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A Comparative Analysis of two Gain- and Offset-Compensated Switched-Capacitor Integrators

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Abstract: Two high-performance switched-capacitor (SC) integrators which use different approaches for the compensation of the operational amplifier finite dc gain and offset voltage are considered. Analytical expressions for the gain, phase and offset voltage errors of the Baschirotto-90 integrator are derived and compared with the corresponding errors of the Shafeeu-91 integrator. Both the integrators are used as building blocks of a high-Q bandpass SC biquad. The resultant filters are compared in terms of the percent deviations from the ideal case of the central frequency and the quality factor. Subsequently, the slight shift in the frequency response of the biquad with Shafeeu-91 integrator is eliminated by modifying the values of two capacitors.

Keywords: Gain compensation, offset compensation, operational amplifiers, switched-capacitor integrators, filters.

1 Introduction

For high-Q high-frequency switched-capacitor (SC) bandpass filters both high-speed and large dc gain operational amplifiers (op amps) are required. Due to the speed-gain trade-off, classic high-speed op amps are often inadequate to guarantee a satisfactory dc gain, and this limits the performances of the filters.

In the literature [1]-[10], different techniques have been suggested to compensate the effects of the finite op amp dc gain Ao in active SC inte-

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grators. The influence of the input-referred op amp offset voltage V_{OS} is also reduced. The most of these gain- and offset- compensated (GOC) approaches need a two-step operation (compensation and integration) where each step requires the results of the previous one. According to the authors knowledge, out of all the GOC integrators, based on this two-step operation, the integrator proposed by Shafeeu et al. [9] yields the best performances. It has simultaneously low gain, phase and offset errors and a low component cost.

One another approach for realizing SC integrators compensated for the op amp dc gain A_0 and offset voltage V_{OS} was presented by Baschirotto et al. in [11], [12]. This technique is maximally effective in a narrow frequency range centered at a specific frequency f_0 such that the ratio f_S/f_0 is an integer (typically from 4 to 8). Here f_S is the sampling frequency. In [11] the case of a high-Q bandpass biquadratic filter was reported as an example. The conventional uncompensated integrators in the original Fleischer and Laker's E-type bandpass BP 10 cell have been consecutively replaced by the GOC Baschirotto-90 integrator [11], by the GOC Haug-85 integrator [2] and by the GOC Nagaraj-85 integrator [1]. For each integrator, the name of the first author was assigned to the circuit, along with the year of publication. The frequency response of the biquad designed with the Baschirotto-90 integrators follows much more closely the ideal response than those of the design with the other compensated integrators.

In this paper analytical expressions for the gain, phase and offset voltage errors of the Baschirotto-90 integrator are derived and compared with the corresponding errors of the Shafeeu-91 integrator. Subsequently both the integrators are used as building blocks of the high-Q bandpass biquad from [11]. The performances of the resultant filters are also compared.

2 Theoretical Results

In the following the op amps are assumed to have finite dc gain $A_0 = 1/\mu$ and infinite bandwidth. This supposition is adequate for the analysis of SC circuits containing fast and relatively low-gain amplifiers.

The Baschirotto-90 integrator [11], [12] is shown in Fig.1. The clock phases outside brackets apply to the inverting integrator, whereas those inside brackets apply to the noninverting integrator. The nonzero input-referred dc offset voltage V_{OS} of the op amp is modeled as a voltage source at the noninverting input terminal. The integrator have been designed to have a unity gain frequency f_0 equal to one-fourth of the sampling frequency f_S (i.e., $C_1 = 1$ and $C_2 = 0.7071$). Furthermore, the following capacitor values have been used: $C_B = 0.5$ and $C_M = 5$. The total capacitance and the capacitance spread are : $\Sigma C = 12.2071$, $C_{max}/C_{min} = 10$.

