

TWO APPROACHES TO DESIGN OF FORWARD ADAPTIVE PIECEWISE UNIFORM SCALAR QUANTIZERS

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Abstract. *In this paper, two forward adaptive piecewise uniform scalar quantizers are proposed for high-quality quantization of speech signals modeled by the Laplacian probability density function. In designing both forward adaptive piecewise uniform scalar quantizers an equidistant support region partition is assumed and a distribution of the number of reproduction levels per segments is optimized. The proposed models differ in the approach of determining the reproduction levels. In particular, one model defines the reproduction levels as the cell centroids and the other one as the cell midpoints. We show that, in the high-resolution case, the proposed quantizers provide approximately the same performance being close to the one of the forward adaptive nonlinear scalar compandor with equal number of quantization levels.*

Key words: *cell centroid, cell midpoint, forward adaptation, Laplacian source, piecewise uniform scalar quantization*

1. INTRODUCTION

For a fixed number of quantization levels N , or equivalently a fixed resolution $R = \log_2 N$ [1]-[5], the reproduction levels and the partition regions or cells of a quantizer can be determined according to a different criteria. The primary goal of a quantizer designing is to divide the support region and to determine reproduction levels so that to obtain minimal possible distortion. An additional goal is usually to provide a straightforward approach to the design of the quantizer and to enable a very simple implementation procedure. A quantizer that has the support region consisting of several segments, each of which containing several quantization cells and reproduction levels corresponding to a uniform quantizer, is the piecewise uniform quantizer [1], [2]. The piecewise uniform scalar quantizers (PUSQs) are widely used in practice due to their simple design and also due to their simple encoding procedure, which does not require the full search of the quantizer code book.

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The prevailing international standard for digital telephony, G.711 standard, defines a symmetric piecewise linear scalar quantizer by 8 bits of resolution and $L = 8$ positive segments increased in length by a factor of 2 for each successive segments having 16 uniform cells [1], [6]. In particular, the G.711 quantizers based on a piecewise linear approximation to the A -law and μ -law compressor characteristics divide the support region into a $2L = 16$ unequal segments with equal number of uniform cells. Along with the support region partition, according to the mentioned piecewise linear compressor characteristics, there are some novel propositions of the support region partition, i.e. of the piecewise uniform distribution of the reproduction levels. For instance, the robustness conditions of the quantizer based on a piecewise linear approximation to the optimal compressor law have been analyzed in [7], [8]. Also, the piecewise uniform vector quantizer that considers an unequal number of cells within the segments has been proposed in [9]. However, in the conclusion of the paper [9], the authors have emphasized that the main drawback of their method is that they do not have a manner for deciding how to determine the segments into which to divide the support region of a piecewise uniform quantizer for an arbitrary signal distribution. This comment has motivated us to research the performance of the PUSQ that defines the equidistant support region partition and the optimized distribution of the cells (or reproduction levels) within such defined segments.

In this paper we propose two models of a PUSQ having the reproduction levels as midpoints or centroids on the corresponding cells, respectively. We actually propose the PUSQs composed of L uniform quantizers having equal support regions, but possibly different number of quantization cells. In order to provide an optimal manner of reproduction level distribution per segments, the granular distortion optimization is proposed, while such a constrained optimization problem is solved using the method of Lagrange multipliers [1], [2]. As the main trade off in scalar quantizer design is making the quantization step size large enough to accommodate the maximum peak-to-peak range of the input signal while keeping this step size small enough to minimize the quantizer distortion, the application of forward adaptation is often preferable [1]. Accordingly, we also provide performance analysis of the forward adaptive version of the proposed PUSQ models.

2. DESIGN OF PIECEWISE UNIFORM SCALAR QUANTIZERS

This section contains the short theory background of a PUSQ followed by the description of the novel PUSQ models. Additionally, the brief theory of the forward adaptive technique is presented and it is applied in designing the proposed PUSQs. A PUSQ quantizer partitions the quantizer support region into $2L$ quantization segments [1], [2]. Within each segment, the quantizer may be considered as a uniform one with the cell length that may differ from segment to segment. Each uniform quantizer, from the set of L quantizers composing the PUSQ, is designated to quantize the signals belonging to the corresponding segments so that, when the sample of the input signal to be quantized is within the i -th and i -th segment, the corresponding i -th uniform quantizer is then used.

