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# Comparison of Different Optimisation Strategies Applied to Electromagnetic Devices

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**Abstract:** In this paper, different optimisation methods have been compared concerning some test problems of optimal electromagnetic field design, when Finite Element Method has been applied for electromagnetic field determination. Optimal shape design has been modelled. The optimisation criterion used was the quadratic sum of the differences of the prescribed and the actual field at certain points of the region of interest. Numerical experiments indicate some preferences for choosing optimisation method by comparing their robustness, speed and convergence.

**Keywords:** Optimal design, finite element method, response surface methodology, design of experiments.

## 1 Introduction

The most important task of the design engineers is to develop a high-quality, market competitive products, which ensure effective work with respect to given requirements and limitations. Obtaining of the final technical product is complicated synthesis process of detailed investigations, optimisations, expert consultations and ranking [1].

A numerical optimisation combined with field computation methods is found to be an important and powerful engineering tool for the design of electromagnetic devices. It is well known that there is no universal optimisation strategy which can guarantee "the best of the best solutions" and the solution

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of the optimisation problem usually depends on the researcher's experience and available programme tools. But developments in numerical field analysis methods, as well as powerful computers offer the opportunity to attack realistic problems of technical importance. During the past decade important researcher's efforts have been devoted to the development of efficient optimisation techniques, which could amplify the power, brought by modern numerical analysis tools such as the finite element method. Many publications have been reported about successful solutions applying new modern stochastic optimisation technique, combined with FEM [2, 3, 4, 5, 6].

The aim of this paper is to compare some different optimisation techniques concerning their robustness, speed and convergence. Some typical test problems of optimal electromagnetic field design have been considered, when finite element method analysis was applied for objective function determination. Optimal shape design has been modelled. The optimisation criterion used was the quadratic sum of the differences of the prescribed and the actual field at certain points of the region of interest.

## 2 Electromagnetic Field Optimisation Problems - Main Features

In order to compare and select the methods appropriate for solving the electromagnetic field optimisation problems it is reasonable to point out the properties of typical electromagnetic optimisation problem. The main common features of this type of problems are [1]:

- Type of the optimisation problem is parameter or static optimisation $f(\mathbf{x}) = f(x_1, x_2, \ldots, x_n) \to \min$. Nowadays optimisations are performed mainly as static problems. Optimisation of dynamic system behaviour, combined with detailed numerical field analysis will be too time-consuming.
- Complexity, which means a high number of design parameters.
- Constrained - there are constraints concerning device behaviour, as well as constraints concerning device geometry. Any of them can be determined as equality $g(\mathbf{x}) = 0$ or inequality constraints $h(\mathbf{x}) \leq 0$.
- The objective function is non-linear.
- Design variables are real.
- There is no derivative information available (interdependencies between design variables and quality function are unknown) .

- The quality function is disturbed by stochastic errors caused by the truncation errors of numerical field computation.

## 3 Optimisation Methods-Possibilities For Application In Electromagnetic Field Problems

Generally, thousands of different optimisation methods exist, but unfortunately many of them can be successfully applied only for certain types of problems. Despite their huge number, optimisation methods has many common features and even they can be considered as improved versions of each other. Using these common features many different classifications are possible. The most popular classifications of the optimisation algorithms are:

- Deterministic or Stochastic methods
- Direct or Indirect methods

The aim of this work is to make a comparison between optimisation methods applied for electromagnetic problems in the case when FEM is used for determining the objective function. Therefore the main features of these groups of methods have to be analysed regarding the mentioned specific problems, taking into account their:

- Reliability
- Robustness
- Insensibility to stochastic disturbances
- Application range
- Stable solutions
- Performance

The literature observation on this topic shows the following situation: The deterministic methods such as Conjugate Gradient(CG), Newton, Quasi Newton and etc. are classical optimisation methods.

- They are basically local optimisation methods and often are convening to a local minimum
- They are often based on the construction of the derivatives or approximations of the derivative of the objective function. But when using FEM for electromagnetic field analysis it is often difficult to obtain derivatives. It requires human interaction in the optimisation procedure.