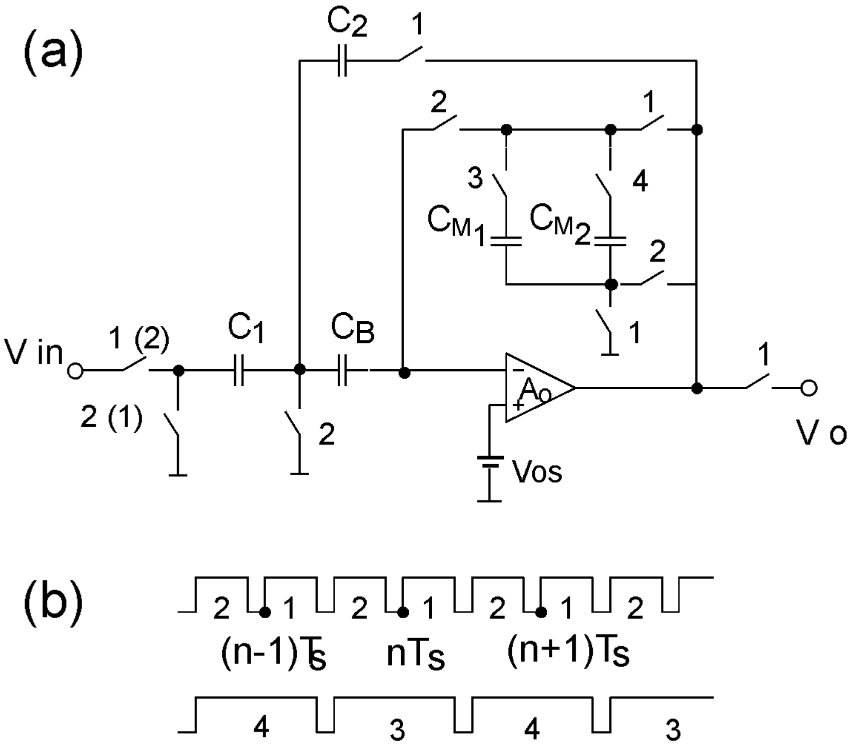

Fig. 1. Baschirotto-90 GOC integrator. (a) Circuit architecture. (b) Clocking scheme.

The z-domain transfer function of the nonideal inverting integrator can be expressed as

$$H_a(z) = -\frac{C_1}{C_2}\frac{1}{1-z^{-1}}E(z) = H_{id}(z)E(z). \tag{1}$$

In the continuous-time domain the error function $E(z)$ has the form

$$E(j\omega) = \frac{N(j\omega)}{D(j\omega)} = [1 + m_B(\omega)]e^{j\theta_B(\omega)} \tag{2}$$

Here, $m_B(\omega)$ is the gain error and $\theta_B(\omega)$ is the phase error due to the finite amplifier gain $A_0 = 1/\mu$. After detailed manipulations, the following expressions are obtained

$$
\begin{aligned}
N(j\omega) =& 1 + (1+k_1)\mu - k_1\mu\cos(\omega T_S) + jk_1\mu\sin(\omega T_S) \\
\Re\{D(j\omega)\} =& 1 + (2+k_1)\mu + (1 + 0.5k + k_1 + kk_1)\mu^2 \\
& - \mu(k + k_1 + k_1\mu)\cos(\omega T_s) + \mu\cos(2\omega T_S) \\
\Im\{D(j\omega)\} =& \mu(k + 0.5k\mu)\cot an(0.5\omega T_S) \\
& - \mu(k + k_1 + k_1\mu)\sin(0.5\omega T_S) + \mu\sin(2\omega T_S)
\end{aligned} \tag{3}
$$

where $k = C_1/C_2$, $k_1 = C_B/C_M$.