In designing the novel PUSQs we assume the equidistant support region partition so that it holds

$$t_i^{\text{seg}} = i \frac{x_{\text{max}}}{L}, \quad i = 0, 1, \dots, L, \quad (1)$$

where $t_i^{\text{seg}}, i = 0, 1, \dots, L$ denote the segment thresholds and x_{max} denotes the support region threshold. We propose equal lengths of the segments, but possibly different number of reproduction levels within them. Due to the symmetry of the PUSQ we define only the positive segment thresholds. Each cell length of the considered PUSQs can be determined by

$$\Delta_i = \frac{t_i^{\text{seg}} - t_{i-1}^{\text{seg}}}{N_i} = \frac{x_{\text{max}}}{LN_i}, \quad i = 1, \dots, L, \quad (2)$$

where N_i is the reproduction level number within the corresponding i -th segment. Observe that index i indicates the cells within the i th segment $(t_{i-1}^{\text{seg}}, t_i^{\text{seg}}]$.

It is commonly accepted that the Laplacian probability density function (pdf) is a good approximation to the actual distribution of speech samples [1]-[5]. For the assumed Laplacian pdf

$$p(x) = \frac{\sqrt{2}}{2\sigma} \exp\left(-\frac{|x|\sqrt{2}}{\sigma}\right), \quad (3)$$

we can derive the following closed form expressions for $P_i, i = 1, \dots, L$ denoting the probability of belonging the input sample x of variance σ^2 to the i th segment

$$P_i = \int_{t_{i-1}^{\text{seg}}}^{t_i^{\text{seg}}} p(x) dx = \frac{1}{2} \left(\exp\left(-\frac{\sqrt{2}t_{i-1}^{\text{seg}}}{\sigma}\right) - \exp\left(-\frac{\sqrt{2}t_i^{\text{seg}}}{\sigma}\right) \right). \quad (4)$$

Granular and overload distortion of the PUSQ in the cell midpoint case is given by [1], [2]

$$D_g^{\text{m}} = 2 \sum_{i=1}^L \frac{\Delta_i^2}{12} P_i, \quad (5)$$

$$D_o^{\text{m}} = 2 \int_{x_{\text{max}}}^{\infty} (x - y_{L, N_L}^{\text{m}})^2 p(x) dx, \quad (6)$$

where $y_{L, N_L}^{\text{m}} = x_{\text{max}} - \Delta_L / 2$. Eqs (5) and (6) further specify signal to quantization noise ratio (SQNR) [1]-[5]

$$\text{SQNR} = 10 \log\left(\frac{\sigma^2}{D}\right) = 10 \log\left(\frac{\sigma^2}{D_g + D_o}\right). \quad (7)$$

The common approach to solving constrained optimization problems is based on the method of Lagrange multipliers [1], [2]. With this method, the constrained optimization problem:

$$\min_{N_i} \{D_g\} \text{ subject to the constraint } 2 \sum_{j=1}^L N_j = N, \quad (8)$$

is converted into an unconstrained one:

$$J = D_g + \lambda \sum_{j=1}^L N_j, \quad \frac{\partial J}{\partial N_i} = 0, \quad i = 1, \dots, L, \quad (9)$$

$$\frac{\partial J}{\partial N_i} = \frac{\partial \left(\frac{x_{\max}^2}{6L^2} \sum_{j=1}^L \frac{P_j}{N_j^2} \right)}{\partial N_i} + \frac{\partial \left(\lambda \sum_{j=1}^L N_j \right)}{\partial N_i} = 0, \quad i = 1, \dots, L, \quad (10)$$

where λ is a Lagrange multiplier. Taking that the probabilities P_i , $i = 1, \dots, L$ are not a function of N_i , we can simply obtain the following expression

$$N_i = \sqrt[3]{\frac{P_i x_{\max}^2}{3\lambda L^2}}, \quad i = 1, \dots, L, \quad (11)$$

which further, in combination with the constraint given by (8) yield the closed form expression for the optimal number of reproduction levels per segments

$$N_i = \frac{NP_i^{1/3}}{2 \sum_{i=1}^L P_i^{1/3}}, \quad i = 1, \dots, L. \quad (12)$$

If a current amplitude value of the input signal falls in the j th cell within the i th segment ($t_{i,j-1}, t_{i,j}$]

$$t_{i,j} = t_{i-1}^{\text{seg}} + j\Delta_i, \quad i = 1, \dots, L, \quad j = 0, \dots, N_i, \quad (13)$$

the quantization rule provides its coping onto the near allowed value $y_{i,j}$ defined by the code book of the PUSQs that in the cell midpoint case and the cell centroid case are respectively given by [10]

$$y_{i,j}^m = t_{i-1}^{\text{seg}} + \frac{(2j-1)}{2} \Delta_i, \quad i = 1, \dots, L, \quad j = 1, \dots, N_i, \quad (14)$$

$$y_{i,j}^c = \frac{\int_{t_{i,j-1}}^{t_{i,j}} xp(x)dx}{\int_{t_{i,j-1}}^{t_{i,j}} p(x)dx}, \quad i = 1, \dots, L, \quad j = 1, \dots, N_i. \quad (15)$$

