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Comparison of the FDTD and Direct-Integrating Methods for Electrodynamic Problem in Time-Varing Medium

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Abstract: In this paper the finite differences in time domain and the integral equation in time domain approaches for calculation of an electromagnetic signal propagation in an active media are investigated, accuracy and stability comparison of these methods are examined. Special computer modeling software is developed for realization of the numerical experiments.

Keywords: Volterra Integral equations of second kind, Time domain, FDTD, nonstationary, transient.

1 Introduction

Electromagnetic propagation in media with time-varying parameters is of interest for many applications: shock waves, strong laser fields, nuclear burst, atmospheric or environmental change and others [1, 2, 3]. In order that these phenomena can be exploited for engineering benefit there is a need to simulate them. The investigation of nonstationary electromagnetic signal propagation is made by various methods based on integral transformations from a frequency domain into a time domain as well as by handling Maxwell equations or equivalent in the immediate time domain. The latter is preferable, especially in that cases when nonstationarity is determined not only by

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an initial signal but also by change of medium parameters (for example, a permittivity and a conductivity). As a rule, an exact consideration of electromagnetic processes in media with the time-varying parameters is possible in special cases only. In general case, an initial electromagnetic problem can be investigated by numerical methods.

For a problem solving in the differential statement the FDTD method has received wide recognition [4]. FDTD methods are popular due to their clarity and elegant realization in numerical algorithms. Indeed, it is difficult to identify areas in electromagnetic theory where FDTD-based methods have not been applied. However, an undesirable feature of FDTD schemes is the time instability during long integration times [5].

An alternative approach reduces a problem to integral equations [6, 7, 8, 9], which are equivalent to Maxwell's equations. The integral equations approach seems to have a more promising future in terms of both reflection-free realization and stability. An advantage of the integral equation method is that it can be generalized to nonlinear dispersive materials. This is potentially of importance for non-linear applications of semiconductor optical amplifiers. The aim of this work is to compare both differential and integral approaches for calculation of a nonstationary electrodynamic problems by solving test problems using both methods and comparing results with the exact analytical solutions.

2 Problem Formulation

An electromagnetic field exists in unbounded medium while. Permittivity of the medium is and conductivity equals to zero. After initial moment of time the permittivity and conductivity depend on time as and respectively. The electrical field has to be computed after by using two algorithms Finite-Differences in Time-Domain (FDTD) [4] and the Volterra integral equation in time-domain method (VIETD) [6, 7, 8, 9]. The differential and the integral equation formulations of the 1D nonstationary electromagnetic problem follow.

2.1 Differential form

The differential equation for the problem can be obtained from the Maxwell electrodynamics equations

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\varepsilon_1 E) - \frac{\partial^2}{\partial x^2}E + \frac{1}{c^2}\frac{\partial}{\partial t}(\sigma E) = 0. \tag{1}$$

Initial conditions are

$$\varepsilon E_0 = \varepsilon_1 E \quad \frac{\partial}{\partial t}(\varepsilon E_0) = \frac{\partial}{\partial t}(\varepsilon_1 E), \tag{2}$$

where c is speed of light in free space.

It's more convenient for calculation to use dimensionless quantities τ, ξ instead of the time and spatial coordinates t, x:

$$\begin{aligned} \tau = k\nu t, \quad \xi = kx, \quad b(\tau) = \frac{1}{\varepsilon\nu k}\sigma\left(\frac{\tau}{\nu k}\right), \\ a^2(\tau) = \frac{\varepsilon}{\varepsilon_1(\frac{\tau}{\nu k})}, \quad E(\tau,\xi) = \tilde{E}(\frac{\tau}{\nu k}, \frac{\xi}{k}), \end{aligned} \tag{3}$$

where k is the scaling factor with the wave-number dimension, $\nu = c/\sqrt{\varepsilon}$.