In the vicinity of the ratio $f_0/f_S = 0.25$ the errors $m_B(\omega)$ and $\theta_B(\omega)$ are approximated by the expressions

$$
\begin{aligned}
m_B(f) \approx& -2\mu\cos^2(2\pi\frac{f}{f_S}) - \mu^2(1 + 0.5k + k_1 + kk_1) \\
& + \mu(k + k_1\mu)\cos(2\pi\frac{f}{f_S}) \\
\theta_B(f) \approx& \mu((k + 0.5k\mu)\cot an(\pi\frac{f}{f_S}) + \mu\sin(4\pi\frac{f}{f_S}) \\
& - \mu(k + k_1\mu)\sin(2\pi\frac{f}{f_S})
\end{aligned} \tag{4}
$$

The Shafeeu-91 GOC integrator [9] is shown in Fig. 2. The value of the holding capacitor C_h is not critical and can be set equal to the smallest filter capacitance. The z-transfer function of the nonideal inverting integrator (clock phases outside brackets) is given by (1). For $C_1 = 5$, $C_2 = 3.5355$ and $C_h = 0.5$ the Shafeeu-91 integrator has the unity gain frequency $f_0 = f_S/4$ and the capacitance spread of the Baschirotto-90 integrator. But the total capacitance is 25% smaller.

The corresponding gain and phase errors are found to be [13]

$$
\begin{aligned}
m_S(f) \approx& -k'\mu - 0.5[(1+k)(2+k) \\
& - 2k'(1 + k + k')]\mu^2 \\
\theta_S(f) \approx& \frac{k(1+k)\mu^2}{2\tan(\pi\frac{f}{f_S})}
\end{aligned} \tag{5}
$$

where $k = C_1/C_2$, $k' = C_h/C_2$.

The error responses of the two integrators for $C_1/C_2 = \sqrt{2}$ and $A_0 = 100$ are compared in Fig.3. The errors are computed from (4) and (5).

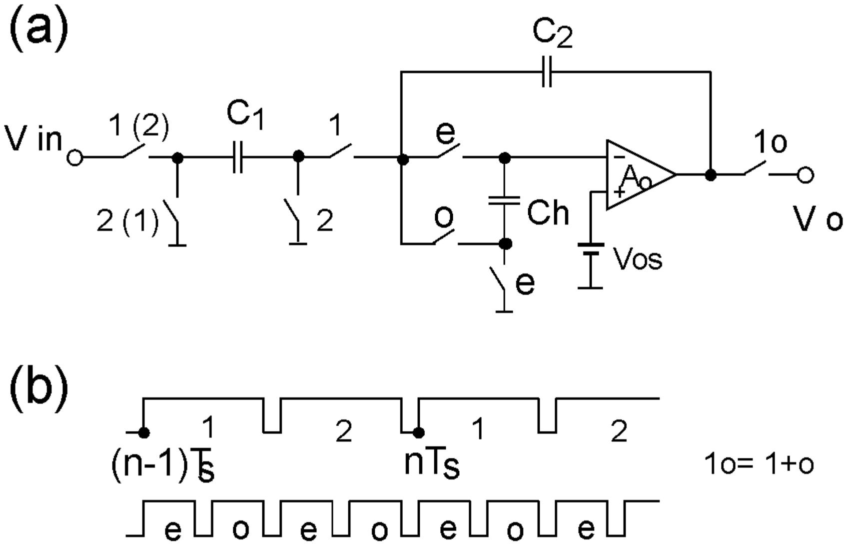

Fig. 2. Shafeeu-91 GOC integrator. (a) Circuit architecture. (b) Clocking scheme.

It can be seen that the errors m_B and θ_B are very small in a narrow-band centered around the specific frequency $f_0 = f_S/4$. In the frequency range $0.22f_S \leq f \leq 0.266f_S$ the gain error of the Baschirotto-90 integrator is still smaller in absolute value than that of the Shafeeu-91 integrator. For the phase errors the corresponding very narrow frequency range is $0.2495f_S \leq f \leq 0.251f_S$.

The compensation of the amplifier dc offset can be manifested in the suppression factor γ of the offset voltage V_{OS} [6] as listed in Table 1. The factor by which the offset voltage error for the Baschirotto-90 integrator is reduced, when compared with the Shafeeu-91 integrator, is approximately given by $(1 + k)$.

Table 1. Comparison of the two integrators in terms of offset voltage error γ.