- They are very sensitive to stochastic disturbances, especially contained in the derivative information they are based on.
- These gradient based methods are very popular and very effective and converge to the local minimum in a small number of steps when analytical objective function exists.

Stochastic optimisation methods, such as simulated annealing (SA), evolution strategy (ES) and genetic algorithms (GA) are modern methods, based on statistical analysis.

- These methods utilise random processes to scan the whole parameter space. That is way they can reach the global optima with high probability.
- They accept deterioration in the objective function during the solving process and this enables them to escape local minimum and find the region of the global optimum no matter where the strategy is started from.
- They do not require derivative information
- They are rather simple to implement.
- They are stable in convergence.
- The main drawback of this methods is the fact that they need a very high number of function evaluations during the solving process. It means that when applying FEM for electromagnetic field analysis the process becomes too time consuming for the complex realistic problems.

These preliminary, based on the literature observation, evaluations and comparisons lead to the conclusion that both groups-deterministic and stochastic methods faced problems in solving electromagnetic optimisation problems when applying FEM. There are some reports [7] for combined optimisation strategy. It begins with stochastic method and after localisation of the global optimum region searching process continues with deterministic method. The problem is how to choose the switching point between the two methods. There is also another possibility which can overcome some of the mentioned problems. This is the method based on the response surface methodology (RSM) and design of experiments (DOE).

This method allows both using the time expensive FEM field analysis and reducing significantly the total optimisation time.

The combination of the Response Surface Methodology and the Design of Experiments offers possibility to reduce the number of FEM function evaluations by applying statistical methods to sample the search space efficiently. The approximation of the real objective function can be obtained in this way using numerical FEM analysis experiments. In this way it can be described by first or second orders polynomials expression-for example of the following type

$$\hat{\alpha}^j = b_0^j + \sum_{k=1}^{p} b_k^j h_k + \sum_{k=1}^{p} b_{kk}^j h_k^2 + \sum_{\substack{k=1 \\ r=k+1}}^{p} b_{kr}^j h_k h_r, \tag{1}$$

where $h_k$ is the $k$-th normalised design variable , $b_0$, $b_k$, $b_{kk}$ and $b_{kr}$ are the unknown coefficients that can be found using the method of least squares and p is the number of the variables. Thus the number of numerical experiments when using the Central Composite Design (CCD) is:

$$M = 2^p + 2p + 1 \tag{2}$$

This secondary function model can be optimised by means of non-linear programming. This combination of the RSM and DOE offers opportunity not only to optimise the design, but also to evaluate the main and interactive effects of the design parameters. Computation of the coefficients also gives information about the significance of the interactions ($b_{kr}$) and the main effects ($b_k$, $b_{kk}$). They provide a tool for reducing the dimensionality of the problem by excluding non-significant design parameters from the optimisation task being solved.

The major drawback of these methods is the fact that due to the use of first or second orders polynomials there is possibility of finding the local instead of global optimum. But if the surface region in the neighbourhood of the global optima is flat, falling into local optima is not so dangerous and it can be acceptable.

## 4 Numerical Examples

The best way to estimate the possibilities of different optimisation methods is to compare solutions of the test problems.

The first example is shape optimisation of the pole face of a motor, as shown in Fig. 1. This example has been solved in [3, 4, 5] using FEM combined with different stochastic optimisation techniques. The stator and pole face are treated as magnetic materials of magnetic permeability $\mu = 2000$. The winding carries a current of density $J$=10 A/m$^2$. The dimensions in millimetres are given in Fig. 1 The goal of the optimisation is set as to achieve a sinusoidal magnetic flux density distribution along the line AB positioned at 1mm below the stator. At point A the magnetic flux density is maximal and it is expected to follow a cosine function to become zero at point B, which represents a 90° distance from point A. The points $P_1$ to $P_4$ are allowed to move in the y-direction between $y$ =22.5mm and $y$ =28 mm. They are fixed in the $x$-direction. The pole face is defined by a curve connecting $P_1$ to $P_4$.