Substituting (3) to the equation (1) we obtain

$$\frac{\partial^2}{\partial \tau^2}\left[\frac{1}{a^2}\tilde{E}(\tau,\xi)\right] - \frac{\partial^2}{\partial \xi^2}\tilde{E}(\tau\xi) + \frac{\partial}{\partial \tau}\left[b\tilde{E}(\tau,\xi)\right] = 0 \tag{4}$$

and the initial conditions

$$\tilde{E}(0,\xi) = a(0)\tilde{E}(0,\xi) \tag{5}$$

and

$$\left.\frac{\partial \tilde{E}(\tau,\xi)}{\partial t}\right|_{t=0} = \left.\frac{\partial}{\partial t}\left[a(\tau)\tilde{E}(\tau,\xi)\right]\right|_{t=0} \tag{6}$$

According to [4] the stability criteria for the FDTD algorithm is: $\Delta\tau < \Delta\xi$ in passive media. In this work the $2\Delta\tau = \Delta\xi$ relationship is used.

2.2 Integral form

The Volterra integral equation for the problem is obtained from the Maxwell electrodynamics equations [10, 11]

$$E(t,x) = E_0(t,x) - \frac{1}{2\nu}\int_0^t \mathrm{d}t' \int_{-\infty}^{\infty} \mathrm{d}x' \left\{\frac{\sigma}{\varepsilon} + \frac{\varepsilon_1 - \varepsilon}{\varepsilon}\frac{\partial}{\partial t}\right\}\delta\left(t - t' - \frac{|x - x'|}{\nu}\right)E(t',x') \tag{7}$$

In dimensionless quantities (3) it has the form

$$E(\tau,\xi) = \tilde{E}_0(\tau,\xi) - \frac{1}{2}\int_0^t \mathrm{d}\tau' \int_{-\infty}^{\infty} \mathrm{d}\xi' \left\{b + \frac{1 - a^2}{a^2}\frac{\partial}{\partial \tau}\right\}\delta\left(\tau - \tau' - |\xi - \xi'|\right)E(\tau',\xi') \tag{8}$$

after integration by spatial axis we obtain

$$\tilde{E}(\tau,\xi) = a^2\Big\{\tilde{E}_0(\tau,\xi) - \frac{1}{2}\int\limits_0^t \mathrm{d}\tau' b\big[\tilde{E}(\tau',\xi+\tau-\tau) + \tilde{E}(\tau',\xi-\tau+\tau')\big] + \frac{1-a^2}{a^2}\frac{\partial}{\partial\xi}\big[\tilde{E}(\tau',\xi+\tau-\tau) - \tilde{E}(\tau',\xi-\tau-\tau')\big]\Big\} \qquad (9)$$

As in [11], we assume a uniform external grid on the coordinate plane (τ,ξ) with $\delta\tau = \Delta\xi$. The integration paths in equation (8) are then straight diagonal lines, which pass through the nodes in the grid. More detailed this approach is described in [11].

3 The Description of the Software

For the automation of the calculation, original software based on abovementioned methods is created. The program has sophisticated graphical user interface, which is realized in Borland C++ Builder environment and is very effective for the modeling of wave propagation in the nonstationary media with different laws of parameter change. This software is a further development of those proposed in [12] and presents some uniform tool for analysis of one- dimensional nonstationary problems in both dispersive and nondispersive media whose properties depend on temporal coordinates. The software includes different numerical algorithms for solving the problem and allows to compare results and to analyze algorithm's accuracy on test problems. The idea of software architecture is a high flexibility for further development and update. This is practically realized by using object oriented programming concept where all types of input data, algorithms of numerical calculations and modules of analysis are realized as special class libraries.

4 Numerical Results

In general, the considered methods and the developed software enable us to consider an initial field with an arbitrary time?spatial dependence. Here for the test purposes we consider only a plane wave as an initial field: $E_0(\tau,\xi) = \cos\omega(\tau-\xi)$, $\omega = 1$. For all experiments the grid spacing is $\Delta\tau = 0.05$. In followed sections different types of the medium change are investigated.

