FACTA UNIVERSITATIS (NIŠ)
SER.: ELEC. ENERG. vol. 15, Aug. 2002, 195-204

Some Thermodynamic Aspects of Viscoplasticity of Ferromagnetics

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Abstract: The paper deals with viscoplasticity of ferromagnetic materials. Tensor representation is applied to a set of evolution equations comprising the plastic stretching and residual magnetization tensors. Small magnetoelastic strains of isotropic insulators are considered in detail in two special cases of finite as well as small plastic strain. A special emphasis is given to piezomagnetism effects in the case of uniaxial cycling strain. Vakulenko's irreversible thermodynamics is applied to irreversible magnetic phenomena by means of a hereditary evolution function. Purely mechanical as well as purely magnetic irreversible phenomena are considered in detail.

Keywords: Ferromagnetics, endochronic thermodynamics, magnetic phenomena, piezomagnetism.

1 Introduction

The principal issue in this work is how to establish relationship between irreversible magnetisation and inelastic strains serving primarily to subsequent nondestructive electromagnetic examination of inelastic behavior of nuclear reactor steels (cf. [1]).

In this paper like in [2, 3, 4] evolution equations are based on the appropriate geometry of deformation and the endochronic irreversible thermodynamics. Consider a crystalline body in a real configuration (k) with dislocations and an inhomogeneous temperature field $T(X, t)$ (where t stands

Manuscript received February 3, 2002. A version of this paper was presented at the Fifth International Conference on Applied Electromagnetics, ΠΕC 2001, October 8 -10, 2001, Niš, Serbia.

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for time and X for the considered particle of the body) subject to surface tractions. Corresponding to (k) there exists, usually, an initial reference configuration (K) with (differently distributed) dislocations at a homogeneous temperature T_0 without surface tractions. It is generally accepted that linear mapping function $\mathbf{F}(.,t) : (K) \to (k)$ is compatible second rank *total deformation gradient tensor.* In the papers dealing with continuum representations of dislocation distributions configuration (k) is imagined to be cut into small elements denoted by (n), these being subsequently brought to the temperature of (K) free of neighbors. The deformation tensor $\mathbf{F}_E(.,t) : (n) \to (k)$ obtained in such a way is incompatible and should be called the *thermoelastic distortion tensor* whereas (n)-elements are commonly named as *natural state local reference configurations* (cf. for instance [5]). The corresponding *plastic distortion tensor*

$$\mathbf{F}_P\,(.,t) := \mathbf{F}_E\,(.,t)^{-1} \cdot \mathbf{F}\,(.,t), \tag{1}$$

is also incompatible. Herein $\mathbf{F}$ is found by comparison of material fibres in (K) and (k) while $\mathbf{F}_E$ is determined by crystallographic vectors in (n) and (k).

In the paper [6] the authors connected magnetization vectors to the natural state elements in such a way that they are isoclinic in (n) and inhomogeneous in (k) the inhomogeneity being responsible for magnetostrictive strains. Such an assumption is very much in accord with the above geometrical argument and is accepted in the sequel.

2 Isotropic Viscoplastically Deformed Insulators

Let us consider an isotropic body under the following very simplifying assumptions:

(A1) elastic strain, reversible and irreversible magnetization are small of the same order but plastic strain itself is finite (cf. also[2]); and

(A2) thermal and electric effects are neglected.

Such assumptions correspond to the so called piezomagnetism processes when magnetization is generated by straining processes (cf. eg. [8] and [9]).

It is taken that by its very nature the mechanical stress disappears when pure elastic strain vanishes and, similarly, the local magnetic field equals to zero if the reversible magnetization vanishes. Then, according to [6] it is reasonable to introduce magnetostrictive strain by means of following two relationships

$$\mathbf{E}_E := \mathbf{E} - \mathbf{E}_P = \mathbf{F}_P^T \cdot (\mathbf{F}_E^T \cdot \mathbf{F}_E - \mathbf{1}) \cdot \mathbf{F}_P, \tag{2}$$

$$\mathbf{E}_E \equiv \mathbf{E}_e + \mathcal{L} : (\overrightarrow{\mathcal{M}_0} \otimes \overrightarrow{\mathcal{M}_0}) \equiv \mathbf{E}_e + \mathbf{E}_{mag}. \tag{3}$$

