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Direct Synthesis of the Digital FIR Full-Band Differentiators

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Abstract: In this paper, two methods for designing digital full-band FIR differentiators are presented. First of them, named Immediate, represents the efficient and general method for brusque designing the first and higher degree digital full-band FIR differentiators of even and odd order, with simultaneous approximation of the prescribed magnitude and group delay responses, using originally modified eigenfilter method. The proposed method presents an approach for the FIR differentiator frequency response approximation directly in the complex, and not in the real domain. The second method, named Intermediary, represents the efficient approach for oblique designing only the second and higher degree digital full-band FIR differentiators of odd order. The comparison of the characteristics and results of these two presented methods is performed. In order to illustrate the presented methods effectiveness, the numerical design examples of the first second degree full-band digital differentiators are given, too.

Keywords: FIR differentiator, group delay, eigenfilter.

1 Introduction

Digital FIR differentiators are devices whose output signal samples are equal to the samples of the input signal derivative of the first or higher degree. They have a broad application in a various practical signal processing systems, such as: control systems, various communication systems, seismic systems, biomedical electronic devices etc.

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Digital differentiator, in a general case, can be designed by using many available numerical differentiation formulas, such as: Gregory-Newton forward and backward difference formulas or Bessel, Everett and Stirling central difference formulas. They can also be designed as nonrecursive digital filters on the basis of the frequency response of an ideal differentiator, or by Fourrier series method in conjunction with the Kaiser window function.

Some of very good and interesting techniques for the synthesis of the linear phase digital differentiators of first and higher orders, are presented in [1]-[8] All techniques proposed in these papers represent approaches for the frequency response complex function approximation in the real (and not directly in the complex) domain. However, for the design of the FIR filter, having lower time delay than the linear phase FIR filter, and approximately constant passband group delay level, it is needed to solve the complex approximation problem. Very good techniques for this purpose are presented in [9]-[16]. In some of them [15], [16] the complex approximation problem is (again) converted in a problem of real approximation, but most of the rest can be considered as the iterative linear programming techniques. The common characteristic for all those methods is that their algorithms are rather complex and demand significant computer memory space and computing times.

In this paper, two efficient methods for designing digital full-band FIR differentiators are presented, which are very fast, easy and effective, because they don't involve any iterative calculation. First of them, named Immediate method, represents the powerful and general method for immediate designing the first and higher degree digital full-band FIR differentiators, directly in the complex domain [17], [18]. The second method, named Intermediary method, represents the efficient approach for intermediary designing only the second and higher degree digital full-band FIR differentiators of odd order [19].

2 Development of the Proposed Immediate Method

In a general case, the frequency response of an ideal full-band differentiator ($\omega \in [0, \pi]$) with linear phase, of degree k, is given by :

$$F(\omega) = (j\omega)^k e^{-j\omega\tau} = M_k(\omega)e^{-j\omega\tau} = \begin{cases} j(-1)^{\frac{k-1}{2}}\omega^k e^{-j\omega\tau}, & k-\text{odd}, \\ j(-1)^{\frac{k}{2}}\omega^k e^{-j\omega\tau}, & k-\text{even}. \end{cases} \quad (1)$$

In the case of the FIR structure with order N, the passband group delay level is: $(N-1)/2$. For the case of the first degree differentiator ($k=1$), expression (1) is reduced to

$$\begin{aligned} F_1(\omega) &= j\omega e^{-j\omega\tau} = \omega\sin(\omega\tau) + j\omega\cos(\omega\tau) \\ &= F_{R1}(\omega) + jF_{I1}(\omega) = M_1(\omega)e^{jP_1(\omega)}, \end{aligned} \tag{2}$$

while, for the second degree differentiator ($k=2$), it becomes

$$\begin{aligned} F_2(\omega) &= \omega^2 e^{-j\omega\tau} = -\omega^2\cos(\omega\tau) + j\omega^2\sin(\omega\tau) \\ &= F_{R2}(\omega) + jF_{I2}(\omega) = M_2(\omega)e^{jP_2(\omega)}. \end{aligned} \tag{3}$$

The frequency response of the designed FIR full-band differentiator with order N and real impulse response $a(n), n = 0, 1, \ldots, N-1$, regardless of its degree k, is given by

$$\begin{aligned} H(\omega) &= \sum_{n=0}^{N-1} a(n)e^{-j\omega n} = H_R(\omega) + jH_I(\omega) \\ &= \sum_{n=0}^{N-1} a(n)\cos(n\omega) - j\sum_{n=0}^{N-1} a(n)\sin(n\omega). \end{aligned} \tag{4}$$