Note that in the many of the given formulas the index m or c is omitted with the goal to avoid copying the same formulas that holds for the both PUSQ models. Eventually, in the case when the reproduction levels are cell centroids, the granular and the overload distortion of the PUSQ are respectively given by

$$D_g^c = 2 \sum_{i=1}^L \sum_{j=1}^{N_i} \int_{t_{i,j-1}}^{t_{i,j}} (x - y_{i,j}^c)^2 p(x) dx, \quad (16)$$

$$D_o^c = 2 \int_{x_{\max}}^{\infty} (x - y_{L,N_L}^c)^2 p(x) dx . \quad (17)$$

In this paper, we apply forward adaptive scheme, shown in Fig. 1, composed of a buffer, an adaptive N -level PUSQ, the gain estimator and the N_g -level scalar quantizer for gain quantizing (SQ_{gq}). Particularly, in this paper, the log-uniform scalar quantizer for gain quantizing rather than the uniform scalar quantizer is implemented, since we have recently demonstrated that it could provide higher SQNR [11]. The design procedure of the proposed forward adaptive PUSQ is composed of a few steps. Initially, the design of non adaptive (fixed) PUSQ for the reference variance ($\sigma_{\text{ref}}^2=1$) is based on determining the thresholds $t_{i,j}$, $i = 1, \dots, L$, $j = 0, \dots, N_i$ (3) and the reproduction levels $y_{i,j}$ $i = 1, \dots, L$, $j = 1, \dots, N_i$ (4) or (5). Buffering a frame after frame enables an estimation of the gain, defined as $g = \sigma / \sigma_{\text{ref}}$, i.e. as a ratio of the squared root of the current frame variance and squared root of the reference variance. The estimated gain is then quantized by using the N_g -level log-uniform scalar quantizer SQ_{gq}

$$20 \log(\hat{g} = \hat{g}_k) = 20 \log(\sigma_{\min}) + (2k-1) \frac{\Delta^u}{2}, k = 1, \dots, N_g, \quad (18)$$

where the variance range of the input signal in decibels $|B| = 20 \log(\sigma_{\max}/\sigma_{\min})$ is divided into N_g cells having equal lengths $\Delta^u = |B| / N_g$. Eventually, the determining decision thresholds and the reproduction levels of the adaptive PUSQ is performed by multiplying the appropriate thresholds and the reproduction levels of the non adaptive PUSQ with the quantized gain \hat{g}

$$t_{i,j}^a = \hat{g} t_{i,j}, y_{i,j}^a = \hat{g} y_{i,j}, i = 1, \dots, L, j = 1, \dots, N_i. \quad (19)$$

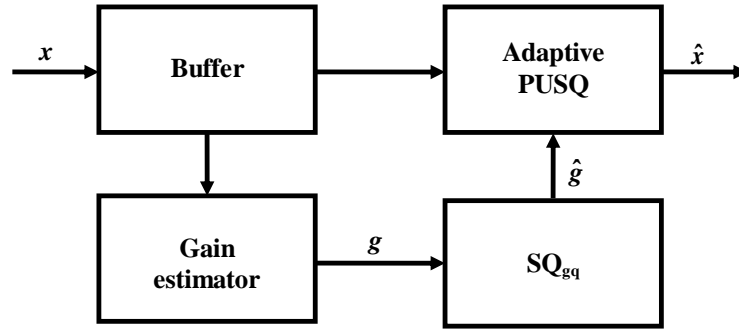


Fig. 1 Forward adaptive quantization scheme

3. RESULTS AND CONCLUSIONS

The performances that we have ascertained by applying the considered forward adaptive PUSQs in quantization of signals having Laplacian pdf and a wide variance range ($|B|=40$ dB) are presented in Figs. 2 and 3. Since the support region width has a great in-

fluence on the quantizer design and, consequently, on its performance [12], we have started our research with the support region optimization of the fixed PUSQs having $L = 8$ positive segments and $N = 128$ quantization levels. By designing the PUSQs for the unit variance and different support regions, we have numerically determined the values of the optimal support region thresholds that minimize distortions of fixed PUSQ.