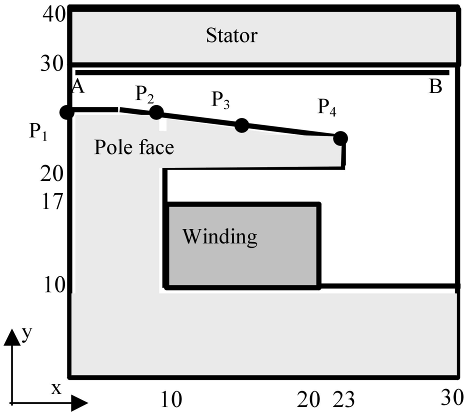

Fig. 1. Problem geometry.

The objective function value $F$ that is used by the search process as the decision criteria is obtained from the desired field values (the cosine function) and the calculated field values at 21 points along the line AB using the function

$$F = \sum_{k=1}^{n} (B_{desired,k} - B_{calculated,k})^2 \tag{3}$$

The following constraints are imposed:

- The pole face has to be smooth, without zigzags;
- Outer end of the pole face (point $P_4$ should be lower than the point at the inner end (point $P_1$);and
- The curve segment between $P_1$ and $P_4$ must be monotonically increasing.

Four design parameters determine the shape of the pole face. These are $y$-co-ordinates of 4 points placed at fixed $x$-co-ordinates.

The optimisation problem has been solved using the advantage of the response surface methodology and design of experiments. For FEM evaluations Student's QuickField®-package has been used. The results obtained for the problem are given in Fig. 2 and Fig. 3 and Table 1.

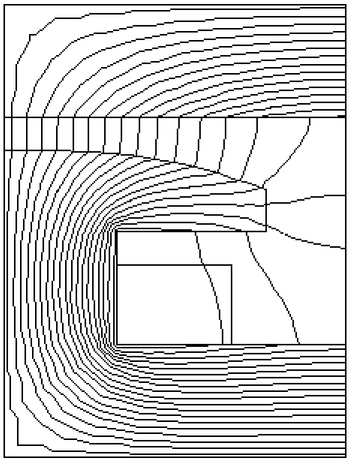

Fig. 2. Optimised pole shape and the field lines.

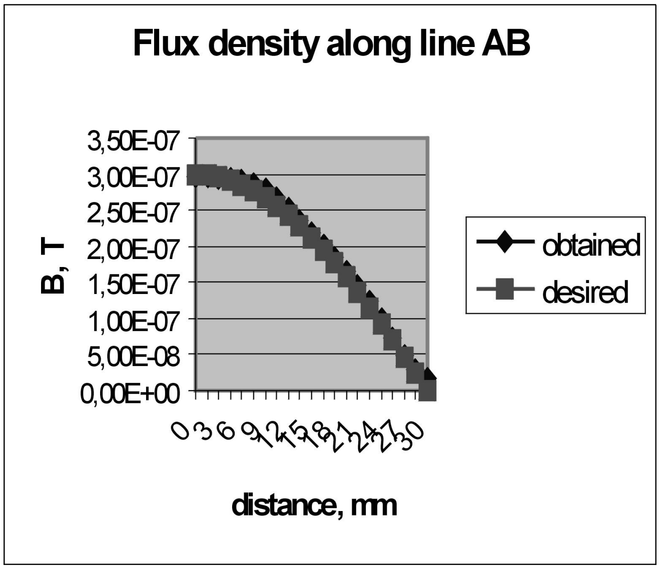

Fig. 3. Flux density along the line AB.

Table 1.

| Point $N$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $x$-coord. | 0 | 7.667 | 15.333 | 23.0 |
| $y$-coord. | 27.12 | 27.04 | 26.0 | 23.74 |

The relative error, defined by the expression

$$\frac{1}{n}\sum_{1=1}^{n}\Big(\frac{B_{computed}}{B_{desired}}-1\Big) \tag{4}$$

is 4.37%.

