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3D Electric Field Analysis of a Voltage Transformer Using Boundary Integral Equation Method and General-Purpose CAD System

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Abstract: The paper presents an application of indirect Boundary Integral Equation Method to the analysis of three-dimensional electric field of a voltage transformer. The employed mathematical model consists of Fredholm boundary integral equations of the first and second kind. The real geometry of the voltage transformer has been taken into account using parametric description. General-purpose CAD system has been used at pre- and post- processing level. Results have been obtained for the distribution of electric potential and the field intensity. The influence of additional high voltage shields has also been estimated.

Keywords: Electric fields, Fredholm integral equations, boundary integral equation method, voltage transformers, general-purpose CAD system.

1 Introduction

Boundary integral equation methods are widely used for computation of three-dimensional electric fields [1-7]. This is mainly due to their ability to solve problems in unbounded regions and easier manner of building surface

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mesh than volume mesh, as needed when the finite element method is employed for solving 3D problems. In the present paper, an indirect variant of the boundary integral equation method is used for the analysis of the three-dimensional electric field of a voltage transformer.

2 Mathematical Model

The idea of the method is first to introduce surface charges of unknown density over all boundaries - electrodes and dielectric boundaries. Then the surface charge distribution over all the boundaries is obtained after solving a system of integral equations. The dielectrics are removed and all needed results are obtained with the help of integrals over the boundaries in free space.

Thus the mathematical model of the electric field consists of Fredholm integral equations of the first and second kind [1, 3]. They are obtained after defining the surface charge density for both electrodes and dielectric boundaries and forming the system of integral equations for this charge density.

The surface charge density σ for electrodes is defined as

$$\sigma = \varepsilon_0 E_n \tag{1}$$

where E_n is the normal component of the field intensity on the electrode boundary.

The surface charge density in this case directly corresponds to the electric field intensity, which at the boundary of an electrode has only normal component.

For a dielectric boundary the surface charge density is defined as

$$\sigma = 2\varepsilon_0 \lambda E_n \tag{2}$$

where

$$\lambda = \frac{\varepsilon^{(i)} - \varepsilon^{(e)}}{\varepsilon^{(i)} + \varepsilon^{(e)}}$$

$\varepsilon^{(i)}$ and $\varepsilon^{(e)}$ are dielectric permittivities of the inner and outer medium.

The normal vector is directed to the medium of $\varepsilon^{(e)}$.

Consider a general system containing n boundaries, of which m are boundaries of electrodes of known potential and $n - m$ are dielectric boundaries.

Assuming surface charge density distribution as a simple layer potential, the following boundary integral equations are obtained:

- For electrode boundaries - Fredholm integral equation of the first kind

$$\frac{1}{4\pi\varepsilon_0}\sum_{j=1}^{n}\int_{S_j}\sigma(M_j)\frac{1}{r_{Q_kM_j}}\mathrm{d}S_j = U_k, \quad k = 1, \dots, m \tag{3}$$

- For dielectric boundaries - Fredholm integral equation of the second kind

$$\sigma(Q_k) - \frac{\lambda_k}{2\pi}\sum_{j=1}^{n}\int_{S_j}\sigma(M_j)\frac{\cos(\boldsymbol{r}_{Q_kM_j}, \boldsymbol{n}_{Q_k})}{r^2_{Q_kM_j}}\mathrm{d}S_j = 0, \quad k = m+1, \dots, n \tag{4}$$

In (3) and (4) the following notations are used: k is number of the boundary, U_k is potential of the k-th electrode, S_j is boundary j, M_j is influence point, Q_k is observation point, $\boldsymbol{r}_{Q_kM_j}$ is radius vector from point Q_k to point M_j, $\boldsymbol{n}_{Q_k}$ is outward unit normal vector to the boundary S_j and $\lambda_k = (\varepsilon_k^{(i)} - \varepsilon_k^{(e)})/(\varepsilon_k^{(i)} + \varepsilon_k^{(e)})$.

