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Electromagnetic Field of Current Conductor Above Semi-Conducting Half-Space

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Abstract: Using Charge Simulation Method the electromagnetic field distribution of the line current conductor above semi-conducting half-space is determined.

Keywords: Electromagnetic field, line current conductor, semi-conducting earth, charge simulation method.

1 Introduction

Very often, especially in practice, there is necessity for calculation electromagnetic (EM) field in surround of a power conductor, which represents thin, current conductor, located in the air, above the semi-conducting earth.

An analitical approach for solving the problem of the current conductor above semi-conducting half-space is supposed in [1]. It is based on some transformations, which substitute Somerfeld's integral, the solution obtained by the integral transformation method, with Henkel's function and their asymptotic expansion. Henkel's functions represent linear solution combinations of the Bessel's functions of the first and of the second kind [2]. Somerfeld's solution is in the form of non-elementary integral, which is very difficult to be calculated, even numerically. The main difficulties of the numerical calculation of this integral are result of the character of the sub integral

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function. All methods for calculations have some limitations regarding sub integral parameters or calculation time.

The obtained solutions in [1] are compared with known results [3] and excellent agreement is obtained. New numerical procedure for calculation of EM field of the current conductor above semi-conducting half-space is proposed in [4,6]. Charge Simulation Method (CSM)[5] is suggested for numerical determination of this problem and accuracy of this method is estimated. The obtained results show that boundary conditions are excellent satisfied with small number of fictitious sources and that CSM ensures fast convergence of results.

In this paper, a lot of the results obtained by using CSM applied for calculation of the EM field of power conductor under semi-conducting earth are shown. This problem is very interesting, because it has direct application in the practice, for calculation of the EM field of the power line conductors under semi-conducting earth [6].

2 CSM Application

Thin, large conductor, with uniform current amplitude, I, and angular frequency, ω, is located in the air parallel above homogeneous, isotropic and linear semi-conducting half-space, with electric and magnetic permitivity $\varepsilon = \varepsilon_0 \varepsilon_r$ and $\mu = \mu_0 \mu_r$ and conductivity σ (Fig. 1). It is placed in the cilindrical coordinate system r, θ, z, along z-axis, on the direction $r = h$, $\theta = \pi/2$.

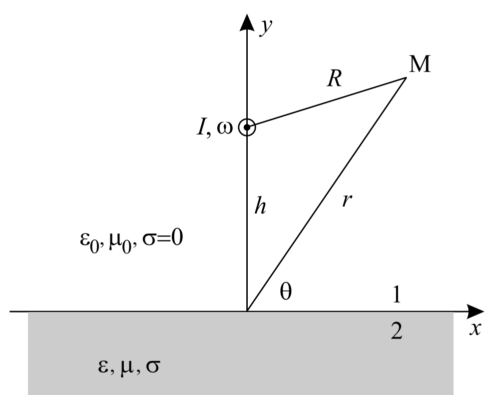

Fig. 1. Linear confuctor above semiconducting half-space.

CSM is suggested for numerical determination of this problem, because of the nature of the problem, geometrical characteristics of the system and the influence of the finite earth conductivity. CSM is based on the theorem of equivalence of different EM systems. Two independent, equivalent systems are suggested here (Fig. 2). The first one is applied for EM field components determination in the air and the second one for EM field components determination in the semi-conducting half-space. First equivalent system consist of primary, linear, very large, current conductor, with uniform amplitude current I and N fictitious parallel conductors, also linear, very large, with uniform amplitude currents I_{1n}, $n = 1, 2, \ldots, N$. Second equivalent system consists of M fictitious, linear, very large, current conductors, with uniform amplitude currents I_{2n}, $n = 1, 2, \ldots, M$. The intensities of the unknown current's amplitudes are determined by using points matching method, satisfying boundary conditions in a finite number of matching points located on the bound between two spaces.