4.1 Modulation of the medium by power low

Consider medium with parameters $a^2 = 1/(1+g\tau)^4$ and $b(\tau) = 0$. Let $g = 0.2$. An exact solution for this case is known [13]

$$E(\tau,\xi) = \frac{e^{-i\xi}}{(1+g\tau)^3}\Big[\cos\Big(\frac{\tau}{1+g\tau}\Big) + (i-g)\sin\Big(\frac{\tau}{1+g\tau}\Big)\Big].$$

The absolute and relative errors parameters for the computed fields are presented in Fig. 1.

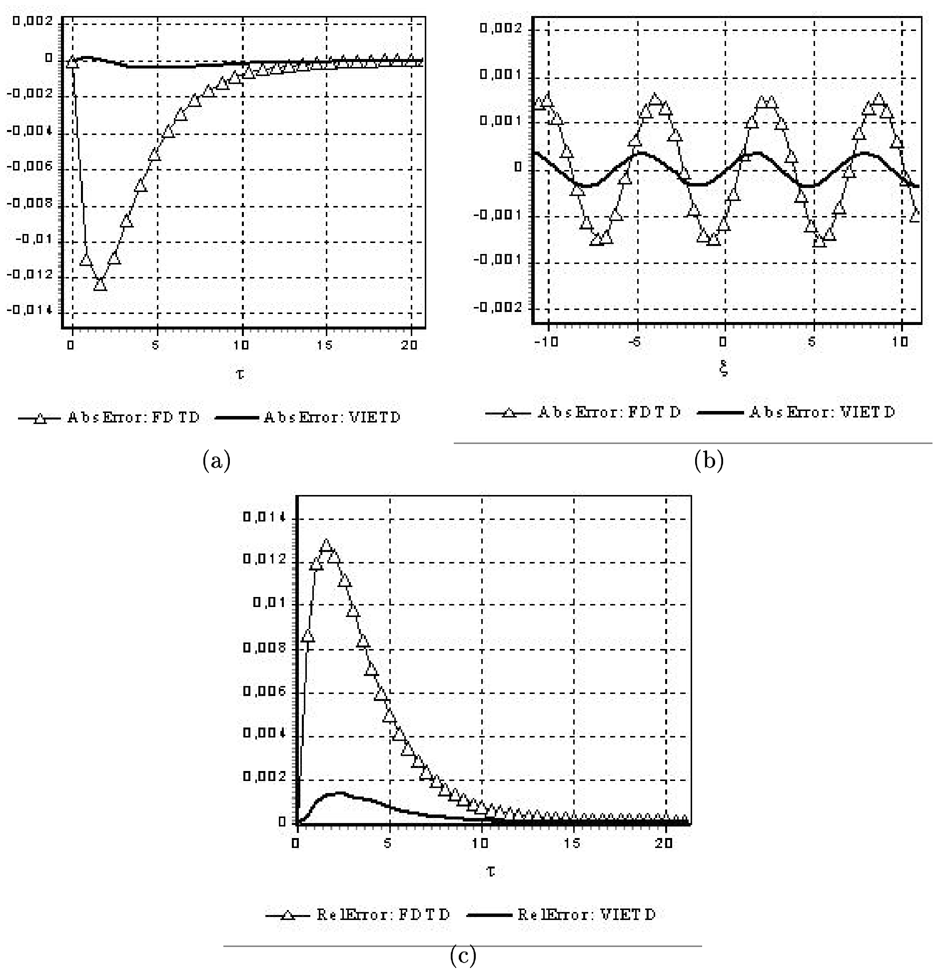


Fig. 1. (a) Absolute errors for $x = -1.0$. (b) Absolute errors for $t = 10.0$. (c) Relative errors for $x = -1.0$.

In this paper the relative numerical error is calculated as the ratio of the absolute error to the greatest value of field. The computation results show that VIETD algorithm has better accuracy than FDTD, which is more sensitive to significant changes of the medium parameters. But calculation time for the FDTD program is smaller.