3 Numerical Approach Employed

The system of integral equations (3)-(4) is solved by the method of mechanical quadratures and constant boundary elements.

The discrete analogues of (3) and (4) are obtained dividing all the boundaries into boundary elements and representing the integrals over each boundary as a sum of integrals over its boundary elements:

$$\frac{1}{4\pi\varepsilon_0}\sum_{j=1}^{n}\sum_{s=1}^{N_j}\int_{S_{js}}\sigma(M_{js})\frac{1}{r_{Q_{ki}M_{js}}}\mathrm{d}S_{js} = U_k, \tag{5}$$

$$i = 1, \dots, m_{be}$$

$$\sigma(Q_{ki}) - \frac{\lambda_k}{2\pi}\sum_{j=1}^{n}\sum_{s=1}^{N_j}\int_{S_{js}}\sigma(M_{js})\frac{\cos(\boldsymbol{r}_{Q_{ki}M_{js}}, \boldsymbol{n}_{Q_k})}{r^2_{Q_{ki}M_{js}}}\mathrm{d}S_{js} = 0, \tag{6}$$

$$i = m_{be}+1, \dots, n_{be}$$

where N_j is the number of boundary elements on the boundary j, S_{js} is the s-th boundary element of the j-th boundary, m_{be} is the total number of boundary elements on electrode boundaries and n_{be} is the total number of boundary elements.

Equations (5) and (6) constitute a system of linear equations with respect to the values of the surface charge density at the nodes of the boundary elements. For overcoming the singularities appearing in the integral equations of the first kind - in the linear equation system they cause singularities in the diagonal coefficients of the matrix - analytical expressions for rectangular elements are used. For a rectangular boundary element of sides a and b, the integral used for diagonal coefficients is

$$\int\limits_{S_{ii}} \frac{1}{r_{QM}} \mathrm{d}S_M = 2a \ln \frac{\sqrt{a^2+b^2}+b}{\sqrt{a^2+b^2}-b} + 2b \ln \frac{\sqrt{a^2+b^2}+a}{\sqrt{a^2+b^2}-a} \tag{7}$$

For all other integrals numerical integration using Gaussian quadratures is used.

After solving the system (5)-(6) and obtaining the surface charge density σ, the potential U and the electric field intensity $\boldsymbol{E}$ at an arbitrary point P are obtained by the well-known expressions

$$U(P) = \frac{1}{4\pi\varepsilon_0} \sum_{j=1}^{n} \int\limits_{S_j} \sigma(M_j) \frac{1}{r_{PM_j}} \mathrm{d}S_j \tag{8}$$

$$\boldsymbol{E}(P) = \frac{1}{4\pi\varepsilon_0} \sum_{j=1}^{n} \int\limits_{S_j} \sigma(M_j) \frac{\boldsymbol{r}_{PM_j}}{r_{PM_j}^3} \mathrm{d}S_j \tag{9}$$

4 Approach Employed for Building and Analysing 3D-model

Creation of a proper three-dimensional model for numerical computation is one of the most important tasks of the study and design. The geometry should be described in a way suitable for the numerical technique. The Boundary Integral Equation Method for solving three-dimensional problems requires specific geometry description. In earlier work [8, 9], approach and tools were developed for describing geometry, applying boundary conditions, physical properties and loads in three-dimensional problems solved by the Finite Element Method. These tools were expanded for use with BIEM. The essence of the approach is that pre- and postprocessor levels are carried out in the environment of general-purpose CAD program with the help of additional software for coupling with the processor. For building the model of the studied voltage transformer a mixed approach with solid and surface geometrical objects was employed. The approach is illustrated in Fig. 1.

Solids are transformed automatically to surface objects taking into account the direction of the normal vector.

The system AutoCAD® has been employed. Additional software tools have been developed for visualisation at pre- and post-processor level, making use of ActiveX Automation technology in AutoCAD® environment.

