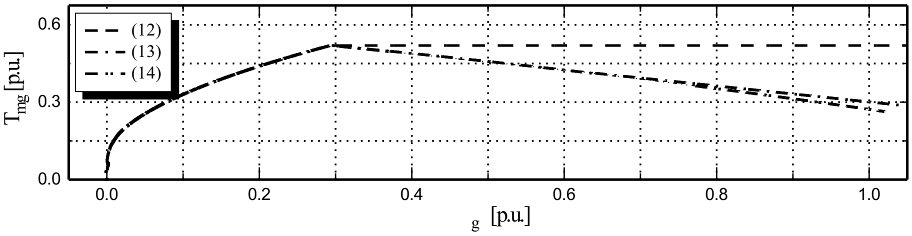


Fig. 5. SG driving torque versus its rotor velocity.

5 Conclusions

Significance of the starting generator (SG) driving torque modelling in the studies of synchronous starting of synchronous machines (SSSM) is analysed in this paper. For this reason, detailed mathematical model of SSSM in RPSP "Bajina Bašta" is formed. Three different ways of SG driving torque modelling are explored. Parameter identification of SG driving torque functions is done using recordings of test starting. In this way, comparison between numerical and experimental results is enabled.

Results of comparison show that application of the most frequently used model for SG driving torque function (12) leads to the erroneous results in all characteristic variable waveshapes. To avoid such situations in SSSM modelling, polynomial approximation of SG driving torque function (14) is suggested in the paper. Obtained numerical and experimental results coincide very well after applying suggested approach. In this way, validity of the formed model is demonstrated.

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