By introducing the following vectors

$$\begin{aligned} \mathbf{a} &= [a(0)a(1)\ldots a(n-1)]^T \\ \mathbf{c} &= [1\cos(\omega)\ldots\cos[(N-1)\omega]]^T \\ \mathbf{s} &= [0\sin(\omega)\ldots\sin[(N-1)\omega]]^T, \end{aligned}$$

where superscript **T** denotes the vector transpose operation, expression (4) becomes

$$H(\omega) = \mathbf{a^T c}(\omega) - j\mathbf{a^T s}(\omega) = H_R(\omega) + jH_I(\omega). \tag{5}$$

Expression (5) is used now for the approximation both the desired magnitude, $M_k(\omega)$, and phase, $P_k(\omega), (k = 1, 2, \ldots)$ responses of the ideal full-band differentiator, from (1). More exactly, in the defined, full frequency band, the real, $H_R(\omega)$, and the imaginary, $H_I(\omega)$, parts of the designed frequency response, from (5), are designed to respectively approximate the real, $F_{Rk}(\omega)$, and imaginary, $F_{Ik}(\omega)$, parts of the ideal frequency responses, from (2) and (3). This approximation is performed by the minimization of the quadratic measure error, which in a general case is defined as

$$\begin{aligned} E(\alpha_R;\alpha_I) &= \alpha_R \int_0^{\pi} [F_{RK}(\omega)H_R(\omega_0) - H_R(\omega)F_{RK}(\omega_0)]^2 d\omega \\ &+ \alpha_I \int_0^{\pi} [F_{RI}(\omega)H_I(\omega_0) - H_I(\omega)F_{RI}(\omega_0)]^2 d\omega, \end{aligned} \tag{6}$$

which can be written as

$$E(\alpha_R;\alpha_I) = \alpha_R E_R + \alpha_I E_I, \tag{7}$$

where is the total approximation error (of the total frequency response), $E(\alpha_R;\alpha_I)$ is the passband reference frequency, α_R is the weighting coefficient of the frequency response real part approximation,α_I is the weighting coefficient of the frequency response imaginary part approximation, and E_R and E_I are the approximation errors of the frequency response real and imaginary parts, respectively.

Equation (7) obviously shows that the contributions of the frequency response real and imaginary parts approximation errors,E_R and E_I, to the total approximation error $E(\alpha_R;\alpha_I)$, can be adjusted (decreased) by pertinent choice of the numerical values of free parameters α_R and α_I. This is the reason of their introducing in expression (6), i.e.(7). Substituting expressions for $F_{Rk}(\omega)$, and $F_{Ik}(\omega)$, from (2) or (3), and expressions for $H_R(\omega)$, and $H_I(\omega)$, from (5), into equation (6), one can obtain

$$E(\alpha_R;\alpha_I) = \mathbf{a}^{\mathbf{T}}[\alpha_R \mathbf{Q_R} + \alpha_I \mathbf{Q_R}]\mathbf{a}. \tag{8}$$

From (7) and (8), it is obviously that

$$E_R = \mathbf{a}^{\mathbf{T}}\mathbf{Q_R}\mathbf{a} \text{ and } E_I = \mathbf{a}^{\mathbf{T}}\mathbf{Q_I}\mathbf{a}, \tag{9}$$

where Q_R and Q_I are NxN quadratic, real and symmetric matrices of the frequency response real and imaginary parts approximation, respectively, while

$$\mathbf{Q} = \alpha_R \mathbf{Q_R} + \alpha_I \mathbf{Q_I}, \tag{10}$$

in the general case represents NxN quadratic, real and symmetric matrix, whose elements and eigen-system is necessary to determine. On this way, expression (8) becomes

$$E(\alpha_R;\alpha_I) = \mathbf{a}^{\mathbf{T}}\mathbf{Q_R}\mathbf{a}, \tag{11}$$

which represents the classical formulation of the eigenfilter problem in the least-squares sense. For the calculation of integrals in equation (6), the program written in the programming language MATHEMATICA was used. Obtained expressions for the elements of matrix Q, have showed that their values depend on the values of the specification parameters. When the numerical values of the matrix Q elements are computed, its eigen-values and