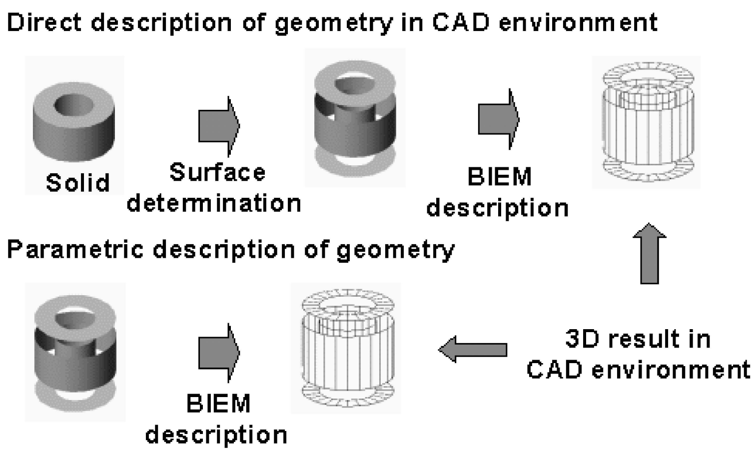


Fig. 1. Approaches employed for building and verification of the 3D-model.

For verification of newly developed tools, procedures for parametric description of geometry of the model and automatic building of it in the AutoCAD® environment have been created. These procedures are the base for mesh generation and creation of appropriate postprocessor. For defining the geometry, parametric description of all boundaries is employed. Each surface is described as a function of two parameters - u and v. After generating the mesh, each boundary element is defined by lower and upper values of u and v. Thus, each boundary element is a part of a surface, defined by four points. The following basic types of elements are implemented so far:

- rectangle (parameters u and v are the co-ordinates along the two sides);
- part of a cylinder (u is the angle, v is the height co-ordinate);
- part of a disc (u is the radius, v is angle);
- part of a torus (u and v are angles).

The system is open and any other surface type described parametrically can be added. Regular and cosine distribution of the boundary elements can be utilised.

The quality of generated mesh affects directly the quality of the post-processor results. For better presentation of the results, additional mesh refinement is carried out at postprocessor level, followed by approximation of the surface charge density over the refined mesh. This improves the visualisation of the results, especially of the colour maps. An example of the mesh refinement in azimuthal direction is shown in Fig. 2.

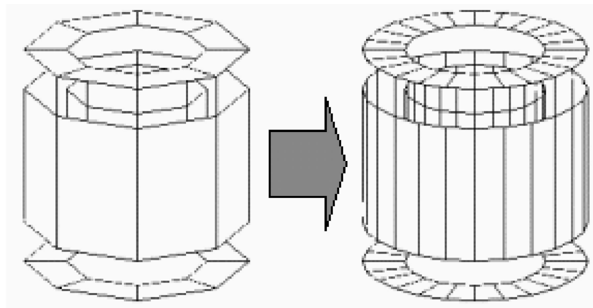


Fig. 2. Improvement of postprocessor results based on parametrical output.

