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THERMODYNAMIC MODEL OF THE PROTECTOR COOLING SYSTEM WITH APPLICATIONS

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Abstract. *This paper presents thermodynamic model of the system for rubber strip (protector) cooling. This model is used for determining the number of cooling system cascades, and rubber contraction coefficient, important parameters in mechanical model of the system which is the starting point of designing control system for rubber strip cooling. The correlation between the working velocity and rubber strip dimension, as well as the relation for the cooling water flow per cascade is also given.*

Key words: *transportation system, cascade, heat exchanger, contraction coefficient, stochastic parameter, working velocity*

1. INTRODUCTION

Rubber strip cooling systems were developed more than thirty years ago and they have been applied ever since in tire industry [1]. These systems are very complex, difficult to control, because of the large number of cascades and many stochastic parameters (rubber strip contraction coefficients, some time constants in cascades) [2, 3]. There are several thousand similar systems all over the world. Mechanical model of the system for protector (external part of tire) cooling was first made three decades ago [4-7]. Parameters of this model depend on temperatures in specific cascades. Values of these parameters are mostly determined experimentally, either in laboratory or directly on the designed system. In this paper, thermodynamic model of the system is designed. This model allows us to calculate parameters of mechanical model of the system. This mechanic-dynamical model is necessary for the design of control system for rubber strip cooling process. The method for determining parameters of the model is described in the paper. First, a well known mechanical model is presented in order to indicate the problems which occur during

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system control. Then thermodynamic model is derived. At the end, method for calculating necessary parameters and values for design the system control is described.

2. MATHEMATICAL MODEL OF THE PROTECTOR COOLING SYSTEM

The length change of the rubber strip between two transporters is given by the following equations:

$$\begin{aligned}
&\frac{dl_i}{dt} = V_{R,i-1}^{(2)} - V_{R,i}^{(1)}, i = 1,2,...,n \\
&V_{R,i-1}^{(2)} = V_{i-1}, \quad V_{R,i}^{(1)} = \frac{1}{\mu_i} V_i \\
&\mu_i = \frac{V_{R,i}^{(1)} - V_{R,i}^{(2)}}{L} \\
&\frac{dl_i}{dt} = V_{i-1} - \frac{1}{\mu_i} V_i \\
&\Delta l_i = \frac{1}{s}\left(V_{i-1} - \frac{1}{\mu_i} V_i \right)
\end{aligned} \tag{1}$$

where the used notation represents follows:

i – cascade ordinal number,
l_i – the length of rubber strip between i -th and $i+1$ -th transporter,
$V_{R,i}^{(1)}$ – rubber velocity at the beginning of the i -th transporter,
$V_{R,i}^{(2)}$ – rubber velocity at the end of the i -th transporter,
Δl_i – length change of rubber strip between two consecutive transporters,
V_i – the velocity of the i-th transporter,
L – transporter length,
μ_i – the rubber contraction coefficient for the i -th transporter.

Characteristic of measurer (potentiometer) for rubber between transporters is:

$$\beta_i = \phi(\Delta l_i), \tag{2}$$

where β_i is potentiometer angle. Potentiometer voltage is:

$$u_i = K_P \beta_i , \tag{3}$$

where K_P is the potentiometer coefficient [V/rad].

The block diagram of the derived model of cascade-connected system is shown in Fig. 1.

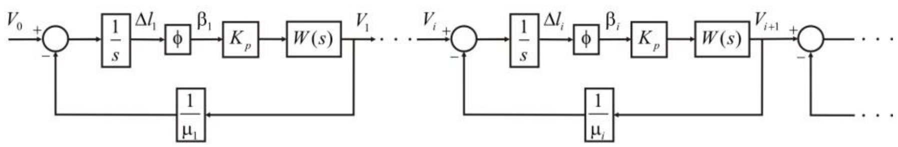

Fig. 1 Block diagram of the cascade-connected system

Transfer function $W(s)$ of the drive motor with load in Fig.1 has the following form:

$$W(s) = \frac{V_i(s)}{u_i(s)} = \frac{k}{s^2 + as + b}, , \tag{4}$$

where k is gain, a and b are some functions of time constants [5].

Problems during designing control systems which can appear are:

- Determine number of cascades necessary for the rubber to cool down, i.e. become relaxed,
- Determine rubber contraction coefficients in specific cascades.

In order to calculate these parameters, thermodynamic model of the system for rubber cooling can be designed.