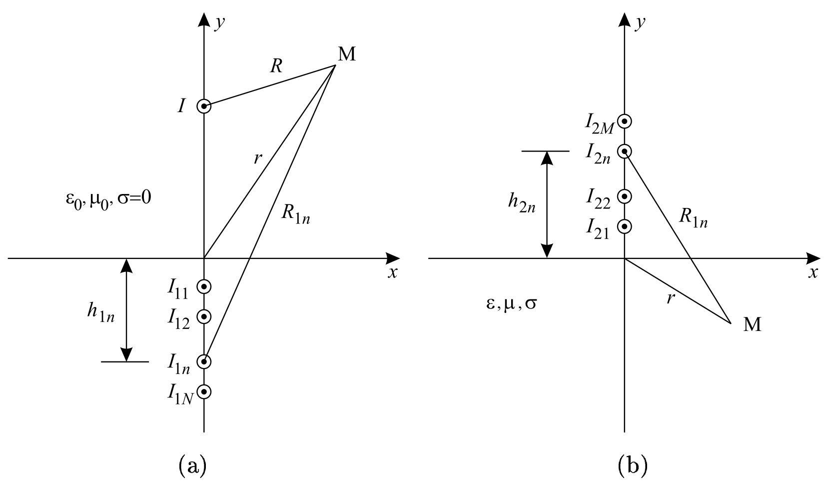

Fig. 2. Equivalent systems for EM field calculation. (a) In the air. (b) In the semi cunducting halgspace.

Using procedure given in [6] EM field components in semi-conducting half-space are determined with

$$E_{2z} = -\frac{\omega\mu}{4} \sum_{n=1}^{M} I_{2n} H_0^{(2)}(kR_{2n}), \tag{1}$$

$$H_{2r} = -j\frac{k}{4} \sum_{n=1}^{M} \frac{h_{2n} \cos\theta}{R_{2n}} I_{2n} H_1^{(2)}(kR_{2n}) \tag{2}$$

and

$$H_{2\theta} = -j\frac{k}{4}\sum_{n=1}^{M}\frac{r - h_{2n}\sin\theta}{R_{2n}}I_{2n}H_1^{(2)}(kR_{2n}), \tag{3}$$

and, in the air,

$$E_{1z} = -\frac{\omega\mu_0 I}{4}H_0^{(2)}(k_0R) - \frac{\omega\mu_0}{4}\sum_{n=1}^{N}I_{1n}H_0^{(2)}(k_0R_{1n}), \tag{4}$$

$$\begin{aligned} H_{1r} = &- j\frac{k_0 I}{4}\frac{h\cos\theta}{R}H_1^{(2)}(k_0R) \\ &- j\frac{k_0}{4}\sum_{n=1}^{N}\frac{h_{1n}\cos\theta}{R_{1n}}I_{1n}H_1^{(2)}(k_0R_{1n}) \end{aligned} \tag{5}$$

and

$$\begin{aligned} H_{1\theta} = &- j\frac{k_0 I}{4}\frac{r - h\sin\theta}{R}H_1^{(2)}(k_0R) \\ &- j\frac{k_0}{4}\sum_{n=1}^{N}\frac{r - h_{1n}\sin\theta}{R_{1n}}I_{1n}H_1^{(2)}(k_0R_{1n}), \end{aligned} \tag{6}$$

where

$$k_0 = \omega\sqrt{\varepsilon_0\mu_0}\ , \tag{7}$$

$$\begin{aligned} k = &\omega\sqrt{\varepsilon\mu}\sqrt{\frac{\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1}{2}} \\ &- j\omega\sqrt{\varepsilon\mu}\sqrt{\frac{\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1}{2}}, \end{aligned} \tag{8}$$

$$R = \sqrt{r^2 + h^2 - 2rh\sin\theta}, \tag{9}$$

$$R_{1n} = \sqrt{r^2 + h_{1n}^2 - 2rh_{1n}\sin\theta} \tag{10}$$

and

$$R_{2n} = \sqrt{r^2 + h_{2n}^2 - 2rh_{2n}\sin\theta}. \tag{11}$$

The intensities of the unknown currents are determined by using points matching method and satisfying boundary conditions in a finite number of the matching points located on the bound betwen two spaces. Here are satisfied boundary condition for tangential components of the magnetic field and normal component of the magnetic flux densities, so:

$$1.\ H_{1r} = H_{2r};$$

and

$$2.\ \mu_0 H_{1\theta} = \mu H_{2\theta}. \tag{12}$$

Total number of the unknown currents is $L = N + M$. Boundary conditions are satisfied in L points on the boundary surface, in N points first one, and in M second boundary condition. Matching points are placed on boundary surface between spaces, with respect of the existing symmetry.

On Fig. 3 circles define points that satisfy first boundary condition and cross denote points that satisfy second boundary condition. The position of the matching points is not strictly determined, so two of more possible choices are given here.

