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DESIGNING OF THE FORWARD ADAPTIVE COMPANDING QUANTIZER WITH VARIABLE LENGTH CODEWORDS FOR STOHASTIC MEASUREMENT SIGNALS*

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Abstract. *This paper deals with the designing of the forward adaptive μ-law companding quantizer whose levels are coded with the Golomb-Rice code. The designing is performed for measurement signals with the Gaussian distribution and applied for the speech signal. The model satisfies the G. 712 standard and achieves the decreasing of the bit-rate for 1.34 bps (bits per sample) compared to the G. 711 standard.*

Key words: *forward adaptation, Golomb-Rice code, companding quantizer with μ-law companding function*

1. INTRODUCTION

Digital processing and transmission of measurement signals are dominant nowadays, while the most of real signals are analog. As one of the main parts of analog-to-digital converters, quantizers are important and inevitable part of almost all modern telecommunication systems. Two main types of scalar quantizers are uniform and nonuniform quantizers [1]. In uniform quantizers, quantization levels are equally spaced. Uniform quantizers are optimal for signals with the uniform distribution. However, the most of real signals are described with some nonuniform distribution [1] (e.g. Gaussian, Laplacian, etc.). For these signals, nonuniform quantizers have to be used, where quantization levels are not equally spaced. Nonuniform quantizers can achieve higher values of SQNR compared to uniform quantizers [1].

There are two ways of the realization of nonuniform quantizers [1]: the iterative Lloyd-Max algorithm and the companding technique. The iterative Lloyd-Max algorithm is computationally exhausted, thus it is used for the designing of quantizers with small number of levels. The companding technique, which represents the realization of the nonuniform quantizer as a serial connection of a compressor, a uniform quantizer and an expandor, is used for the designing of nonuniform quantizers with a high number of levels.

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In the companding technique, a companding function is applied on the input signal. One of the most used companding function is the logarithmic μ-law function, which depends on a parameter μ.

The most of real measurement signals are nonstationary, which means that power (i.e. variance) of these signals changes in the wide range during the time. For these signals, the robust quantizers have to be used. Robust quantizers are quantizers with the almost constant SQNR in the wide range of variances. Logarithmic μ-law companding quantizers are robust. However, the best way to achieve robustness is to use adaptation. There are two types of adaptation: forward and backward [1]. It was highlighted in [2] that the forward adaptation can achieve for 1 dB higher SQNR than the backward adaptation.

One of the main demands in the process of the design of telecommunication systems, beside the achievement of the high signal quality, is the decreasing of the bit-rate (i.e. the signal compression). It can be achieved by coding the quantization levels with codes with the variable-length codewords (those codes are also called lossless or entropy codes) [3]. The main idea of the entropy codes is to code high probable quantization levels with short codewords and to code small probable quantization levels with longer codewords [3]. The most used entropy code is the Huffman code. However, the Huffman code is very complex for realization, especially for quantizers with high number of levels, since it requires the forming of the code tree. The Golomb-Rice code [3, 4] is much simpler than the Huffman code but very effective in the decreasing of the bit-rate. Therefore, the Golomb-Rice code is used in many modern telecommunication systems: JPEG-LS [5], MPEG-ALS (Audio lossless Coding) [6] and CCSDS (for space communications) [7].

In this paper, the design of the forward adaptive companding μ-law quantizer with the Golomb-Rice code is presented, for the speech signal. The proposed model satisfies the G.712 standard [8] with 6.66 bps, providing the bit-rate decrease of 1.34 bps and 0.1 bps compared to quantizers proposed in the G.711 standard [9] and one presented in [10], respectively.

This paper is organized in the following way. In section 2, the descriptions of main components (companding μ-law quantizer, the Golomb-Rice code and the forward adaptation) of the model are presented. In section 3, the designing of the forward adaptive companding μ-law quantizer with the Golomb-Rice code for the speech signal is presented. Section 4 concludes the paper.