3. Thermodynamic Model of the System

The system for the rubber strip cooling, used for forming protector is, from a thermodynamic point of view, multiple heat exchanger [8]. This heat exchanger consists of several cascades. Rubber strip is formed in extruder, then squeezed out and then using the existing mold, the defined rubber strip profile can be produced. In that moment, rubber temperature is around 136°C. Rubber strip comes to the first cascade of the cooling system, i.e. the first transporter then moves from the first to second one, and so on until it finally comes to the last cascade. The number of cascades depends on the rubber strip thickness and rubber strip velocity. Each transporter consists of transporting strip, in the form of a metal grid, which carries the rubber strip. Both strips run through the cooling water. Cooling water is injected at one part of the metal tub and discharged at the opposite one. While the rubber strip moves through the water, heat is being transferred from rubber to water. One part of the heat is transferred to metal tub by conduction, second to air by radiation and the last one by convection to the rubber in the next cascade, i.e. to the next transporter. In this way, temperature is decreasing, while rubber moves from one to another transporter. Rubber is cooled down to the desired, predefined temperature, where the rubber is completely relaxed (no inner strains). The reason is that, at the output of the system rubber strip must be cut into precisely determined pieces equal to the volume of the tire torus. If rubber is not fully relaxed, after winding the strip around torus (an inner part of the tire), rubber can contract afterwards which causes tire imbalance. Such tire does not have the required quality and cannot be used.

Deriving thermodynamic model starts from elementary equations for stationary heat transfer by conduction and convection [9].

The amount of heat transferred by conduction is described by the Fourier's heat conduction law:

$$H = -KA\frac{d\theta}{dx}. \tag{5}$$

The amount of heat transferred by convection is:

$$H = Ah(\theta_2 - \theta_1). \tag{6}$$

The amount of heat transferred by fluid:

$$H = \rho c Q \theta \,. \tag{7}$$

Heat balance equation has the following form:

$$mc\frac{d\theta}{dt} = \sum_j H_j \;. \tag{8}$$

When we apply the above relations to the unit length of the rubber strip we obtain a thermodynamic model for each cascade of the system:

$$\begin{aligned} &m_1 c_R \frac{d\theta_{xi}}{dt} = -Ah(\theta_{xi} - \theta_V) \\ &dt = \frac{1}{V}dx \\ &\frac{d\theta_{xi}}{dx} = -\frac{K_1}{V}(\theta_{xi} - \theta_V), \quad i = 1,2,..n \end{aligned} \tag{9}$$

with the following notation:

$K_1 = \dfrac{Ah}{m_1 c_R}$ – coefficient which depends only on rubber parameters,

A – one meter of rubber strip area, one meter long,
h – surface conductivity coefficient from rubber to water,
m_1 – mass of strip, one meter long,
c_R – specific heat of rubber,
V – transporter velocity.

Appropriate temperatures are:
θ_W – water temperature in the system,
θ_0 – rubber temperature at the system input,
θ_i – rubber temperature at the end of the *i*-th transporter,
θ_{xi} – rubber temperature in the *i*-th transporter on *x*(*m*) from the beginning.

Relation for θ_{xi} can be obtained from (9):

$$\theta_{xi} = \theta_W + c\exp\left(-\frac{K_1}{V}x\right). \tag{10}$$

At the transporter beginning ($x = 0$); $\theta_{xi} = \theta_{i-1}$. Using this initial condition, we can determine integration constant and obtain:

$$\theta_{xi} = \theta_W + (\theta_{i-1} - \theta_W)\exp\left(-\frac{K_1}{V}x\right). \tag{11}$$

For $x = L$:

$$\theta_{xi} = \theta_W + (\theta_{i-1} - \theta_W)\exp\left(-\frac{K_1}{V}L\right), \;\; 1,2,..n\,. \tag{12}$$

Equation (12) represents recurrent relation for finding temperatures at cascades outputs. Relation (12) allows us to determine rubber temperatures at the end of each cascade and temperature at the system output as the most important value. Successively substituting, we obtain equation system:

$$
\begin{aligned}
&\theta_1 = \theta_W + (\theta_0 - \theta_W)\exp\left(-\frac{K_1}{V}L\right),\\
&\theta_2 = \theta_W + (\theta_1 - \theta_W)\exp\left(-\frac{K_1}{V}L\right) = \theta_W + (\theta_0 - \theta_W)\exp\left(-2\frac{K_1}{V}L\right),\\
&.\\
&.\\
&.\\
&\theta_i = \theta_W + (\theta_0 - \theta_W)\exp\left(-i\frac{K_1}{V}L\right).
\end{aligned}
\tag{13}
$$

Huge problem occurs in determining coefficient K_1 because it is difficult to analytically determine h. In practice, that is the reason why h is usually experimentally obtained. The advantage of this method is that the experiment can be done in laboratory, before designing the system.

3.1. Determining the number of system cascades

If we want to keep rubber temperature at the system output in the desired limits:

$$\left|\theta_i - \theta_W\right| \le \varepsilon \,, \tag{14}$$

(ε – tolerance), minimal number of transporters can be obtained from (13):

$$N = \left\lceil \frac{V}{K_1 L}\ln\frac{\theta_0 - \theta_W}{\varepsilon} \right\rceil. \tag{15}$$

Note 1: Symbol $\lceil x \rceil$ represents rounding up. Relation (15) is valid when all cascades are the same. In the case when cascade are different, transporter number is determined for each system in particular, using the relations above.