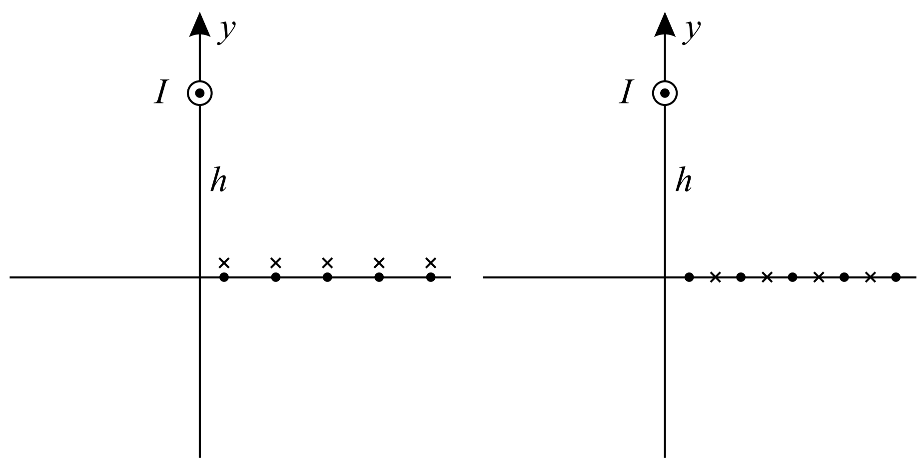

Fig. 3. Different ways for choosing position of the matching points.

3 Numerical results

Using this procedure the EM field components can be calculated, in any point, which belongs to one, or other space, for different parameters of semi-conducting half-space.

It is supposed that conductor is infinite and thin, with current amplitude $I = 1$ A, $\Psi = 0$, located on $h = 1$ m high from the semi-conducting half-space.

In this paper the case when semi-conducting half-space is substituted with semi-conducting earth is considered. Investigations showed that most types of the real earth have $3 \leq \varepsilon_r \leq 11$ and $\mu_r = 1$. For frequencies of few MHz, the conductivity σ is in the range of $0.001\text{S/m} \leq \sigma \leq 0.1\text{S/m}$.

Here is considered an example when semi-conducting half-space is substituted with real earth with characteristics $\sigma = 10^{-3}\text{S/m}$, $\varepsilon_r = 6$, $\mu_r = 1$ and $f = 3\text{MHz}$ The optimal number of used fictitious sources is $N = 6$, $M = 6$ and $L = 12$, for this example. Then the obtained values for intensities of currents of fictitious sources in the air and in the earth are presented in Table 1.

Table 1. Calculated currents intensities of the fictitious sources, for $N = M = 6$.

	In the air	In the earth
I_1	$0.00253-j0.00949$	$0.00314-j0.01341$
I_2	$-0.02089+j0.06972$	$-0.03230+j0.14224$
I_3	$0.09119-j0.25641$	$0.17990-j0.79776$
I_4	$-0.20367+j0.43337$	$-0.53825+j2.32128$
I_5	$0.21091-j0.24911$	$0.76146-j3.25332$
I_6	$-0.07512-j0.02609$	$0.57586+j1.71885$

Table 2 shows the values of the magnetic field components in the air (space 1), when $r = \text{const}$ and $0^0 \leq \theta \leq 90^0$.

Table 2. EM field components in the air, in the point $M(r, \theta)$, when $0^0 \leq \theta \leq 90^0$ and r=0.5 m.

$\theta[^0]$	H_{r1} [A/m]	$H_{\theta 1}$ [A/m]
0	1.267E-01+j3.564E-03	1.267E-01+j3.564E-03
15	1.545E-01+j3.024E-03	1.545E-01+j3.024E-03
30	1.833E-01+j2.427E-03	1.833E-01+j2.427E-03
45	2.070E-01+j1.816E-03	2.070E-01+j1.816E-03
60	2.070E-01+j1.205E-03	2.070E-01+j1.205E-03
75	1.449E-01+j6.010E-04	1.449E-01+j6.0104E-04
90	3.895E-17+j1.404E-19	3.895E-17+j1.4041E-19

Table 3 shows the distribution of the magnetic field components in the earth (space 2), when $r = \text{const}$ and $0^0 \leq \theta \leq -90^0$.

Magnetic field components in the air (space 1), when r is changeable, $0.25\text{m} \leq r \leq 3\text{m}$ and $\theta = const$ are given in Table 4. Analogue results in the earth (space 2) are given in Table 5.

The distribution of the intensity of the magnetic field versus r, when θ takes values 20°, 45° and 60°, is presented in Fig. 4(a).

Table 3. Magnetic field components in the earth, in the point $M(r,\theta)$, $0^0 \leq \theta \leq -90^0$, $r = 0.5$m.