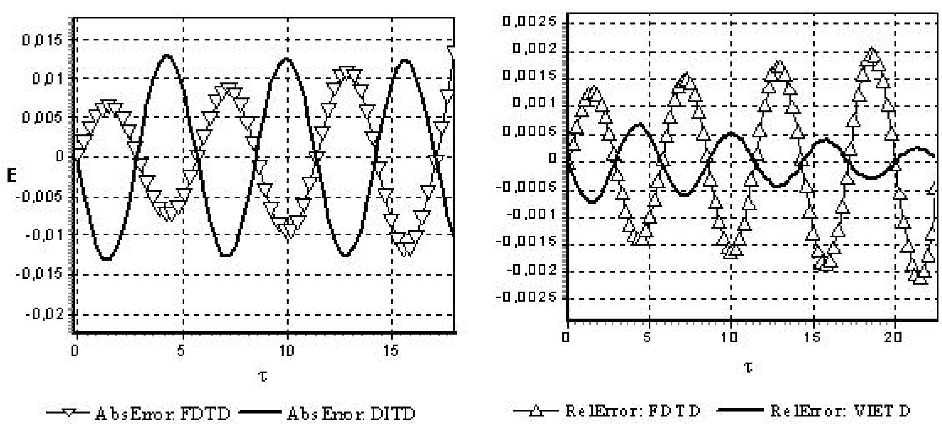
4.2 Step changing of the medium parameters

Consider medium with parameters $a^2 = (\tau) = A\theta(\tau)$ and $b(\tau) = 0$. Let $A = 1.1$. An exact solution for this case is known [11]

$$E(\tau,\xi) = \frac{a^2}{2h} e^{-\frac{a^2 b}{2}\tau} \Big\{ (h+1)\cos\omega(h\tau - \xi) + (h-1)\cos\omega(h\tau+\xi) - \frac{a^2 b}{2\omega}\Big[\sin\omega(h\tau-\xi) + \sin\omega(h\tau+\xi)\Big]\Big\}$$

where $h = a\sqrt{1-(ab/2\omega)^2}$ and ω is the wave frequency of $E_0 \cos\omega(\tau - \xi)$.

The absolute and relative errors parameters for the resulting fields are presented in Fig. 2.



(a) (b)

Fig. 2. (a) Absolute errors for $x = 0.0$. (b) Relative errors for $x = 0.0$.

Obtained results show that when medium parameters are constant there is almost no difference in accuracy for the realized algorithms and both methods give similar results.

4.3 Modulation of the medium by periodical impulses

Consider temporal variation of medium parameters as a finite sequence of the periodic rectangular pulses originating at the zero moment of time. We assume that the medium parameter $a(\tau)$ acquires the value A_1 respectively on the disturbance intervals $(n-1)T < \tau < T_1 + (n-1)T$, $n = 1, \ldots, N$ and the value A_0 on the quiescence intervals $T_1 + (n-1)T < \tau < nT$, $n = 1, \ldots, N$

$$a(\tau) = A_0 + (A_1 - A_0) \sum_{k=1}^{N} \{\theta[\tau - (k-1)T] - \theta[\tau - T_1 - (k-1)]\},$$

$$b(\tau) = 0.$$

Here, $\theta(t)$ is the Heaviside step function, T is the duration of the period of change in parameters, T_1 is the duration of the disturbance interval. The exact solution to this problem is given in [9].

Let $A_0 = 1$, $A_1 = 1.2$, $T_1 = 2$, $_2 = 0.5$, $N = 20$. The resulting fields are presented in Fig. 6.

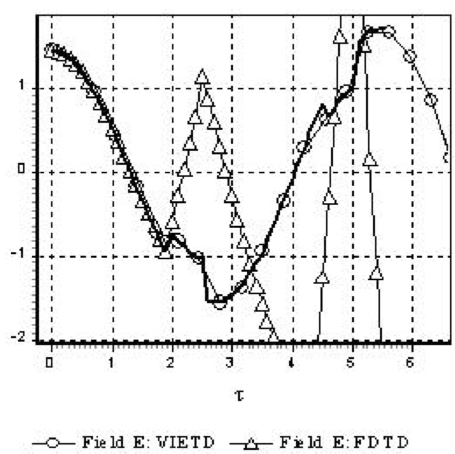


Fig. 3. Electric field E for $x = 0.0$ (Bold line is exact solution).