Integrator	Offset voltage error γ
Baschuroto-90	$\dfrac{k\mu}{[1+(1+k)\mu][1+(1+k_1)\mu]}$
Shafeeu-91	$\dfrac{k(1+k)\mu}{[1+(1+k+k')\mu][1+(k+k)\mu]}$

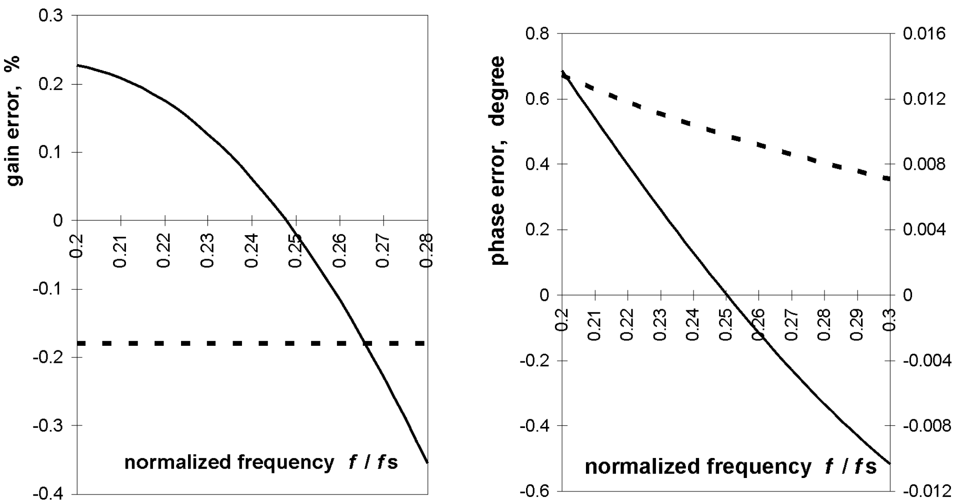

Fig. 3. Gain error responses (a) and phase error responses (b) of different integrators: —— Baschirotto - 90 integrator and ······ Shafeeu - 91 integrator (right-hand scale for phase error)

3 High-Q Bandpass Filter with Different Integrators

As an example providing a comparison of reduction in non-ideal effects for the two integrators we consider a second order bandpass filter designed from the Fleischer and Laker biquad cell in its E-type form (Fig.1(b) of [14]). The filter specifications are:

- quality factor $Q_0 = 25$, central frequency $f_0 = 25$ kHz, peak gain $H_0 = 34$ (30.63 dB),
- sampling frequency $f_S = 100$ kHz, ratio $f_S/f_0 = 4$.

Both of the standard uncompensated integrators in the original structure have been consecutively replaced by the above GOC integrators from Fig. 1 and Fig. 2.

The scheme with the Baschirotto-90 integrators is presented in Fig. 4. It is necessary to introduce two capacitors A_1 and A_2 in place of the original capacitor A and two additional clock 5 and 6 . The relative capacitor values are: $A = 13.926305$, $B = 10$, $C = 31.831462$, $D = 22.86801$, $E = 1$, $G = H = 17.00085$, $I = 10.353285$, $C_B = 1$, $C_M = 5$.

The circuit schema of the bandpass biquad with the Shafeeu-91 integrators is shown in Fig. 5. The required clock waveforms are depicted in Fig.2(b). The values of the holding capacitors are $C_{h_1} = C_{h_2} = 1$.