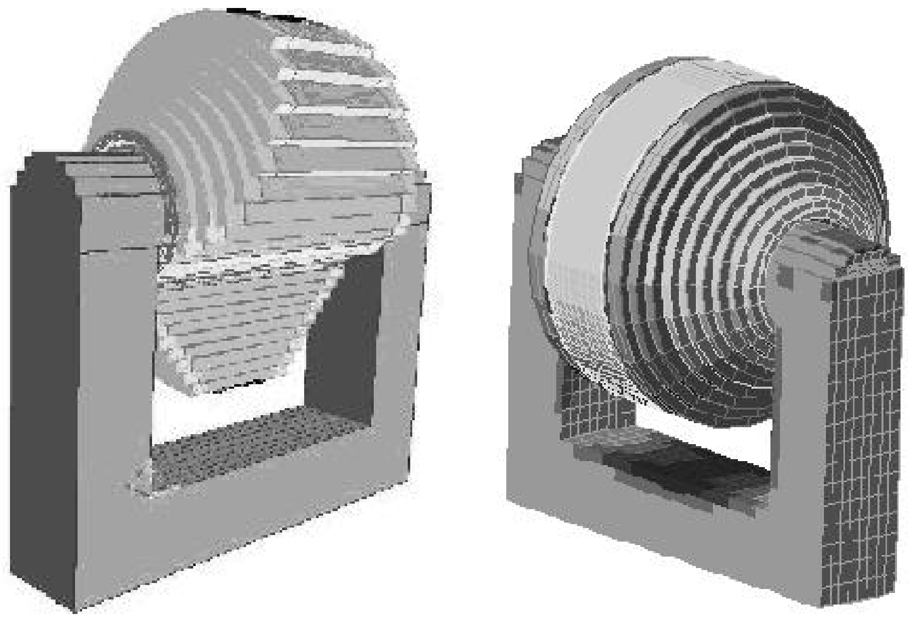
Employment of general-purpose CAD system for solving problems of postprocessor enables receiving a lot of different viewpoints in three-dimensional space, real time visualisation and manipulation. It is possible to obtain images representing geometry of the model and the mesh, contours, equipotentials and areas in two- and three-dimensional space. For the post-processor level, a tool for visualisation of the surface charge density has also been developed.

Another advantage of using general purpose CAD system is the fact that many of the design engineers use this system in their everyday work. Having additional tools at their disposal, it becomes easy for them to deal with field analysis as a real opportunity.

5 Electric Field of a Voltage Transformer

A 126 kV voltage transformer has been modelled and its electric field has been analysed with the help of the approach presented. An idea about the geometry of the voltage transformer can be obtained from Fig. 3(a) where part of the high voltage winding has been removed. The orientation of the co-ordinate system is also shown.

The cross section of the high voltage winding is stepwise, covered by solid insulation. The geometry of the transformer is such that it cannot be modelled as two-dimensional and that is why three-dimensional analysis is needed. For building the geometric model a total of about 130 macroele-



(a) (b)

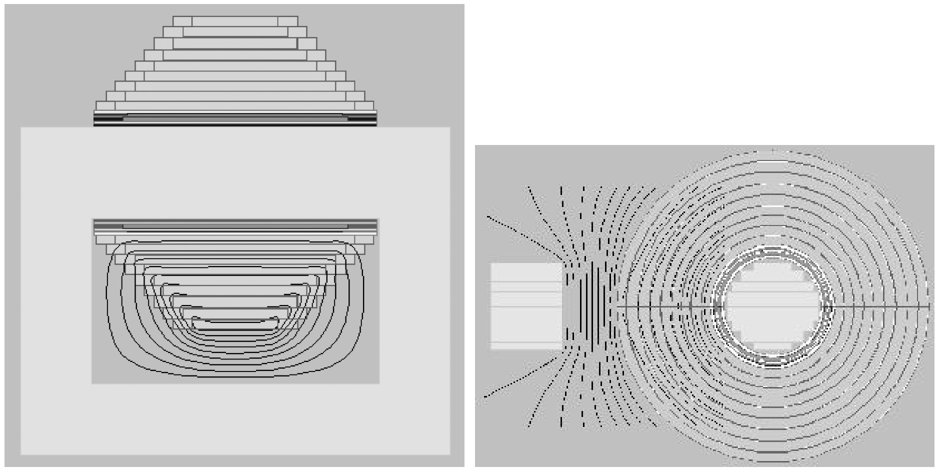
Fig. 3. (a) Geometry of the voltage transformer (VT). (b) Surface charge density for the basic VT.

ments are used, resulting in about 4500 boundary elements after the mesh generation. Cosine discretization was used in x-direction of the winding elements.