Table 1. Impulse response coefficients of the first order FIR full-band differentiator

n	a(n)	n	a(n)
0	0.628379409E−03	16	0.705382380E−02
1	−0.159056390E−02	17	−0.523220686E−02
2	0.330944404E−02	18	0.400321043E−02
3	−0.552923990E−02	19	−0.313262771E−02
4	0.881209349E−02	20	0.252435390E−02
5	−0.142607412E−01	21	2.083410E−002
6	0.247141338E−01	22	0.173084849E−02
7	−0.498141902E−01	23	−0.143943804E−02
8	0.140568566E+00	24	0.123051256E−02
9	−0.127345252E+01	25	−0.107678328E−02
10	0.127353719E+01	26	0.929005254E−03
11	−0.140838642E+00	27	−0.792886244E−03
12	0.502920642E−01	28	0.702281650E−03
13	−0.254034690E−01	29	−0.640230141E−03
14	0.151739155E−01	30	0.564529843E−03
15	−0.999862984E−02	31	−0.485819555E−03

eigen-vectors can be easy computed by using some of, in that purpose, available mathematical methods. In that aim we have used the FORTRAN program IMSL EIGRS. Eigenvector a of the matrix Q, from the expression (11), corresponding to its smallest eigenvalue, is the vector which minimizes the error (11), i.e. (7), and, thus, the desired impulse response coefficient vector of the designed second degree FIR full-band differentiator, from (4).

3 Design Examples

Example 1

Design of the first degree FIR full-band differentiator with even order and the following specification parameters values: $N = 32$, $\tau = 9.5$, $\omega_0 = 0.5\pi$, $\alpha_I = 99\alpha_R$.

Example 2

Design of the second degree FIR full-band differentiator with even order and the following specification parameters values: $N = 31$, $\tau = 11.5$, $\omega_0 = 0.5\pi$ $\alpha_I = 99\alpha_R$.

The technique, presented in this immediate method, is general and it is successfully used for the design of the most frequently used differentiators

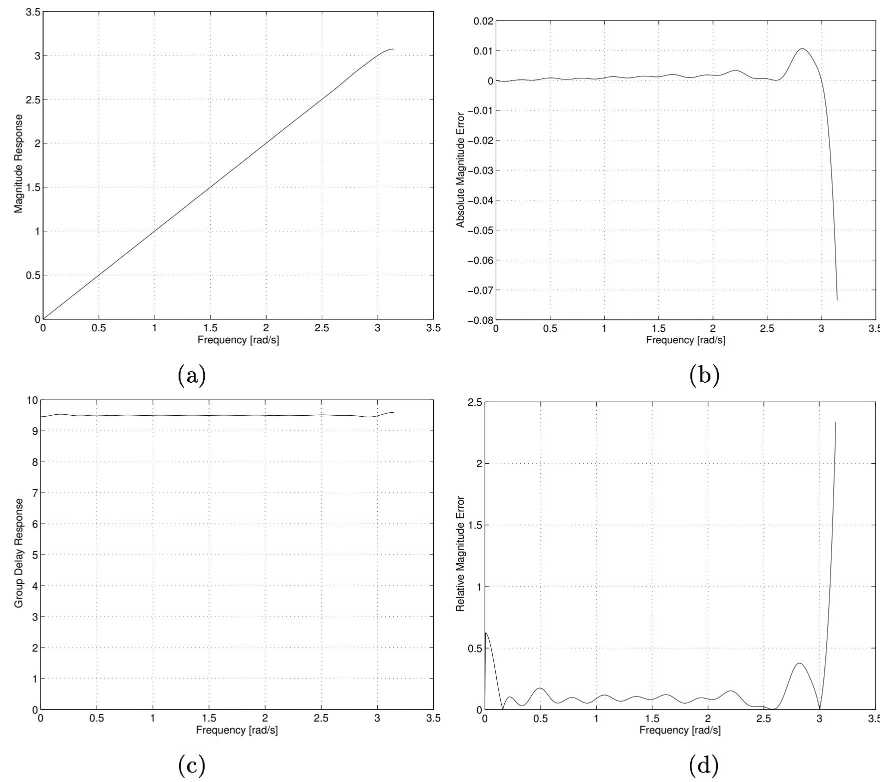

Fig. 1. (a) Magnitude response of the first degree FIR full-band differentiator. (b) Absolute magnitude error of the first degree FIR full-band differentiator. (c) Group delay response of the first degree FIR full-band differentiator. (d) Relative magnitude error of the first degree FIR full-band differentiator.