2. DESCRIPTION OF THE MODEL

2.1. Description of the nonuniform μ-law companding quantizer

In this paper we consider the memoryless Gaussian source, whose probability density function is given with the following expression:

$$p(x,\sigma)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{x^2}{2\sigma^2}\right), \tag{1}$$

where σ^2 is the variance (power) of the input signal. We consider the nonuniform μ-law companding quantizer. Let us define some parameters of this quantizer: N – the number of quantization levels; $x_{\max}$ – the maximal amplitude; x_i, $i = -N/2,...,N/2$ – the decision thresholds. The companding function of this quantizer is defined as [1]:

$$c(x) = \frac{x_{\max}}{\ln(1+\mu)} \ln\left(1 + \mu \frac{|x|}{x_{\max}}\right) \operatorname{sgn}(x), \quad -x_{\max} \le x \le x_{\max}, \tag{2}$$

where μ is a parameter. In this paper the value $\mu = 255$ is assumed, since this value is used in the G.711 standard [9]. The thresholds of the nonuniform µ-law companding quantizer in the positive part of the real axis can be expressed as:

$$x_i = \frac{x_{\max}}{\mu}((1+\mu)^{2i/N} - 1), \; i = 0,..., N/2. \tag{3}$$

The thresholds in the negative part of the real axis are symmetric to the ones in the positive part. We can see that the thresholds of the quantizer in question are expressed in the analytical (i.e. closed) form, which significantly simplifies the quantizer design. This is an important advantage of the μ-law companding quantizer compared to the optimal companding quantizer, where for the observed Gaussian pdf, integral equations have to be solved to calculate the thresholds.

During quantization, an irreversible error is made, which can be expressed by the distortion. The total distortion D is equal to the sum of the granular distortion D_g and the overload distortion D_{ov}, i.e. $D(\sigma) = D_g(\sigma) + D_{ov}(\sigma)$. In companding quantization, the granular distortion can be calculated using the Bennett integral [1]:

$$D_g(\sigma) = \frac{2x_{\max}^2}{3N^2} \int_0^{x_{\max}} \frac{p(x,\sigma)}{(c'(x))^2} dx. \tag{4}$$

For $p(x,\sigma)$ defined with (1) and $c(x)$ defined with (2), the granular distortion becomes:

$$D_g(\sigma) = \frac{\ln^2(1+\mu)}{3N^2\mu^2\sqrt{2\pi}} \left(2x_{\max}\mu \left(2 - e^{-\frac{x^2}{2\sigma^2}} (2+\mu) \right) \sigma + \sqrt{2\pi} (x_{\max}^2 + \mu^2\sigma^2) \operatorname{erf}\left(\frac{x_{\max}}{\sqrt{2}\sigma} \right) \right). \tag{5}$$

The overload distortion is defined with expression $D_{ov}(\sigma) = 2\int_{x_{\max}}^{+\infty} (x - x_{\max})^2 p(x,\sigma) dx$ [1]. For $p(x,\sigma)$ defined with (1), the expression for the overload distortion becomes:

$$D_{ov}(\sigma) = -e^{-\frac{x^2}{2\sigma^2}} \sqrt{\frac{2}{\pi}} x_{\max}\sigma + (x_{\max}^2 + \sigma^2) \operatorname{erfc}\left(\frac{x_{\max}}{\sqrt{2}\sigma} \right). \tag{6}$$

The quality of the output signal from the quantizer is defined with the signal-to-quantization noise ratio (SQNR) [1]:

$$\mathrm{SQNR}(\sigma) = 10 \log_{10} \frac{\sigma^2}{D(\sigma)}. \tag{7}$$

2.2. The Golomb-Rice code

Quantization levels of the nonuniform μ-law companding quantizer are coded with the Golomb-Rice code [3]. This is a code with the variable-length codewords. To apply the

Golomb-Rice code, the range of the quantizer ($-x_{\max}$, $x_{\max}$) is divided into L segments. In each segment there are m quantization levels ($m/2$ in the positive part of the real axis and $m/2$ in the negative part). The parameter m has to be a power of two, i.e. $m = 2^k$, where k is an integer. The optimal value of the parameter k will be found in the designing process. It holds that $L \cdot m = N$. Let's $t_1, t_2, \ldots, t_{L-1}$ denote thresholds between segments in the positive part of the real axis. The first segment is placed in the interval $(-t_1, t_1)$; the j-th segment, $j = 2, \ldots L-1$, is placed in the interval $(-t_j, -t_{j-1}) \cup (t_{j-1}, t_j)$; the L-th segment is placed in the interval $(-x_{\max}, -t_{L-1}) \cup (t_{L-1}, x_{max})$. It holds that $t_j = x_{jm/2}$, $j = 1, \ldots, L-1$. Based on (3), it is obtained that:

$$t_j \equiv x_{jm/2} = \frac{x_{\max}}{\mu}((1+\mu)^{j/L} - 1), \quad j = 1, ..., L-1 \,. \tag{8}$$