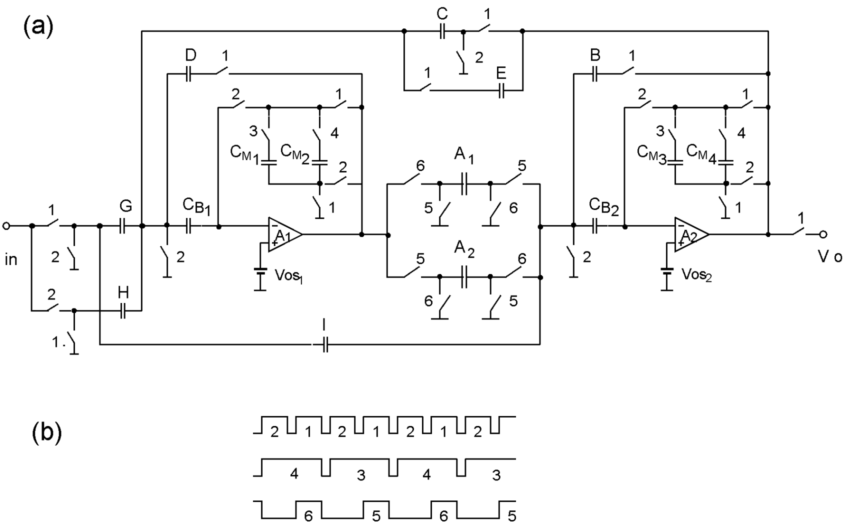

Fig. 4. High-Q bandpass biquad with Baschirotto-90 GOC integrators. (a) Circuit architecture. (b) Clocking scheme.

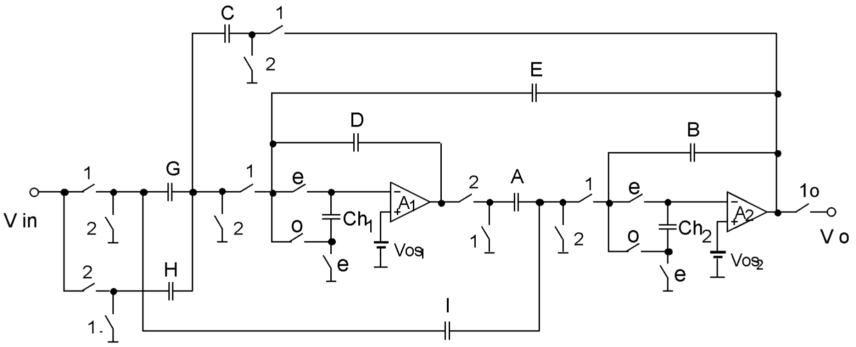

Fig. 5. High-Q bandpass biquad with Shafeeu-91 GOC integrators.

Fig. 6 shows the simulated magnitude responses of the bandpass biquad designed with the standard uncompensated integrators, with the Baschirotto-90 integrators, and with the Shafeeu-91 integrators for $A_0 = 100$. The frequency responses of the biquads with the Baschirotto-90 integrators and with the Shafeeu-91 integrators follow closely the ideal response.

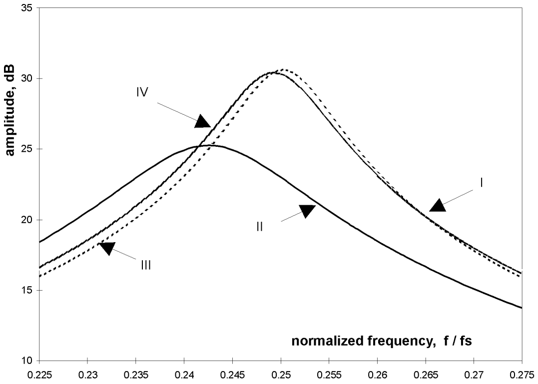

Fig. 6. Magnitude responses of the bandpass biquad. (I) ideal response ($A_0 \to \infty$). (II) with standard uncompensated integrators ($A_0 = 100$). (III) with GOC Baschirotto-90 integrators ($A_0 = 100$). (IV) with GOC Shafeeu-91 integrators ($A_0 = 100$).

The performance parameters of the filter with different integrators are summarized in Table 2.

Table 2. Performance parameters of the biquad with different integrators and finite op amp gain $A_0 = 100$.

Integrator	f_0/f_S	δf_0, %	Q_0	δQ_0, %
Standard non-GOC	0.2427	−2.920	13.45	−46.2
Baschuroto-90	0.25029	0.116	26.346	5.384
Shafeeu-91	0.24941	−0.236	24.28	−2.88

Compared to the biquad with Baschirotto-90 integrators, the biquad based on the Shafeeu-91 integrators has approximately twice larger relative error δf_0 and twice smaller relative error δQ_0. The positive relative error δQ_0 of the Baschirotto's biquad is due to the negative phase error for $f > f_0$.