Here $\mathcal{L}$ is the fourth rank tensor of magnetostriction constants symmetric only in indices of the first as well as the second pair whereas the notation $\overrightarrow{\mathcal{M}_0}$ stands for the unit vector of the magnetization vector $\overrightarrow{\mathcal{M}}$. The constituents of the *Lagrangian elastic strain tensor* $\mathbf{E}_E$, namely, $\mathbf{E}_e$ as well as $\mathbf{E}_{mag}$ are both incompatible and are referred to as *pure elastic strain* and *magnetostrictive strain.*

With these facts taken into account and the assumptions *(A1-A2)* the constitutive equations for mechanical part of the stress tensor and the local magnetic field become [3]

$$\begin{aligned}\mathbf{T} =& (c_1 \mathbf{1} + c_2 \mathbf{E}_P + c_3 \mathbf{E}_P^2)\ tr\mathbf{E}_e + 2c_4\ \mathbf{E}_e \\ &+ c_5\ (\mathbf{E}_P \cdot \mathbf{E}_e + \mathbf{E}_e \cdot \mathbf{E}_P) \\ &+ c_6\ (\mathbf{E}_P^2 \cdot \mathbf{E}_e + \mathbf{E}_e \cdot \mathbf{E}_P^2),\end{aligned} \tag{4}$$

$$\begin{aligned}\mathbf{H} =& c_7 \mathbf{M}_r + c_8 (\mathbf{E}_P \cdot \mathbf{M}_r + \mathbf{M}_r \cdot \mathbf{E}_P) \\ &+ c_9 (\mathbf{E}_P^2 \cdot \mathbf{M}_r + \mathbf{M}_r \cdot \mathbf{E}_P^2).\end{aligned} \tag{5}$$

The antisymmetric second rank tensors $\mathbf{H}, \mathbf{M}_r$ and $\mathbf{M}_R$, made from the corresponding axial vectors $\vec{H}$, $\vec{\mathcal{M}}_r$ and $\vec{\mathcal{M}}_R$ are favored for convenience and more compact representation. Instead of magnetic induction field the internal magnetic field tensor (opposing the local magnetic field tensor under assumption *(A1)*) has been introduced. In the above "magnetic" constitutive equation

$$\mathbf{M}_r := \mathbf{M} - \mathbf{M}_R, \tag{6}$$

is the reversible magnetization tensor[1]. Equation (4) is the plastic strain dependent generalized Hooke's law wheras the constitutive equation for internal magnetic field predicts magnetic anisotropy induced by plastic strain. The free energy function generating linear forms of (4) and (5) reads

$$\begin{aligned}F \;=\;& \frac{1}{2}c_1\ i_1^2 + \frac{1}{2}c_2\ i_2^2 + \frac{1}{2}c_3\ i_3^2 + c_4\ i_4 + c_5\ i_5 \\ &+ c_6\ i_6 + \frac{1}{2}c_7\ i_7 + \frac{1}{2}c_8\ i_8 + \frac{1}{2}c_9\ i_9,\end{aligned} \tag{7}$$

[1] Instead of (5) an equivalent formulation using cross products of vectors $\vec{\mathcal{M}}_r$ and $\vec{\mathcal{M}}_R$ with tensor $\mathbf{E}_P$ is also possible.

where $i_1, ..., i_9$ are proper and mixed invariants of $\mathbf{E}_e, \mathbf{M}_r$ and $\mathbf{E}_P$ and their mutual products.

Similarly, the evolution equations for plastic strain rate and residual magnetization rate are explicitly stated by the following formulae

$$D\mathbf{E}_P = \sum_{k=1}^{12} d_k \mathbf{G}_k^{E_P}, \tag{8}$$

$$D\mathbf{M}_R = \sum_{k=1}^{9} e_k \mathbf{G}_k^{M_R}. \tag{9}$$

where tensor generators $\mathbf{G}_k^{E_P}$ and $\mathbf{G}_k^{M_R}$ are similar to those in (4 - 5). The scalar coefficients $\{d_1, ..., d_{12}\}$ as well as $\{e_1, ..., e_9\}$ depend on proper and mixed invariants of the tensors $\mathbf{H}, \mathbf{T}$ and $\mathbf{E}_P$ according to assumptions *(A1)* and *(A2)*. They are not written here for the sake of brevity and are listed explicitly in [3] on the basis of representation theory for tensor functions.