Probabilities of segments are determined with the following expressions:

$$\begin{aligned} P_1(\sigma) &= 2\int_0^{t_1} p(x,\sigma)dx = \operatorname{erf}\left(\frac{t_1}{\sqrt{2}\sigma}\right); \\ P_j(\sigma) &= 2\int_{t_{j-1}}^{t_j} p(x,\sigma)dx = \operatorname{erf}\left(\frac{t_j}{\sqrt{2}\sigma}\right) - \operatorname{erf}\left(\frac{t_{j-1}}{\sqrt{2}\sigma}\right), \; j = 2, \ldots, L-1; . \\ P_L(\sigma) &= 2\int_{t_{L-1}}^{+\infty} p(x,\sigma)dx = \operatorname{erfc}\left(\frac{t_{L-1}}{\sqrt{2}\sigma}\right); \end{aligned} \tag{9}$$

The Golomb-Rice code works in the following way. Codewords for quantization levels within the j-th segment, $j = 1, \ldots, L-1$, have the following form: $\underbrace{11\ldots1}_{j-1}0\underbrace{x\ldots x}_{k}$. Zero after $(j-1)$ ones is inserted to fulfill the prefix rule, according to which no one codeword cannot be prefix of any other codeword. Codewords for quantization levels within the L-th segment have the form $\underbrace{11\ldots1}_{L-1}\underbrace{x\ldots x}_{k}$. For L-th segment there is no need to insert one zero after $(L-1)$ ones, since there is no any other segment where codewords begin with $(L-1)$ ones. The length of codewords for quantization levels within the j-th segment is $l_j = s_j + k$, where

$$s_j = \begin{cases} j, & j = 1, \ldots, L-1 \\ L-1, & j = L \end{cases}. \tag{10}$$

We can say that this coding principle belongs to the category of the hierarchical coding: firstly, the segment where quantization level belongs is coded with variable number of bits s_j; after that, the position of the quantization level within the j-th segment is coded with fixed number of bits k, using the natural binary code.

The decoding is very simple. When decoder receives a codeword, it counts consecutive ones before the first zero. Let p denote this number of ones. Then, this codeword corresponds to the $(p + 1)$-th segment. If $p < L - 1$, the decoder knows that the length of the received codeword is $p + 1 + k$. If $p = L - 1$, it means that this codeword corresponds to the L-th segment and in this case the decoder determines the length of the codeword as $L - 1 + k$.

When the segment is determined, the decoder calculates the position of the quantization level within this segment using the last *k* bits of the received codeword.

The bit-rate is calculated using the following expression:

$$R(\sigma) = \sum_{j=1}^{L} l_j P_j(\sigma) = \sum_{j=1}^{L} (s_j + k) P_j(\sigma) . \tag{11}$$

Based on (9) and (10), expression for the bit-rate becomes:

$$R(\sigma) = L + k - 1 - \sum_{j=1}^{L-2} \operatorname{erf}\left(\frac{x_{\max}}{\sqrt{2}\mu\sigma} \left((1+\mu)^{j/L} - 1 \right) \right) . \tag{12}$$

Since the Golomb-Rice code is used, the number of quantization levels *N* need not be a power of two (in contrast to the case when codes with fixed-length codewords are used, where the number of quantization levels has to be a power of two). This fact allows high flexibility of the design of the quantizer.

2.3. Forward adaptation

Forward adaptation of the companding μ-law quantizer whose levels are coded with the Golomb-Rice code is described in this subsection. The model consists of the input buffer, block for the variance calculation, log-uniform quantizer Q_α for the variance quantization, divider, the fixed nonuniform companding μ-law quantizer and the Golomb-Rice coder.