It is well known that for bandpass biquads the deviation in pole frequency f_0 can be expressed in terms of the gain errors of the two integrators in the loop as follows [15]

$$\delta f_0 = \frac{\Delta f_0}{f_0} \approx \frac{1}{2}[m_1(f_0) + m_2(f_0)] \tag{6}$$

Therefore, if A_0 is known, the slight shift in the frequency response of the biquad with Shafeeu-91 integrators can be eliminate by modifying the values of the integrating capacitors D and B according to the relations [2], [3]

$$D' = D(1 - |m_1|), \quad B' = B(1 - |m_2|) \tag{7}$$

The performance parameters of the GOC biquad with Shafeeu-91 integrator for $A_0 = 100$ and for three different values of the capacitor D' and B' are summarized in Table 3. The modified capacitances D' and B' are calculated from (6) and (7) for $m_1 = m_2 = m = k\delta f_{0S}$, where $\delta f_{0S} = -2.4941 \times 10^{-3}$ and $k = 1;\ 0.75;\ 0.5$.

Table 3. Performance parameters of the Shafeeu's GOC biquad with modified capacitors D' and B' for $A_0 = 100$.

m	f_0/f_S	δf_0, %	Q_0	δQ_0, %
$1.0\delta f_{0S}$	0.25018	0.072	24.238	−3.05
$0.75\delta f_{0S}$	0.24999	−0.004	24.248	−3.01
$0.5\delta f_{0S}$	0.2498	−0.080	24.259	−2.96

It is seen, from Table 3, that the relative error δf_{0S} is very small for $m = 0.75\delta f_{0S}$. The corresponding modified capacitor values are $D'= 22.827815$ and $B'= 9.982423$. The performance parameters of the Shafeeu-91 biquad for the above modified capacitance values and with gain variation $A_0 = 100(1 \pm 0.1)$ are given in Table 4.

Table 4. Performance parameters of the Shafeeu's GOC biquad with gain variation.

A_0	f_0/f_S	δf_0, %	Q_0	δQ_0, %
90	0.24989	−0.044	24.091	−3.64
100	0.24999	−0.004	24.248	−3.01
110	0.25007	0.028	24.367	−2.53

The comparison of the results from Table 2 and Table 4 shows that the biquad with Shafeeu- 91 integrators and modified capacitors D' and B' has better performances than the filter with Baschirotto-90 integrators.

The steady state output voltages of the bandpass biquad designed with the standard integrators, with the Baschirotto-90 integrators and with the Shafeeu-91 integrators for $A_0 = 100$, are, respectively

$$\begin{aligned}\lim_{n\to\infty} V_0(n) &= 2.06743V_{OS_1} + 0.03604V_{OS_2}\\ \lim_{n\to\infty} V_0(n) &= 0.02045V_{OS_1} + 0.00071V_{OS_2}\\ \lim_{n\to\infty} V_0(n) &= 0.07471V_{OS_1} - 0.09074V_{OS_2}\end{aligned} \tag{8}$$

Table 5 compares the complexities of the two GOC biquads in terms of component count and area requirement.

Table 5. Comparison of the complexity of the two GOC biquads.

With integrator:	Number capacitors	Number switches	Total C	C spead
Baschirotto-90	15	32	149.90707	31.831462
Shafeeu-91	10	19	125.98076	31.831462

4 Conclusion

Analytical expressions for the gain, phase and offset voltage errors of the Baschirotto-90 integrator have been derived and compared with the corresponding errors of the Shafeeu-91 integrator. On the basis of the results presented, it appears that the two integrators can considerably reduce the finite-gain and offset voltage sensitivities of SC circuits. There are, however, differences between them. The gain error of the Shafeeu-91 integrator is negative and frequency independent. The corresponding phase error is positive and proportional to $1/A_0^2$. The errors of the Baschirotto-90 integrator are strongly frequency dependent and cross through zero at signal frequency $f \simeq f_0 = f_S/4$. These errors are smaller than the errors of the Shafeeu-91 integrator in a narrow-band centered around the specific frequency f_0.