of first and second degree, giving equally well results for the both types of differentiators. It is used, without any exchange in the design procedure and formulas, for the design of differentiators with both the even and odd order N. By introducing the condition that the specification parameter of the group delay level,τ, has the value which is not an integer, it is possible to design even the first degree FIR full-band differentiator with odd order N, as well as the second degree FIR full-band differentiator with even order N, which is not the case when designing the linear phase FIR full-band differentiators. These characteristics are obtained at the expense of the following fact: full-band differentiators, designed by the presented method, do not posses the (anti)symmetric feature of their impulse response coefficients, and so either the strictly linear phase. They have approximately constant

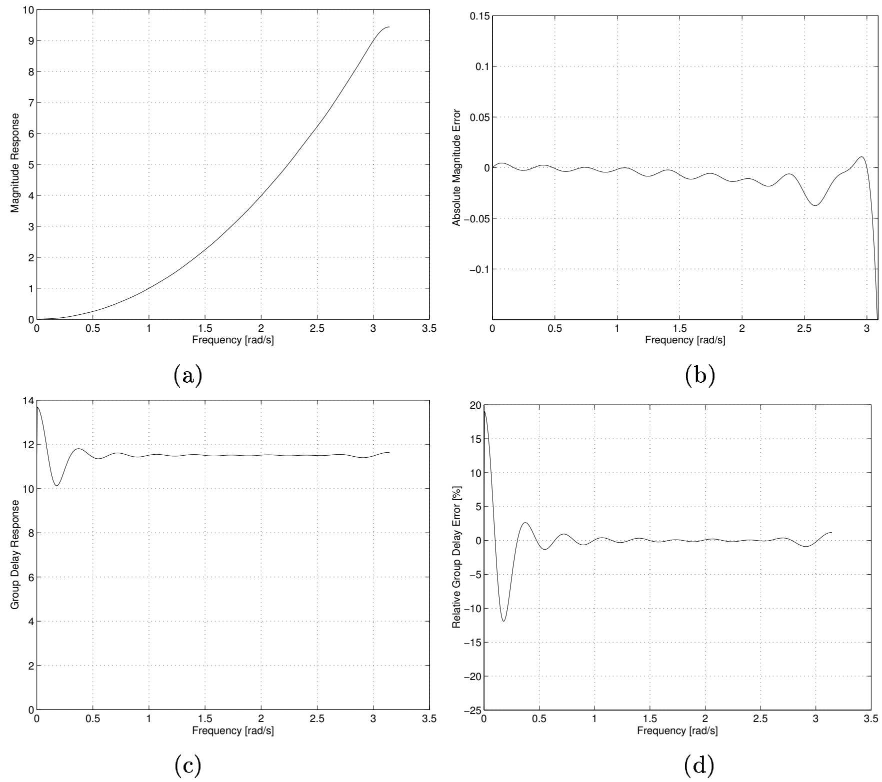

Fig. 2. (a) Magnitude response of the second degree FIR full-band differentiator. (b) Absolute magnitude error of the second degree FIR full-band differentiator. (c) Group delay response of the second degree FIR full-band differentiator. (d) Relative magnitude error of the second degree FIR full-band differentiator

pass-band group delay level that can be distinct (lower or higher) from that of the corresponding linear phase full-band differentiators, simultaneously with extremely low magnitude response error.

When introduced free parameters α_R and α_I have equal values, the particular solution for the designed differentiator frequency response, which is considered as the starting solution, is obtained. By changing the numerical values of these two parameters, the total approximation error (7) can be decreased in the lower (or any other) pass-band part, at the expense of its insignificant increase in the other part of the pass-band; this fact also holds for the magnitude and group delay response characteristics. In addition, performed research have shown that, in a general case, better magnitude response characteristic is corresponding to a slightly inferior group delay

Table 2. Impulse response coefficients of the second order FIR full-band differentiator

n	a(n)	n	a(n)
0	−2.569986E−003	16	−8.867824E−002
1	5.180389E−003	17	6.718717E−002
2	−1.207560E−002	18	−4.283236E−002
3	1.764473E−002	19	3.504568E−002
4	−2.843774E−002	20	−2.436095E−002
5	3.767707E−002	21	2.083410E−002
6	−6.345425E−002	22	−1.599677E−002
7	8.606584E−002	23	1.403822E−002
8	−1.721485E−001	24	−1.065662E−002
9	2.736847E−001	25	9.576038E−003
10	−1.070260E+000	26	−8.090679E−003
11	2.894830E+000	27	7.332384E−003
12	−2.894660E+000	28	−5.704574E−003
13	1.070950E+000	29	5.278305E−003
14	−2.756052E−001	30	−4.798739E−003
15	1.750078E−001		

response characteristic, and vice versa. Due to this, it is obviously that, while choosing these two characteristics, a trade off between their qualities should be madden. This means that by the appropriate analysis of the frequency response real and imaginary parts approximation errors, it is possibly to determine the numerical values of free parametersα_R and α_I , which give the small variation of the group delay response, simultaneously with extremely low magnitude response error, in the pass-band.