The calculation results demonstrate that the solution, obtained by the basic FDTD algorithm, is not stable and crashes when the medium parameters change abruptly, while VIETD algorithm successfully overcomes points of discontinuity.

5 Conclusion

The FDTD and VIETD algorithms were realized. Comparison of the two different approaches for solution of the 1D nonstationary electrodynamics problem was considered. Accuracy and stability of these methods were investigated via comparison of the numerical solutions with the exact one on the test problems. Numerical results for the different medium parameters and initial fields have been presented.

An original software for the computer modeling of electromagnetic wave propagation in a time-varying medium for arbitrary initial fields and different laws of changing in medium parameters has been developed. The software environment allows implementation of different calculation schemes for the 1D nonstationary electrodynamics problem.

The results obtained in this paper show that FDTD algorithm is more sensitive to significant change of the medium parameters, where VIETD algorithm provides better accuracy. Also VIETD algorithm is more stable when medium parameters change abruptly.

References

[1] M. I. Bakunov and A. V. Maslov:*Frequency Upshifting of Electromagnetic Radiation via Oblique Incidence on an Ionization Front.* IEEE Trans. on Plasma Science, vol. 27, No. 3, pp. 655-663, 1999.

[2] S. Kuo and A. Ren: *Experimental Study of Wave propagation Through a Rapidly Created Plasma.* IEEE Trans. of Plasma Science, vol. 21, No 1, pp. 53-56, 1993.

[3] D. K. Kalluri: *Electromagnetics of Complex Media (Frequency Shifting by a Transient Magnetoplasma Medium).* CRC Press: Boca Raton, Boston, New York, 1998.

[4] A. Taflove: *Computational Electrodynamics: The Finite-Difference Time-Domain Method.* Boston, Artech House, 1995.

[5] J.L Young, D. Gaitoude and J.S. Shang: *Toward the construction of a fourth order difference scheme for transient EM wave simulation: Staggered grid approach.* IEEE Trans. on Antennas and Propagation, vol. 45, pp.1573-1580, 1997.

[6] A. G. Nerukh and N. A. Khizhyak: *Contemporary problems of unstationary macroskope electrodinamics*, Test-Radio, Khar'kov, 1991 (in Russian).

[7] A. G. Nerukh: *Evolutionary approach in transient electrodynamics problems.* Radio Science, vol. 30, No 3, pp. 481-491, 1995.

[8] A. Nerukh: *Integral equations and transitive electrodynamic processes*, Foreign radioelectronics, Contemporary radioelectronics, No. 3, pp. 3-24, 1999 (in Russian).

[9] A. G. Nerukh, I. V. Scherbatko and M. Marciniak: *Electromagnetics of modulated media with applications to photonics.* National Institute of Telecommunications of Poland, Warsaw, 2001.

[10] G. Nerukh, I. V. Scherbatko and D. A. Nerukh: *Using evolutionary recursion to solve an electromagnetic problem with time-varying parameters* Microwave

and Optical Technology Letters, vol. 14, No. 1, pp. 31-36, 1997.

[11] A. G. Nerukh, I. V. Scherbatko, O. N. Rybin: *The Direct Numerical Calculation of an Integral Volterra Equation for an Electromagnetic Signal in a Time - Varying Dissipative Medium.* Journal of Electromagnetic Waves and Applications. 1998. Vol. 12, No. 1, pp. 167-176.

[12] A.G. Nerukh, K.M. Yemelyanov, F.V. Fedotov: *Computer Modeling of Electromagnetic Wave Propagation in a Time-Varying Medium.* In: Proc. 3-d Int. Conference on Antenna Theory and Techniques (ICATT'99), Sevastopol, Ukraine, September 8-11, pp. 429-431.

[13] Averkov, S.I., and V.P. Boldin: *Waves in nondispersive nonstationary inhomogeneous media.* Radiophys. Quantum Electron. (English transl.), vol. 23, No 9, pp. 1060-1066, 1980 (in Russian pagination).