The performances of the two integrators have been demonstrated by designing a bandpass biquadratic filter. Compared to the biquad with Baschirotto-90 integrator, the biquad based on the Shafeeu-91 integrator has approximately twice larger relative deviation of the pole frequency and twice smaller relative deviation of the quality factor. The slight shift in the frequency response of the biquad with Shafeeu-91 integrator was eliminated by modifying two capacitance values. After this additional compensation the Shafeeu's biquad has better performances.

References

[1] K. Nagaraj, K. Singhal, T. R. Viswanathan and J. Vlach: *Reduction of finite-gain effect in switched-capacitor filters.* Electron. Lett., vol.21, 1985, pp. 644-645.

[2] K. Haug, F. Maloberti and G. C. Temes: *Switched-capacitor integrators with low finite-gain sensitivity.* Electron. Lett., vol. 21, 1985, pp. 1156-1157.

[3] K. Haug, F. Maloberti and G. C. Temes: *Switched-capacitor circuits with low op-amp gain sensitivity.* In: Proc. IEEE Int. Symp. Circuits Syst., 1986, pp. 797-800.

[4] K. Nagaraj, J. Vlach, T. R. Viswanathan an K. Singhal: *Switched-capacitor integrator with reduced sensitivity to amplifier gain.* Electron. Lett., vol. 22, 1986, pp. 1103-1105.

[5] K. Nagaraj, T. R. Viswanathan, K. Singhal and J. Vlach: *Switched-capacitor circuits with reduced sensitivity to amplifier gain.* IEEE Trans. Circuit Syst., vol. 34, May 1987, pp. 571-574.

[6] W.-H. Ki and G. .C. Temes: *Offset-compensated switched-capacitor integrators.* In: Proc. IEEE Int. Symp. Circuits Syst., 1990, pp. 2829-2832.

[7] W.-H. Ki and G. C. Temes: *Low-phase-error offset-compensated switched-capacitor integrator.* Electron. Lett., vol. 26, 1990, pp. 957-959.

[8] A. K. Betts, H. Shafeeu and J. T. Taylor: *Amplifier gain insensitive SC integrators with "same- sample correction" of both gain and phase errors for singlepath and multipath circuits.* Electron. Lett., vol. 27, 1991, pp. 1424-1426.

[9] H. Shafeeu, A. K. Betts and J. T.Taylor: *Novel amplifier gain insensitive switched capacitor integrator with same sample correction properties.* Electron. Lett., vol. 27, 1991, pp. 2277-2279.

[10] C. C. Enz and G. C. Temes: *Circuit techniques for reducing the effects of op-amp imperfections: Autozeroing, correlated double sampling, and chopper stabilization.* Proceedings of the IEEE, vol. 84, No 11, 1996, pp. 1582-1614.

[11] A. Baschirotto, R. Castello and F. Montecchi: *Finite gain compensation techniques for high-Q bandpass SC filters.* In: Proc. IEEE Int. Symp. Circuit Syst., 1990, pp. 2813-2816.

[12] A. Baschirotto, R. Castello and F. Montecchi: *Finite gain compensated double-sampled switched- capacitor integrators for high-Q bandpass filters.* IEEE Trans. Circuits Syst., vol.39, June 1992, pp. 425-431.

[13] N. Radev, K. Ivanov and K. Stoykov: *Gain- and offset compensated switched-capacitor integrator.* In: Proc. of the XI Int. Symp. on Theoretical Electrical Engineering- ISTET'01, Linz, Austria, August 19-22, 2001, pp. 125-132.

[14] P. E. Fleischer and K. R. Laker: *A family of active switched capacitor biquad building blocks.* Bell Syst. Tech. J., vol. 58, 1979, pp. 2235-2268.

[15] K. Martin and A. S. Sedra: *Effects of finite gain and bandwidth on the performance of switched- capacitor filters.* IEEE Trans. Circuits Syst., vol. 28, 1981, pp. 822-829.