4 Development Of The Proposed Intermediary Method

The method for designing the second degree full-band digital differentiator that we shall present here is consisting of two steps : I step: Design of the first degree full-band digital differentiator; II step: Realization of the second degree full-band digital differentiator, by cascade connecting two first degree full-band digital differentiator, designed in the first step of the method.

I step of the design procedure

In the first step of this, intermediary method, the procedure which is used for designing the first degree FIR full-band differentiators with order N1, in fact is the previously presented and described immediate method, which is already illustrated with the design Example 1.

II step of the design procedure

By cascade connecting (i.e. cascading) two ideal first degree full-band DD with orders N_1 and linear phase, one can obtain

$$F_2(\omega) = F_1(\omega)F_1(\omega) = M_1(\omega)e^{-j\omega\tau 1}M_1(\omega)e^{-j\omega\tau 1} = M_2(\omega)e^{-j\omega\tau 2}, \quad (12)$$

where, according to (3)

$$|M_2(\omega)| = |M_1(\omega)M_1(\omega)| = \omega^2$$

and

$$\tau_2 = \tau_1 + \tau_1 = 2\tau_1 = 2\frac{N_1 - 1}{2} = N_1 - 1$$

Notice that cascading two first degree full-band digital differentiator, each of the orderN_1, results in the second degree full-band digital differentiator with order $N_2 = 2N_1 - 1$, which, due to this, is always odd. This fact makes the cascade implementation very convenient for the realization of the second (even) degree full-band digital differentiator. Above expression gives the same value for the group delay level of the cascade realized second degree digital differentiator as theoretical

$$\tau_2 = 2\frac{N_1 - 1}{2} = N_1 - 1. \quad (13)$$

Fore mentioned consideration is used for realization of the second degree full-band digital differentiator, by cascading two first degree full-band digital differentiators designed in the first part of this method

$$\begin{aligned} H_2(\omega) = H_1^{'}(\omega)H_1^{''}(\omega) &= (\sum_{n=0}^{N_1-1} a_1^{'}(n)e^{j\omega n})(\sum_{n=0}^{N_1-1} a_1^{''}(n)e^{j\omega n}) \\ &= (\sum_{n=0}^{N_2-1} a_2(n)e^{j\omega n}). \end{aligned} \quad (14)$$

Using the presented method, two cases of the second degree full-band digital differentiators design, with $N_1 = 32$ (from Example 1) are considered. In the first design case the impulse response coefficients are taken to be identical

$$a_1^{'}(n) = a_1^{''}(n). \quad (15)$$

This case is illustrated by Example 3, in which the figures Fig. 3 shows the characteristics of the obtained second degree full-band digital differentiator. In the second design case the impulse response coefficients are taken to be complementary:

$$a_1^{'}(n) = a_1^{''}(N_1 - 1 - n), n = 0, 1, \ldots, N_1 - 1. \tag{16}$$

This case is illustrated by Example 4, in which the figures 4 shows the characteristics of the obtained second degree full-band digital differentiator.

Example 3

Design of the second degree FIR full-band differentiator by cascade connecting two first degree FIR full-band differentiator designed in Example 1, i.e. in the I step of the design procedure (with the following specification parameters values: $N_1 = 32; \tau = 9.5; \omega_0 = 0.5\pi; \alpha_I = 99\alpha_R$. The relation between their impulse response coefficients is given by (15).

Example 4

Design of the second degree FIR full-band differentiator by cascade connecting two first degree FIR full-band differentiator designed in Example 1, i.e. in the I step of the design procedure (with the following specification parameters values: $N_1 = 32; \tau = 9.5; \omega_0 = 0.5\pi; \alpha_I = 99\alpha_R$. The relation between their impulse response coefficients is given by (16).