It is common to consider the variance in the logarithmic domain, defined as α [dB] = $10\log_{10}(\sigma^2/\sigma_0^2)$, where σ^2 is the variance of the signal and σ_0^2 is the referent variance. Without losing the generality, we take that $\sigma_0^2 = 1$. Let's $(-\Delta_\alpha, 0)$ [dB] denotes the range of the variance α in the logarithmic domain, which is of interest for some application. The aim of the forward adaptation is to provide almost constant SQNR in the range of variances $\alpha \in (-\Delta_\alpha, 0)$ [dB].

Forward adaptation works on the frame-by-frame basis. Frame of *M* samples of the input signal $(s_1,\ldots,s_M)$ are loaded into the input buffer. The block for the variance calculation calculates the variance of samples within the buffer as $\sigma^2 = (1/M)\cdot\sum_{q=1}^{M} s_q^2$. Then, the variance in the logarithmic domain is calculated as α [dB] = $10\log_{10}\sigma^2$. The quantizer Q_α performs the uniform quantization of the variance α in the logarithmic domain; hence this quantizer is called the log-uniform quantizer. Let N_g denote the number of levels of the quantizer Q_α. The range of variances in the logarithmic domain α is uniformly divided into N_g intervals. The thresholds of Q_α are $\hat{\alpha}_i = -\Delta_\alpha + i\delta_\alpha$, $i = 0,\ldots,N_g$, where $\delta_\alpha = \Delta_\alpha / N_g$ represents the quantization stepsize.

Let α belong to the interval $(\hat{\alpha}_{i-1}, \hat{\alpha}_i)$. Information about this is transmitted to the receiver as the side information (SI) with $(\log_2 N_g) / M$ bits. For $\alpha \in (\hat{\alpha}_{i-1}, \hat{\alpha}_i)$ the gain is calculated as $g_i = 10^{\hat{\alpha}_i/20}$. All samples $(s_1,\ldots,s_M)$ from the input buffer are divided with g_i. After that, these samples pass through the nonuniform μ-law companding quantizer. Output levels of this quantizer are coded with the Golomb-Rice code.

Functions $R(\alpha)$ and SQNR(α) are obtained from $R(\alpha)$ and SQNR(α), putting $\sigma = 10^{\alpha/20}$. Functions $R(\alpha)$ and SQNR(α) are periodic on the range $\alpha \in (-\Delta_\alpha,0)$ [dB] with the period δ_α. The average SQNR is calculated as:

$$\overline{\text{SQNR}} = \frac{1}{\delta_\alpha} \int_{-\delta_\alpha}^{0} \text{SQNR}(\alpha) d\alpha \, . \tag{13}$$

The average bit-rate is calculated as:

$$\overline{R} = \frac{1}{\delta_\alpha} \int_{-\delta_\alpha}^{0} R(\alpha) d\alpha + \frac{\log_2 N_g}{M} \, , \tag{14}$$

where the last term denotes the side information SI.

3. Design of the Forward Adaptive μ-Law Companding Quantizer with the Golomb-Rice Code for Speech Signal

In this section, the designing of the forward adaptive μ-law companding quantizer with the Golomb-Rice code for speech signal is described. Based on the G.712 standard [8], for speech signal SQNR has to be higher than 34 dB in the range of variances $\alpha \in (-40$ dB, 0 dB) i.e. for the speech signal we have that $\Delta_\alpha = 40$ dB. Since functions $R(\alpha)$ and SQNR(α) are periodic with the period δ_α, the design is done on the interval $\alpha \in (-\delta_\alpha,0)$ [dB]. The aim of the design is the minimization of the average bit-rate $\overline{R}$, with the condition that SQNR(α) has to be higher than 34 dB in the range $\alpha \in (-\delta_\alpha,0)$ [dB]. Since SQNR(α) is an increasing function on the range $\alpha \in (-\delta_\alpha,0)$ [dB], it is enough to satisfy the condition $\text{SQNR}(\alpha = -\delta_\alpha) > 34\,\text{dB}$.

Let us summarize that parameters N, N_g and M are known in advance, i.e. they are input parameters for the design process. The design of the forward adaptive quantizer is done by the minimization of $\overline{R}$ with the condition SQNR($\alpha = -\delta_\alpha$) > 34 dB. As a result of the design process we obtain parameters $x_{\max}$ and k, which completely define the μ-law companding quantizer and the Golomb-Rice coder.