Fig. 3(b) shows the colour map of the obtained charge density on the surfaces of the boundaries. The approach with mesh refinement in azimuthal direction and approximation of the surface charge density has been employed. On electrode boundaries, i.e. on the windings and the core, the charge density is proportional to the field intensity. The most electrically loaded area is on the end part of the high voltage winding on its lower part (i.e. close to the yoke).

In Fig. 4(a) and Fig. 4(b), maps of equipotential lines are shown for the XZ and YZ planes. Both XZ and YZ planes are chosen to coincide the planes of symmetry of the transformer. The difference between potential values of two adjacent equipotential lines is 10 % of the applied voltage.

Similar results are obtained for the same transformer but with two additional shields of the shape of torus.



(a) (b)

Fig. 4. (a) Map of equipotential lines at XZ plane. (b) Map of equipotential lines at YZ plane.

In Fig. 5, colour map of the surface charge density is shown. The presence of shields leads to decreasing about two times the maximal value of the surface charge density (corresponding to the field intensity), which is now moved from the outer HV layer to the shield.

The map of equipotential lines is given in Fig. 6(a) for the XZ plane and in Fig. 6(b) for the YZ plane.

It can be seen that, while the maps in XZ planes are different due to the shields, these in YZ plane are almost the same. The maximal field intensity is obtained in the region of the shields.

6 Conclusion

An approach for 3D electric field analysis has been presented for a system containing electrodes of known potentials and dielectric boundaries. The approach employs BIEM as a mathematical basis, parametric description of the boundaries and general purpose CAD system for visualisation at pre- and post-processor level.

Using the approach presented, the 3D electric field of a 126 kV voltage transformer has been analysed. Two constructions are considered - without and with additional torus shields. The construction with additional shields

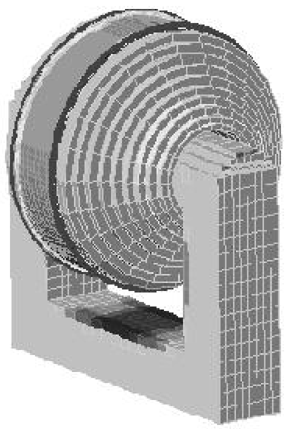
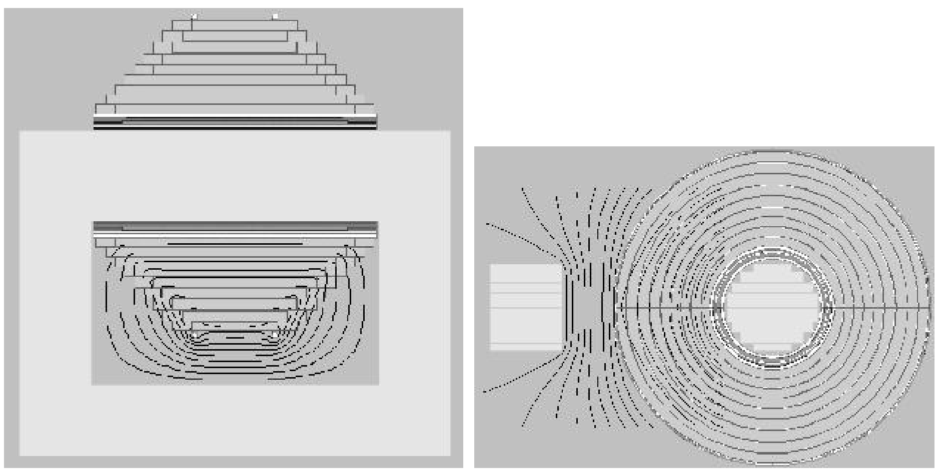


Fig. 5. Surface charge density for the VT with shields.



(a) (b)

Fig. 6. (a) Map of equipotential lines at XZ plane (shield case). (b) Map of equipotential lines at YZ plane (shield case).

features more than two times lower value of the maximal field intensity and it can be subject of further improvement by varying the position and size of the shields.

The approach can be of help to design engineers making easier for them to deal with field analysis in their work.

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