Figures in Fig. 3 shows that the obtained second degree full-band digital differentiator has very good magnitude response in the entire pass-band. In addition, obtained pass-band group delay level is approximately constant and lower ($\tau_1 = \tau_2 = 19$) than the theoretical level of the corresponding second degree digital differentiator with linear phase ($\tau_2 = 31$). Figures in Fig 4 shows that the magnitude response remains the same as in the previous case (Fig 3), but the group delay response is different: its level in the entire pass-band has the strictly constant value, equal (with zero error) to the theoretical value (13) :$\tau_2 = N_1 - 1 = 31$. That means that in this case (with complementary impulse response coefficients of the component first degree differentiators in cascade), the second degree digital FIR full-band differentiators with strictly linear phase are obtained. These two examples (3 and 4) only approve and illustrate practically, what the performed researches have been shown: the group delay level in the entire pass-band can be strictly

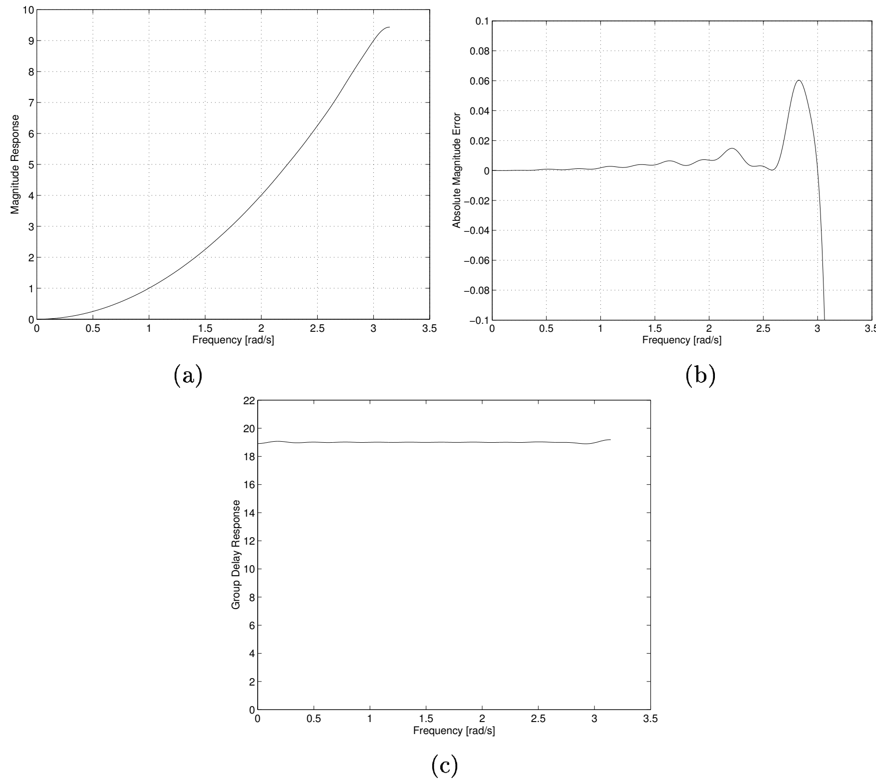

Fig. 3. (a)Magnitude response of the second degree FIR full-band differentiator.(b) Absolute magnitude error of the second degree FIR full-band differentiator.(c) Group delay response of the second degree FIR full-band differentiator

constant and equal to the theoretical value, or it can be approximately constant and distinct (lower or higher) from the theoretical value, depending on the way of choice of the coefficients of the component first-degree differentiators in cascade. On that way, the group delay level of the second degree full-band digital differentiators designed by this method, can be varied in a relatively wide range.

5 Conclusion

In this paper, two methods for designing digital full-band FIR differentiators are presented. First of them, named Iimmediate, represents the efficient and general method for brusque designing the first and higher degree digital full-band FIR differentiators of even and odd order, with simultaneous ap-