θ [^{0}]	H_{r2} [A/m]	$H_{\theta 2}$ [A/m]
0	1.267E-01+j3.564E-03	6.394E-02−j1.156E-03
15	1.015E-01+j2.680E-03	3.911E-02−j1.869E-03
30	7.858E-02+j1.865E-03	4.572E-04−j2.384E-03
45	5.743E-02+j1.198E-03	−6.021E-02−j2.741E-03
60	3.759E-02+j6.911E-04	−1.511E-01−j2.973E-03
75	1.859E-02+j3.109E-04	−2.604E-01−j3.103E-03
90	4.333E-18+j6.994E-20	−3.177E-01−j3.1454E-03

Table 4. Magnetic field components in the air, in the point $M(r,\theta)$, $\theta = 45°$, $0.25\text{m} \leq r \leq 3\text{m}$.

r [m]	H_{r1} [A/m]	$H_{\theta 1}$ [A/m]
0.25	1.58E-01+j2.21E-03	−1.02E-01−j3.02E-03
0.50	2.07E-01+j1.81E-03	−6.02E-02−j2.74E-03
0.75	2.24E-01+j1.49E-03	1.40E-02−j2.55E-03
1.0	1.91E-01+j1.23E-03	7.99E-02−j2.39E-03
1.5	9.96E-02+j8.58E-04	1.12E-01−j2.08E-03
2.0	5.17E-02+j6.21E-04	9.50E-02−j1.82E-03
2.5	3.02E-02+j4.66E-04	7.70E-01−j1.61E-03
3.0	1.95E-02+j3.61E-04	6.35E-02−j1.43E-03

The distribution of the intensity of the magnetic field versus θ, when r takes values 0.2 m, 0.5 m and 0.7 m, is presented in Fig. 4(b).

As it can be inspected the magnetic field intensity is higher in the points which are closer to the conductor and it decreases when the distance from the conductor increases.

Table 5. Magnetic field components in the air, in the point $M(r,\theta)$, $\theta = -45°$, $0.25\text{m} \leq r \leq 3\text{m}$.

r [m]	H_{r2} [A/m]	$H_{\theta 2}$ [A/m]
0.25	7.92E-02+j1.85E-03	1.07E-01+j9.12E-04
0.50	5.74E-02+j1.19E-03	9.84E-02−j1.75E-04
0.75	4.28E-02+j1.19E-05	8.84E-02−j2.71E-05
1.0	3.30E-02+j1.73E-04	8.02E-02−j2.51E-03
1.5	2.11E-02−j5.11E-04	6.60E-02−j4.69E-03
2.0	1.45E-02−j9.40E-04	5.555E-02−j6.66E-03
2.5	1.04E-02−j1.19E-03	4.749E-02−j8.26E-03
3.0	7.89E-03−j1.34E-03	4.10E-02−j9.65E-03

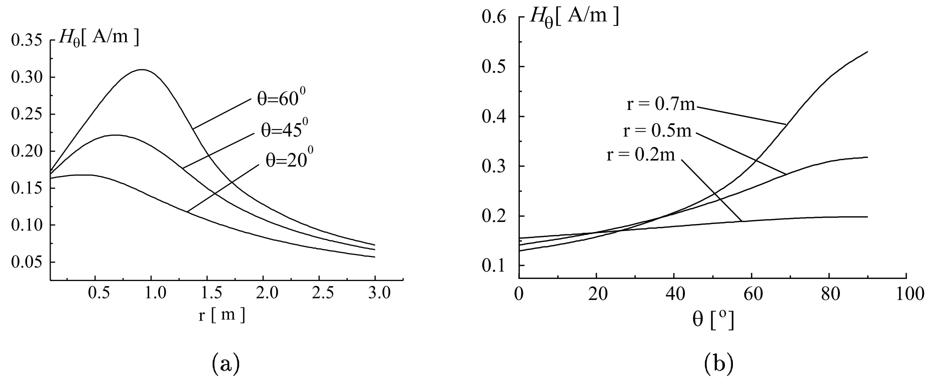

Fig. 4. Intensity of the magnetic field in the air: (a) versus r and (b) versus θ.

4 Conclusion

New numerical procedure for calculation of the EM field of the current conductor above semi-conducting half-space is proposed in this paper. The solution for EM field components contents Henkel's functions, including their asymptotic expansions.

CSM is suggested for numerical determination of this problem, because of the nature of the problem, geometrical characteristics of the system and the influence of the finite earth conductivity. It can be concluded that CSM ensures excellent accuracy and convergence with small number of fictitious sources. This procedure can be very useful for EM field calculation of power conductor above semi-conducting earth.

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