It should be noted here that all the scalar coefficients in above constitutive relations (4)-(5) are functions of the principal invariants of the plastic strain tensor $\mathbf{E}_P$. For advanced magnetizations, a nonlinearity of the magnetic relationship (5) must be taken into account while Hooke's law (4) is always linear in the elastic strain for steels. If plastic strain itself is small, then evolution equations would reduce to Onsager-Casimir reciprocity relations.

3 Small Magneto-Viscoplastic Strains

Let us see what consequences could have an introduction of a generalized loading function Ω with the following orthogonality properties

$$D\mathbf{E}_P = D\Lambda \frac{\partial \Omega}{\partial \mathbf{H}} \quad \text{and} \quad D\mathbf{M}_R = D\Lambda \frac{\partial \Omega}{\partial \mathbf{H}}. \tag{10}$$

where the material time rate of a scalar function Λ vanishes if the yield function f (enclosing the corresponding elastic range) is either negative or zero (cf. eg. [2]). Suppose, for simplicity that the assumption *(A2)* still holds whereas the assumption *(A1)* is replaced by means of the following:

(A3) Elastic and plastic strain, reversible and irreversible magnetization, as well as plastic strain rate and irreversible magnetization rate are all small of the same order.

Then we may assume the loading function in the following polynomial form

$$\Omega = \frac{1}{2}\omega_1\ s_1^2 + \frac{1}{2}\omega_2\ s_4 + \frac{1}{2}\omega_3\ s_7, \tag{11}$$

where $s_1 \equiv tr\mathbf{T}, s_4 \equiv tr\left\{\mathbf{T}^2\right\}, s_7 \equiv tr\left\{\mathbf{H}^2\right\}$. This leads by means of (10) into the following two evolution equations

$$D\mathbf{E}_P = D\Lambda\ [\ \omega_1 \mathbf{1}\ tr\mathbf{T} + \omega_2\ \mathbf{T}\ ], \tag{12}$$

$$D\mathbf{M}_R = D\Lambda\ \omega_3\ \mathbf{H}, \tag{13}$$

whose simplicity follows from the above very special loading scalar function Ω. In addition, the free energy function F becomes very simplified (by means of notations $i_1 \equiv tr\{\mathbf{E}_e\}$, $i_4 \equiv tr\{\mathbf{E}_e^2\}$, $s_7 \equiv tr\{\mathbf{M}_r^2\}$) as follows

$$F = \frac{1}{2}c_1\ i_1^2 + c_4\ i_4 + \frac{1}{2}c_7\ i_7 + F^*(\mathbf{E}_P, \mathbf{M}_R) \tag{14}$$

where F^* would depend on proper and mixed invariants of $\mathbf{E}_P$ and $\mathbf{M}_R$. Such a function allows the following very special constitutive equations

$$\mathbf{T} = c_1 \mathbf{1}\ tr\mathbf{E}_e + 2c_4\ \mathbf{E}_e, \text{ with } c_1 \equiv \lambda; \quad c_2 \equiv \mu, \tag{15}$$

$$\mathbf{H} = c_7 \mathbf{M}_r, \text{ with } \chi \equiv \frac{1}{c_7}. \tag{16}$$

Obviously, the inherent material constants are easily recognized to be *Lame constants* as well as the constant of *magnetic susceptibility* (cf. [7]). It should be noted that if the tensor of magnetostriction constants $\mathcal{L}$ is introduced into (15) then magnetostriction process can be shown explicitly.

The situation described in this section could correspond to piezomagnetism induced by low-cycle fatigue of ferromagnetics. Such a process was investigated experimentally in the paper [8]. A cylindrical specimen of AISI 1018 was uniaxially treated by push-pull tests on MTS-810 servo-hydraulic testing machine such that total strain was periodic and triangularly shaped $\|\mathbf{E}\| \in \{0,\ 0.009\}$ with cycle duration of 2 s. Magnetic induction due to piezomagnetism effect was also almost periodic with very slight changes with increase of relative number of cycles N/N_f (where N_f is number of cycles at failure) and cumulation of phase delay with respect to strain with growth of accumulated plastic strain. Maxima and minima of $\mathbf{E}$ are almost coincident with minima and maxima of the magnetic induction $\mathbf{B}$. Thus, if plastic strain accumulation is calculated by means of