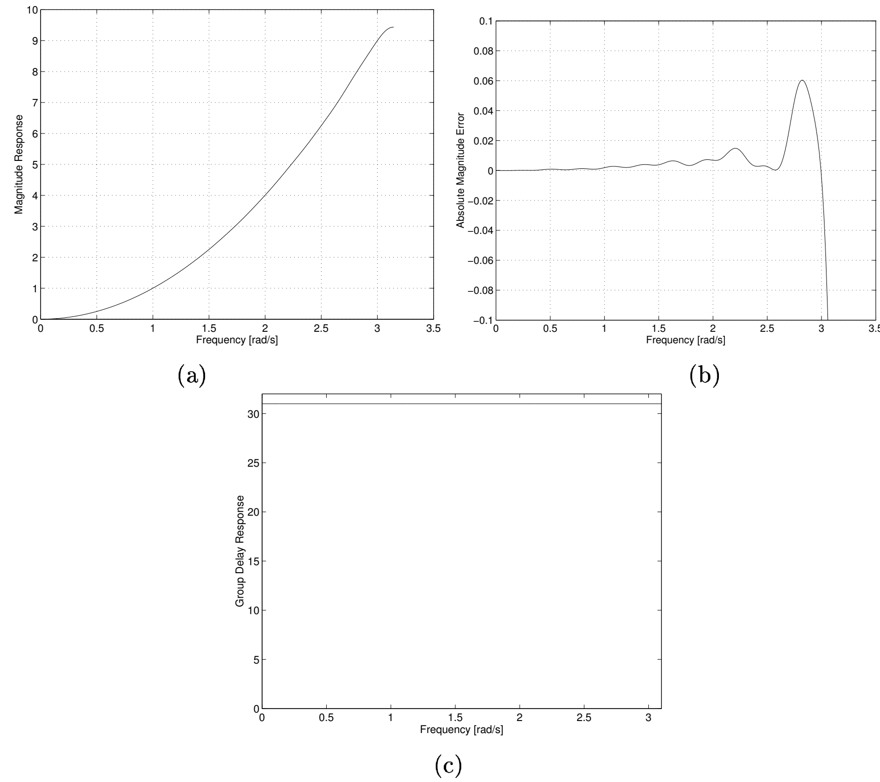

Fig. 4. (a) Magnitude response of the second degree FIR full-band differentiator.(b) Absolute magnitude error of the second degree FIR full-band differentiator.(c) Group delay response of the second degree FIR full-band differentiator

proximation of the prescribed magnitude and group delay responses, using originally modified eigenfilter method.

The proposed method presents an approach for the FIR differentiator frequency response approximation directly in the complex, and not in the real domain. Two weighting coefficients, α_R and α_I, for the approximation of the real and imaginary parts of the frequency response, respectively, are introduced. By the proper analysis of the frequency response real and imaginary parts approximation errors, the values of these parameters giving a small variation of the group delay response, simultaneously with a small magnitude response error in the pass-band, can be obtained. In fact, performed research have shown that, in a general case, better magnitude response characteristic is corresponding to a slightly inferior group delay response characteristic, and vice versa. Due to this, it is obviously that, while choosing these two characteristics, a trade off between their qualities should be madden. FIR full-band differentiators designed by this method posses neither the (anti) symmetric feature of their impulse response coefficients, nor the strictly linear phase. Their pass-band group delay level is approximately constant and distinct from that of the corresponding linear phase full-band differentiators and can be varied in a relatively wide range.

The second method, named Intermediary, represents the efficient approach for oblique designing only the second and higher degree digital full-band FIR differentiators of odd order. The method can be divided in two parts.

The first part includes, in fact, the previously presented (immediate) design method, applied for designing the first degree FIR full-band differentiator.

The second part of the proposed design method includes the realization of the second degree full-band digital differentiator by cascading (i.e. cascade connecting) two first degree full-band digital differentiators, designed in the first part of the method and defined by the proper choice of their impulse response coefficients. If the impulse response coefficients of the component first degree differentiators in the cascade are taken to be mutually equal, then obtained second degree differentiators do not have strictly linear phase; they have the pass-band group delay level which is approximately constant and lower than the theoretical level of the corresponding second degree digital differentiator with linear phase. On the other side, if the impulse response coefficients of the component first degree differentiators in the cascade are taken to be mutually complementary, then the second degree differentiators with strictly linear phase and strictly constant pass-band group delay level are obtained. Both presented methods: immediate and intermediary, represents the methods for the design of the digital FIR differentiators directly in the complex domain. If the quality of the pass-band group delay response characteristic has the advantage over the quality of the pass-band magnitude response characteristic, than the intermediary method should be used for designing the second degree digital FIR differentiators. In the opposite case, when the quality of the pass-band magnitude response characteristic has greater importance than the quality of the pass-band group delay characteristic, the immediate design method should be used for designing the second degree digital FIR differentiators. Of course, the immediate design method has absolute advantage over the intermediary design method when the digital FIR differentiators with degree greater than second has to be designed.

References

[1] B. Kumar and S.C. Dutta Roy, *"Maximally linear FIR digital differentiators for high frequencies"*, IEEE Trans. on Circuit and Systems ,Vol.36, No.6, pp.890-895, 1989.

