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2D CONSTELLATION DESIGN BASED ON GEOMETRIC Z^2 LATTICE AND POLAR QUANTIZATION OF CIRCULARLY SYMMETRIC SOURCES

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Abstract. *In this paper we propose 2D constellation design based on geometric lattice for circularly symmetric sources as it allows easier performance evaluation. Geometric Z^2 lattice quantization of Gaussian source is analyzed. Constellation design provides lower energy for the same error probability. It enables more efficient data transmission and energy saving for wireless systems. The exact error probability is also determined.*

Key words: *constellation design, polar quantization, geometric lattice*

1. Introduction

Quantization is a necessary step in the process of signal digitalization and should be performed in such a way so that quantization error affects the signal reconstruction as little as possible. Different methods for vector quantization can be used for signal constellation design but its efficiency strongly depends on an appropriate choice of a vector set (codebook). Such codebook must minimize the mean square error (distortion). Geometric Z^2 lattice quantization can be used in all applications where polar quantization is in use: syntetic aperture radars (SAR) imaging systems, interferometric and polarimetric applications, QAM and M-QAM constellation design. Multipoint concentric circle mapping may be used to reduce peak-to-average-power ratio (PAPR) of OFDM signal without side information [1].

Geometric Z^2 lattice quantization is simpler than polar quantization and allows easier performance evaluation. We consider quantization of circularly symmetric sources and design constellation of geometric Z^2 lattice type neglecting the edge effect. Circularly symmetric sources are characterized by contours of a constant probability density function which are circles in the two-dimensional space. The support partition of geometric quantizers is defined by Gaussian source geometry. In order to provide better following of the statistical characteristics of a signal a geometric approach of the quantizer design is ap-

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plied. The support region is partitioned into regions with boundaries which represent constant input vector PDFs. For the two dimensional distribution of independent Gaussian variables support regions are concentric rings.

The lattice cell length for each region is obtained on the base of minimum distortion criterion. We use results from [2] where the number of points in each region is obtained by optimizing the granular distortion.

The main goal of this paper is to design a simpler signal constellation than iterative polar quantization (IPQ) but with similar characteristics.

Constellation design should provide lower energy for the same probability of error. It enables more efficient data transmission and energy saving for wireless systems. Decoding is performed by determining the data belonging ring and by the use of minimum distance method.

2. EXACT ERROR PROBABLITY DETERMINATION FOR 2D SIGNAL CONSTELLATION AND POLAR QUANTIZATION OF CIRCULARLY SYMMETRIC SOURCES

In polar quantization a two dimensional random vector is quantized in terms of magnitude and phase which is preferable specially for sources with circulary symmetric densities. Consider a pair of independent identically distributed Gaussian random variables (x_1, x_2) with zero mean value and variance σ_n^2. The joint pdf of Gaussian random variables (x_1, x_2) is a product of individual pdfs [2-5]

$$p(x_1,x_2)=\frac{1}{2\pi\sigma_n^2}e^{-\frac{(x_1^2+x_2^2)}{2\sigma_n^2}},-\infty\le x_1,x_2<\infty \tag{1}$$

We want to quantize variables by independently quantizing the random variables (x_1, x_2) into polar coordinates (r,ϕ) where phase is assumed to be uniform on [0,2π].

$$\begin{aligned} r &= \sqrt{x_1^2+x_2^2} \\ \phi &= \tan^{-1}(x_2/x_1) \end{aligned} \tag{2}$$

The magnitude r is Rayleigh distributed on $[0,\infty]$ with density

$$p(r)=\frac{r}{2\pi\sigma_s^2}e^{-\frac{r^2}{2\sigma_s^2}} \tag{3}$$

while the input vector phase ϕ has uniform distribution on interval [0,2π].

Quantization is performed by first partitioning the region (circle with maximum radius r_{max}) into L concentric rings. Each ring has constant probability density function which is very important condition for the geometric vector quantizer design since it represents the region boundary. The i-th region is a ring having circles of radii r_{i-1} and r_i as boundaries. Region R_i has area

$$S_i=(r_i^2-r_{i-1}^2)\pi,\quad i=1,\ldots,L \tag{4}$$

where L is the number of regions.

The amplitude probability density function remains constant inside the ring after the partition and is given by the following approximation [2,5]

$$f_i = r_i e^{-\frac{r_i^2}{2}}, \quad i = 1,\dots,L \tag{5}$$

where

$$r_i = \frac{i \cdot r_{\max}}{L} \tag{6}$$

$r_{\max}$ represent radius of the support region and its values are taken from paper [6].

The second step in quantization is partitioning each ring into subregions of lattice type. We propose 2D lattice with rectangular cells (Fig.1) where each ring is divided into N_i rectangular subcells Δ_i. N_i should be determined by optimizing granular distortion subject to the constraint given by

$$\sum_{i=1}^{L} N_i = N \tag{7}$$

where the number of points for quantizer N is obtained from the following relation and particular code rate R or the vector quantizer rate is defined by [1]

$$R = \frac{1}{2}\log_2 N \Rightarrow N = 2^{2R} \tag{8}$$

Taking into account that the analysis is asymptotic, the relation between S_i and Δ_i can be written as