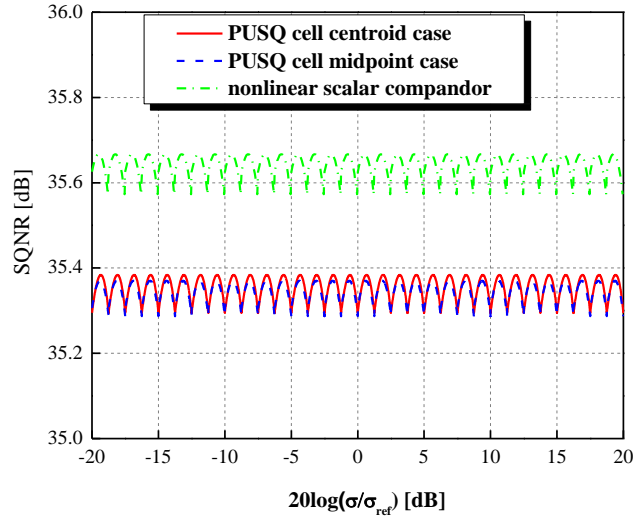


Fig. 2 SQNR in the wide variance range

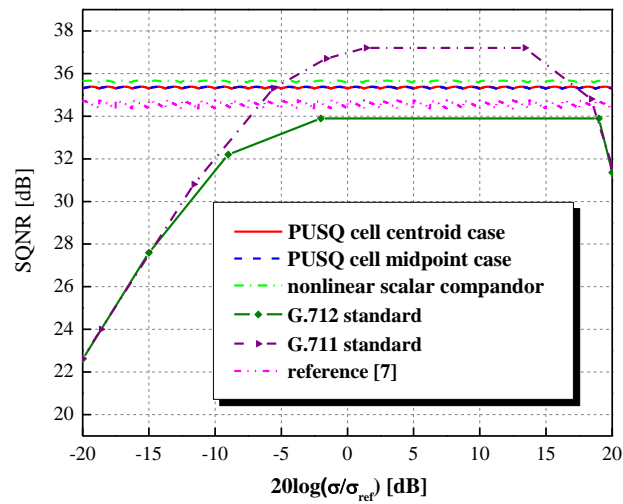


Fig. 3 Comparison of the SQNR characteristics of the PUSQs with the characteristics defined by the G.711 and G.712 standard

In particular, we have determined that these thresholds amount to $x_{\max}^m = 9$ and $x_{\max}^c = 9.6$, in the cell midpoint and the cell centroid case. In the same manner, we have optimized the support region threshold of the asymptotically optimal nonlinear compandor having the same number of quantization levels and we have determined that $x_{\max}^{nc} = 9.5$. For so obtained support region thresholds, we have determined the appropriate SQNR characteristics of the forward adaptive PUSQs and the forward adaptive nonlinear compandor (see Figs. 2 and 3). We have considered the case when the adaptive PUSQs and the log-uniform SQ_{gq} have $N = 128$, $N_g = 32$ quantization levels, respectively. We have assumed $L = 8$ segments, as in the case of the G.711 quantizer design [1], [6]. From Figs. 2 and 3 one can conclude that, in the considered high-resolution domain, the proposed PUSQs provides approximately the same performance, where a little bit higher values of SQNR are obtained with the PUSQ designed for the cell centroid case than in the cell midpoint case. Moreover, one can notice that in both cases of the proposed forward adaptive PUSQ design, the obtained SQNR characteristics are near the one of the forward adaptive asymptotically optimal compandor. It is obvious that in both cases of adaptive PUSQs having $N = 128$ quantization levels, the proposed forward adaptive PUSQs can not completely overreach G.711 Recommendation. However, for the proposed models, this is not an issue in the case of adaptive PUSQs having $N = 256$ quantization levels, as it was assumed in the case of the G.711 quantizer design. In such a case, the proposed forward adaptive quantizers provide 6 dB higher level of SQNR and, therefore, a higher level of SQNR than the one of the G.711 quantizer. Additionally, we have ascertained that the proposed PUSQs provide the gain in the average SQNR of about 0.8 dB over the quantizer designed according to the piecewise linear approximation to the optimal compressor law [7]. Finally, since we have ascertained that the proposed quantizers satisfy the G.712 Recommendation [13] in the considered variance range, one can believe that they will find practical implementation in the high-quality quantization of signals, which, as well as speech signals, have statistics modeled by the Laplacian pdf.

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