[2] B.Kumar and S.C.Dutta Roy, *"Design of efficient FIR digital differentiators and Hilbert Transformers for midband Frequency ranges "* , Int.J.Circuit Theory Appl., Vol.17, (4), pp.483-488, 1989.

[3] B.Kumar and S.C.Dutta Roy, *"Design of digital differentiators for low frequencies"* , Proc IEEE, Vol. 76,(3), pp.287-289, 1988.

[4] M.R. Reddy, S.C.Dutta-Roy and B. Kumar, *"Design of efficient second and higher degree FIR digital differentiators for midband frequecies"*, IEEE Proceedings-G, Vol.138, No.1, pp.29-33, 1991.

[5] S. Sunder and V. Ramachandran, *"Design of equiripple nonrecursive digital differentiators and Hilbert transformers using weighted least-squares technique "* , IEEE Trans. on Signal Processing, Vol.42, No.9, pp.2504-2509, 1994.

[6] S. Sunder, W.S. Lu, A. Antoniou and Y. Su, *"Design of digital differentiators satisfying prescribed specifications using optimisation techniques "*, IEE Proceedings - G. Vol. 138 , No.3, pp.315-320, 1991.

[7] S.C. Pei and J.J. Shyu, *"Design of FIR Hilbert transformers and differentiators by eigenfilter "*, IEEE Trans. on Circuits and Systems, Vol.CAS-35, pp.1457-1461, 1989.

[8] S.C. Pei and J.J. Shyu, *"Eigenfilter design of higher order digital differentiators "*, IEEE Trans. on Acoust., Speech and Signal Proc., Vol.ASSP-37, pp.505-511, 1989.

[9] T.Q.Nguyen, *"The design of arbitrary FIR digital filters using the eigenfilter method"*, IEEE Trans. Signal Processing, Vol.41, No.3, pp. 1128-1139, 1993.

[10] A.S.Alkhairy, K.G.Christian and J.S.Lim, *"Design and characterization of optimal FIR filters with arbitrary phase"*, IEEE Trans. Signal Processing, Vol.41, No.2, pp.559-572, 1993.

[11] K.Preuss, *"On the design of FIR filters by complex Chebyshev approximation"*, IEEE Trans. on Acoust., Speech and Signal Proc., Vol.ASSP-37, No.5, pp.702-712, 1989.

[12] S.S.Kidambi and R.P.Ramachandran, *"Design of nonrecursive filters satisfying arbitrary magnitude and phase specifications using a least-squares approach"*, IEEE Trans. on Circuits and Systems, Vol.42, No.11, pp.711-716, 1995.

[13] S.C.Pei and J.J.Shyu, *"Eigen-approach for designing FIR filters and all-pass phase equalizers with prescribed magnitude and phase responses"*, IEEE Trans. on Circuits and Systems, Vol. 39, No.3, pp.137-146, 1992.

[14] L.J.Karam and J.H.McClellan, *"Complex Chebyshev approximation for FIR filter design"*, IEEE Trans. on Circuits and Systems-II: Analog and Digital Signal Proc. Vol.42, No.3, pp. 207-216, March 1995.

[15] X.Chen and T.W. Parks, *"Design of FIR filters in the complex domain"*, IEEE Trans. on Acoustic, Speech and Signal Processing,Vol.ASSP-35, No.2 , pp.144-153, 1987.

[16] F.Wysocka-Schillak and T.Wysocki, *"Design of nonlinear phase FIR digital filter with equiripple error function"*, 4th UK/Australian International Symposium on DSP for Communication Systems, Proceedings, pp. 469-474, 1996.

[17] V.D.Pavlovic and P.N.Lekic, *"Direct synthesis of the second order FIR full-band differentiator"*, IX International Symposium on Theoretical Electrical Engineering ISTET'97, Proceedings, pp. 315-318, Palermo. Italy, 1997.

[18] V.D.Pavlovic and P.N.Lekic, *"Design of the first and higher order digital FIR full-band differentiators"*, 8th International Conference on Signal Processing Applications and Technology ICSPAT'97, San Diego, California, USA, 14.-17. Sept., 1997.

[19] V.D. Pavlovic and P.N. Lekic, *"Design of the second degree digital FIR full-band differentiator"*, 4th Electronic Devices and System Conference (EDS'96), Proceedings, Brno, Czech Republic,pp. 100-103, 1996.