$$\Delta_i = \left(\frac{S_i}{N_i}\right)^{\frac{1}{2}}, i = 1,\dots,L \tag{9}$$

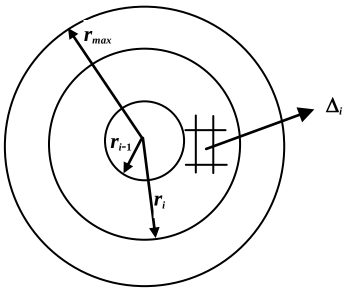

Fig. 1 2D constellation design

Consider the problem of detecting one of L signals in additive white Gaussian noise. The error probability of receiver can be approximated as a sum of probabilities

$$P_e = \sum_{i=1}^{L} P_i P_{ei}, \quad i = 1,\dots,L \tag{10}$$

where P_i is the probability that signal belongs to the i-th ring and is easily calculated as

$$P_i = \int_{r_{i-1}}^{r_i} r e^{-\frac{r^2}{2}} dr = e^{-\frac{r_{i-1}^2}{2}} - e^{-\frac{r_i^2}{2}}, \qquad i = 1,\dots,L \tag{11}$$

P_{ei} is the probability of error for rectangular subcell Δ_i and can be calculated as

$$\begin{aligned} P_{ei} &= 4\int_{\frac{\Delta_i}{2}}^{\infty}\int_{\frac{\Delta_i}{2}}^{\infty} \frac{1}{2\pi\sigma_n^2} e^{-\frac{x^2+y^2}{2\sigma_n^2}} dxdy + 4\int_{\frac{\Delta_i}{2}}^{\infty}\int_{-\frac{\Delta_i}{2}}^{\frac{\Delta_i}{2}} \frac{1}{2\pi\sigma_n^2} e^{-\frac{x^2+y^2}{2\sigma_n^2}} dxdy \\ &= 1 - erf^{\,2}\left(\frac{\Delta_i}{\sqrt{2}\sigma_n}\right) \end{aligned} \tag{12}$$

where

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

Signal to quantization noise ratio is defined as

$$SQNR = 10\log\frac{\sigma_s^2}{\sigma_n^2} \tag{13}$$

where we assume $\sigma_s^2 = 1$. Now we can calculate N_i as

$$N_i = \frac{N \cdot f_i^{\frac{1}{2}} \cdot S_i}{\sum_{k=1}^{L} f_k^{\frac{1}{2}} \cdot S_k} \tag{14}$$

Finally, the error probability per symbol (SER) is calculated from (12) for different bit rates and numerical results are shown in Fig.2.

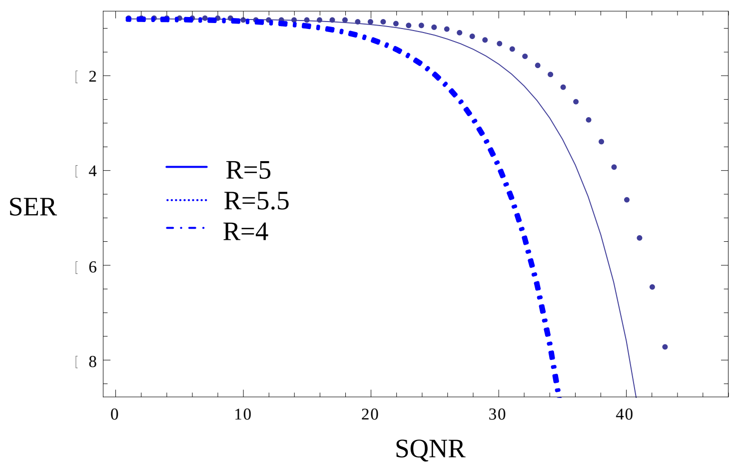

Fig. 2 SER versus SQNR

In Fig. 3, the comparison of the error rate per symbol is made for constellation design proposed in this paper with the results obtained by employing uniform 2D quantization. It can be easily seen that 2D constellation design based on geometric Z^2 lattice and polar quantization requires lower energy for the same error probability per symbol. Thus, the proposed method for constellation design is more efficient and much simpler than iterative polar quantization [8].

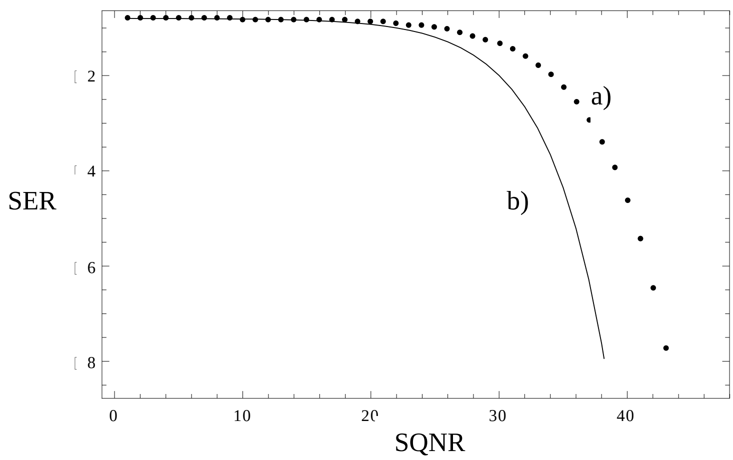

Fig. 3 SER versus SQNR for a) uniform 2D quantization and
b) 2D constellation design based on geometric Z^2 lattice and polar quantization.

2. CONCLUSION

The simple method for constellation design based on geometric Z^2 lattice and polar quantization of circularly symmetric sources is presented in this paper. Geometric Z^2 lattice quantization provides simpler design and performance evaluation. The error probability per symbol is numerically obtained for different bit rates. The proposed constellation diagram design requires lower energy for the same error probability which was the main design challenge.

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