$$\pi(t) := \int_0^t \|D\mathbf{E}_P(\tau)\|\ d\tau \tag{17}$$

then if uniaxial components of $\mathbf{E}$ as well as $\mathbf{B}, \mathbf{M}_r, \mathbf{M}_R$ are denoted by means of E_{11} as well as B_{11},M_{r11}, M_{R11} the following memory-type equation

$$B_{11}(t) := \int_0^t J(\pi, t-\tau)\ DE_{11}(\tau)\ d\tau, \tag{18}$$

would describe fairly well the above explained experimental situation. Time differentiation of the above relationship gives rise to the expression:

$$DB_{11}(t) := J(\pi, 0)\ DE_{11}(t) + \int_0^t \frac{\partial}{\partial t} J(\pi, t-\tau)\ DE_{11}(\tau)\ d\tau. \tag{19}$$

In the above integro-differential equation the second term on the right hand side is responsible for the above mentioned change of time delay and the deflection of pure periodicity of $B_{11}(t)$. Therefore, it is much smaller than the first part. On the other hand, if the constitutive equation $B_{11} = \mu\ H_{11}$ (where μ is *magnetic permeability*) is used, then we have

$$DB_{11} = \frac{\mu}{\chi}(DM_{11} - DM_{R11}) \tag{20}$$

if $\mu/\chi = const$ which holds approximately only for magnitude of magnetic field much smaller than its saturation value. Since in the paper [8] the above splitting has not been made, a more specific comment on simultaneous zeros of $D\mathbf{E}_P$ and $D\mathbf{M}_R$ (following from (12) and (13)) is not possible.

4 Endochronic Thermodynamics

4.1 Purely Mechanical Irreversible Phenomena

In the paper [13] where only inelastic irreversible behaviour of steels was considered the specific free energy of the considered body was taken to be of the form

$$f = f_E(\mathbf{E}_E, T) + f_P(\lambda, T) \tag{21}$$

where λ is the isotropic hardening parameter whose time rate is given by

$$D\lambda := \mathbf{T} : D\mathbf{E}_P, \tag{22}$$

having the meaning of plastic power. Since the free energy is assumed in the form (21) we have $T\vartheta = (1 - \rho\partial_\lambda f)\, D\lambda$. Here the dissipation appearing in the second law of thermodynamics is denoted by ϑ, namely $\vartheta \equiv \rho Ds + \operatorname{div}(\mathbf{q}/T) \geq 0$, where ρ is the mass density, T is the absolute temperature, $\mathbf{q}$ is the heat flux vector and s is the specific entropy.

By making use of this dissipation Vakulenko introduced a concept of *thermodynamic time* [12] by the following hereditary function

$$\zeta(t) := \int_0^t \psi[T\vartheta(t')]\, dt' \tag{23}$$

The function $\zeta(t)$ is piecewise continuous and nondecreasing in the way that $D\zeta(t) = 0$ within elastic ranges and $D\zeta(t) > 0$ when plastic deformation takes place. Splitting the whole time history into a sequence of infinitesimal segments Vakulenko claimed that a superposition and causality exists such that the plastic strain tensor is a functional of stress and stress rate history. Moreover, in the paper [3] the accumulated plastic strain $\pi(\zeta) \equiv \int_0^\zeta \|D\varepsilon_P(\xi)\|\, d\xi$ is added extending in such a way formerly mentioned Vakulenko's arguments.

$$\mathbf{E}_P(\zeta) = \int_0^\zeta \mathbf{\Phi}\left[\zeta - \xi, \mathbf{T}(\xi), D\mathbf{T}(\xi), \pi(\xi)\right]\, d\xi. \tag{24}$$

Of course, this integral equation is adopted to our case of finite plastic strains and absence of plastic rotation. Differentiation of (24) with respect to the thermodynamic time gives

$$\begin{aligned}\partial_\zeta \mathbf{E}_P =& \mathbf{\Phi}\left[0, \mathbf{T}(\zeta), D\mathbf{T}(\zeta), \pi(\zeta)\right] \\ &+ \int_0^\zeta \partial_\zeta \mathbf{\Phi}\left[\zeta - \xi, \mathbf{T}(\xi), D\mathbf{T}(\xi), \pi(\xi)\right]\, d\xi.\end{aligned} \tag{25}$$