The results obtained show that the posed test optimisation problem has been solved successfully and the RSM and DOE approach has tackled with the problem. A second test example is given in order to demonstrate the abilities of RSM and DOE to solve problems with greater number of design variables as this is thought to be a drawback of this approach. This example deals with obtaining of the homogeneous field along the axis of a coil of step cross section - Fig. 4. The design variables are the lengths, the inner and outer radii and the current density of the $N = 5$ and $N = 6$ coil sections.

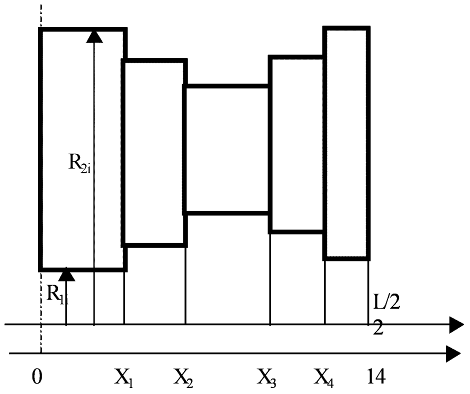

Fig. 4. Coil of stepwise cross section.

In this example, the number of the design variables is $p = 19$ (for $N = 5$) and $p = 23$ (for $N = 6$). This means that the number of numerical experiments would become much greater. For example, for $p = 19$, the number of numerical experiments will be $M = 2^p + 2p + 1 = 524327$ . For avoiding this great number of function evaluations additional time reducing opportunity has been used. Taking into account the fact that the magnetic field problem is linear, the field model along the axis has been obtained as a superposition of the single section models. Each of this single section models depends only in 5 variables. Thus the total number of evaluations has been significantly reduced to:

$$M' = \sum_{1}^{5} (2^5 + 2.5 + 1) = 179 \tag{5}$$

The objective function is used taking into account that required field along the coil axis in normalised units has to be 1. The results obtained are shown in Fig. 5 and Fig. 6.

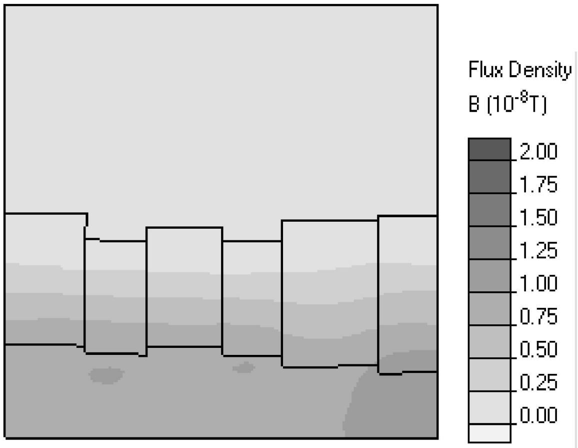

Fig. 5. Magnetic field of the optimised coil.

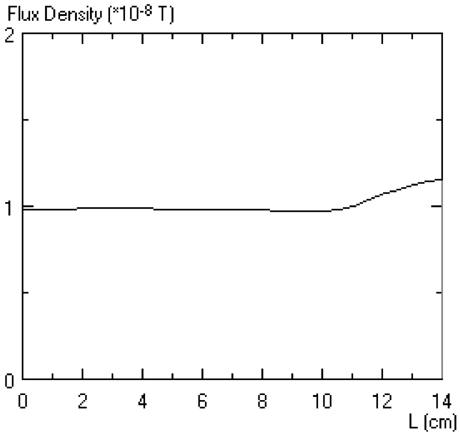

Fig. 6. The obtained homogeneous field along the coil axis.

The results show that RSM and DOE is capable to deal with greater number of design variables having kept the number of function evaluations low in the case of linear field problems. Then the speed feature of the method is maintained.

## 5 Conclusion

In the paper several optimisation techniques have been considered concerning their ability to solve electromagnetic field optimisation problems when finite element method analysis is applied for objective function evaluation. Two typical test problems of optimal electromagnetic field design have been considered.

Special attention has been drawn to the approach using response surface methodology and design of experiments.

The results show that this approach could be successfully applied in the optimisation of electromagnetic devices. The approach is able to keep its advantages, especially the speed for the case of linear field problems.

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