For example, for $N = 256$, $N_g = 16$ and $M = 200$, it is obtained that $x_{\max} = 129.9$, $k = 5$, $\overline{R} = 6.66$ bps and $\overline{\text{SQNR}} = 34.46$ dB. For this case, SQNR in the wide range of variances $\alpha \in$(-40 dB, 0dB) is shown in Fig. 1. We can see that SQNR is almost constant and higher than 34 dB. Our model satisfies the G.712 standard with the bit-rate of 6.66 bps, providing the bit-rate decrease of 1.34 bps compared to quantizers defined with the G.711 standard [9]. Also, the decreasing of the bit-rate of 0.1 bps is achieved compared to the model presented in [10].

It is important to mention that in this case the forward adaptation is done not only to achieve high signal quality, but also to achieve compression. Namely, the forward adaptation allows us to increase $x_{\max}$, which leads to the decreasing of the bit-rate.

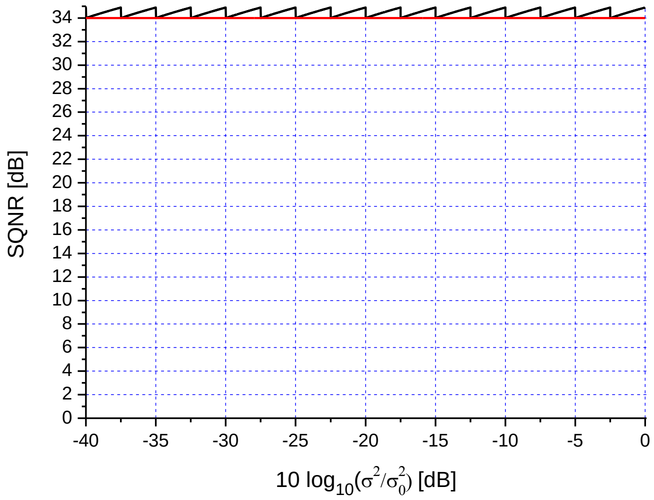

Fig. 1 SQNR in the wide range of variances

4. Conclusion

The design of the forward adaptive μ-law companding quantizer, whose levels are coded with the Golomb-Rice code, was presented in this paper. The model was developed for measurement signals with the Gaussian distribution, and applied for the speech signal. The joint design of the adaptive quantizer and the Golomb-Rice coder was presented, i.e. the adaptation was used for the decreasing of the bit-rate. The presented model satisfies the G.712 standard, achieving the decrease of the bit-rate of 1.34 bps compared to quantizers defined with the G.711 standard.

References

[1] N. S. Jayant and P. Noll, *Digital Coding of Waveforms: Principles and Applications to Speech and Video*. New Jersey: Prentice Hall, 1984, Chapters 4-5, pp. 115–251.

[2] J. Nikolic and Z. Peric, "Lloyd-Max algorithm implementation in speech coding algorithm based on forward adaptive technique", *Informatica*, vol. 19, no. 2, pp. 255-270, 2008.

[3] D. Salomon, Variable-Length Codes for Data Compression. Springer, 2007.

[4] Z. Peric, M. Dincic and M. Petkovic, "Design of a hybrid quantizer with variable length code", *Fundamenta Informaticae*, vol. 98, no. 2-3, pp. 233-256, 2010.

[5] M. Weinberger, G. Seroussi and G. Sapiro, "The LOCO-I lossless image compression algorithm: Principles and standardization into JPEG-LS", *IEEE Trans. Image Processing*, vol. 9, pp. 1309-1324, 2000.

[6] T. Liebchen, Y. Reznik, "MPEG-4 ALS: an emerging standard for lossless audio coding", Proc. Of Data Compression (DCC), *IEEE Computer Society*, pp. 439-448, 2004.

[7] Consultative Committee for Space Data Systems (CCSDS), "Lossless data Compression", blue book, issue 1, 1997.

[8] ITU-T Recommendation G.712, Transmission Performance Characteristics of Pulse Code Modulation (PCM), 1992.

[9] ITU-T Recommendation G.711, Pulse Code Modulation (PCM) of Voice Frequencies, 1972.

[10] Z. Peric, M. Dincic, D. Denic and A. Jocic, "Forward adaptive logarithmic quantizer with new lossless coding method for Laplacian source", *Wireless Personal Communications*, vol. 59, no. 4, pp. 625-641, 2011.