Let the magnitude of the stress be denoted by $\sigma_{eq} \equiv \|\mathbf{T}\|$ (this is so called *equivalent stress*). When the tensorial kernel in (24) is chosen in such a way that

$$\begin{aligned}\mathbf{\Phi}\left[\zeta - \xi, \mathbf{T}(\xi), D\mathbf{T}(\xi), \pi(\xi)\right] =& \mathbf{\Phi}_2\left(\mathbf{T}(\xi), \pi(\xi)\right) \\ &+ J(\zeta - \xi)\;\; \partial_\xi \sigma_{eq}(\xi) \mathbf{\Phi}_1\left(\mathbf{T}(\xi), \pi(\xi)\right)\end{aligned} \tag{26}$$

and a scalar kernel J is introduced by means of $\partial_\zeta\, J(\zeta - \xi) = 0$, then the integral on the right hand side of (25) vanishes. If, moreover the function in

(23) is of the power type i.e. $\psi[T\vartheta] = (T\vartheta)^a$, then a multiplication of (25) by $D\zeta$ transforms this equation into

$$D\mathbf{E}_P = \mathbf{\Phi}_1 J(0)\, D\sigma_{eq} + \mathbf{\Phi}_2 D\zeta. \tag{27}$$

The exponent a is of a great importance since it shows the speed of ageing. Vakulenko accepted only the value $a = 1$. In such a case we get

$$a = 1 \quad \Rightarrow \quad D\zeta = \frac{1 - \rho\partial_\lambda f}{1 - (1 - \rho\partial_\lambda f)\, i_2} i_1 J(0) D\sigma_{eq} \tag{28}$$

such that the correction introduced by means of the tensor $\mathbf{\Phi}_2$ seems unnecessary apart from the stress rate dependent kernel $J(\zeta - \xi)$. However, taking some value for this constant different from Vakulenko's value i.e. $a \neq 1$ the difference becomes significant (cf. [13]). For example $a < 1$ may be named *decelarated ageing* whereas $a > 1$ would define *accelarated ageing.* By such a classification the value $a = 1$ might be termed as *steady ageing.*

4.2 Purely Magnetic Irreversible Phenomena

Let us apply here the above explained concept to evolution of irreversible magnetisation. Again we have non-steady ageing speed (defined by the exponent a) [13]

$$D\zeta = (T\theta)^a \equiv (\mathbf{B} : D\mathbf{M}_R + \mathbf{T} : D\mathbf{E}_P)^a\,,$$

but now irreversible power induced by magnetisation must be taken into account. In the case of negligible mechanical effects the corresponding endochronic memory is characterized by the following integral equation

$$\mathbf{M}_R(\zeta) := \int_0^\zeta \mathbf{\Phi}(\pi, \zeta - z, \mathbf{H}(z))\; dz \tag{29}$$

Choosing a special form of the integral kernel as follows [2]

$$\mathbf{\Phi} = \mathbf{H}(z)\; \omega(\pi)\; \exp\left\{-\rho d\,(\zeta - z)\right\} \tag{30}$$

for purely magnetic case (but plastic strain accumulation dependent) we arrive at the following explicit evolution equation:

$$D\mathbf{M}_R = (\omega\mathbf{H} - \rho d\mathbf{M}_R) \left(\omega \left\|\mathbf{H}\right\|^2 - \mathbf{H} : \mathbf{M}_R\right)^{\frac{a}{1-a}}.$$

[2]Here a material constant d must not be confused with the capital D serving to denote material time differentiation.

Hence, (13) holds only if $\rho d \sim 0$ (i.e. of negligible value). Obviously

$$\mathbf{B} \neq \mu\ \mathbf{H} \Rightarrow \quad D\mathbf{B} \neq \frac{\mu}{\chi}(D\mathbf{M} - D\mathbf{M}_R),$$

which could be used for some nondestructive experimental check of order of magnitude of magnetomechanical interactions at low cycle fatigue or at some other experiments designed to establish characteristic points of inelastic behaviour of steels or some other ferromagnetic materials.

5 Concluding Remarks

The main result of this paper might be stated as follows: *interaction of irreversible magnetization and inelastic strains is naturally given within the framework of the so called endochronic thermodynamics allowing for diverse ageing speeds.* Order of magnitude of such an interaction still has to be determined by experiments.

Acknowledgement

Support of Serbian Ministry of Science and Technology (within grants MM-1309 as well as TR-0035) is gratefully acknowledged.

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