This method for determining number of cascades is applied for three already realized systems. The necessary numbers of cascade for three different types of tires are given in Table 1.

Table 1 Number of cascades for different realized systems

Tire factory	Tire type	Number of cascades N
"Tigar-Michelline" Pirot	diagonal	5
"Tigar-Michelline" Pirot	radial	7
Tire Factory Ruma	tractor	13

3.2. Determining rubber contraction coefficients

Relation (11) can be applied to calculate rubber contraction coefficients of certain cascades. Let ΔL_i denote strip contraction (length L) while running through i -th cascade. Then, we have:

$$\Delta L_i = K_2 \left(\theta_{i-1} - \theta_i\right) L, \tag{16}$$

where K_2 is technological parameter depending on rubber type and experimentally determined.

After replacing (13) into (16) we obtain:

$$\begin{aligned} \Delta L_i &= K_2(\theta_0 - \theta_V)\left(\exp\left(-(i-1)\frac{K_2 L}{V} - \exp\left(-i\frac{K_1 L}{V} \right) \right) \right) L \\ \Delta L_i &= K_2 L (\exp\frac{K_1 L}{V} - 1)(\theta_0 - \theta_V) \exp\left(-i\frac{K_1 L}{V} \right). \end{aligned} \tag{17}$$

Finally, the relation for the rubber contraction coefficient has the following form:

$$\mu_i = \frac{L - \Delta L_i}{L} = 1 - K_2 \exp\left(-i\frac{K_1 L}{V} \right)\left(\exp\frac{K_1 L}{V} - 1 \right)(\theta_0 - \theta_W), \tag{18}$$

i.e.:

$$\mu_i = 1 - \alpha_i(\theta_0 - \theta_W), \; i = 1, 2, ..., n, \tag{19}$$

where $\alpha_i = K_2 \exp\left(-i\frac{K_1 L}{V} \right) \exp\left(\frac{K_1 L}{V} - 1 \right)$.

This part of equation doesn't depend on temperature but only on system parameters. As required $|\Delta\mu_i| \le \varepsilon$, sufficient condition for system control is to keep $\theta_0 - \theta_V$ constant. In practice, variance of this difference should be as little as possible. Parameters μ_i are complex functions of system parameters and cascade ordinal number. Until now, μ_i has been obtained experimentally. Using (19) contraction coefficients are determined in advance, before designing the control system. Coefficient μ_i is the smallest in the first cascade and the largest in the last one.

Relation (19) is experimentally verified in the already realized systems as well as done for determining cascade number. In the case of the largest realized system in Serbia which consists of 13 cascades, the obtained values for contraction coefficients per each cascade are given in Table 2.

Table 2 Contraction coefficients in the system with 13 cascades

i	1	2	3	4	5	6	7	8	9	10	11	12	13
μ_i	0.861	0.903	0.925	0.941	0.955	0.968	0.979	0.984	0.988	0.992	0.995	0.998	0.999

3.3. Correlation between the working velocity and the rubber strip dimension

If we want to keep rubber temperature at the system output in the desired limits regardless of rubber strip dimensions, according to (9), the ratio V / K_1, i.e., V_{m1} / A must be kept constant. When we take into consideration that strip profiles for any size are geometrically similar ($A / d = const.$, d – strip width) we finally obtain:

$$\frac{Vm_1}{d} = const. \tag{20}$$

This relation can be used for determining transport velocity during cooling of different dimensions strip. The largest velocities are when the rubber strip has the smallest dimensions and opposite.

3.4. The flow of cooling water

Cooling water flow in cascades can be determined in the following way. Heat balance of one cascade per unit time is:

$$m_1 c_R V \theta_{i-1} - m_1 c_R V \theta_i + Q_i c_W \theta_{W_i} - Q_i c_W \theta_{W_{i-1}} = 0\,, \tag{21}$$

i.e.:

$$Q_i = \frac{m_1 c_R V(\theta_{i-1} - \theta_i)}{c_W(\theta_{W_{i-1}} - \theta_{W_i})}\,. \tag{22}$$

The relation above represents water flow for i -th transporter. It is an approximated relation because heat radiation to environment is disregarded. Using (22), we obtain slightly higher values of Q_i than needed, but that is recommended for designing system control.

4. Conclusion

Thermodynamic model of protector cooling system is presented in this paper. The system represents multiple cascade type heat exchanger (series of cascade-connected transporters). The relations of thermodynamic model allow us to calculate in advance some system parameters and values necessary for a successful system control design. Relations that provide determining necessary number of cascades of the given system and values for rubber contraction coefficients were derived. The obtained relations were verified on the already realized systems. Thermodynamic model also allow us to determine transporting strip velocities depending on tire strip dimensions (width and thickness), and cooling water flow. Also, using the described method, velocity of water flow through transporters and referent temperature (in the case of automated temperature control) could be calculated.

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