FACTA UNIVERSITATIS
Ser: **Elec. Energ.** Vol. 26, N° 1, April 2013, pp. 31 - 46
DOI: 10.2298/FUEE1301031P

ANALYSIS OF FINITE TOLERANCE EFFECT OF CRITICAL POLE ON CHARACTERISTICS OF SELECTIVE RC ACTIVE SPECIAL GEGENBAUER FILTERS

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Abstract. *A precise analytical method for finding the explicit expression for the characteristic function of special Gegenbauer filters applicable to the design of RC active filters is suggested in this paper. The adverse parasite effects of limited finite gain-bandwidth product of operational amplifiers are decreased by using filters with the low pass-band attenuation. The new class of continual filter functions generated by analytical method by extremal Christoffel-Darboux formula for orthogonal Gegenbauer polynomials has two parameters. One is the filter order, n, and the second one is real free parameter, v, which provides a wide range of the amplitude responses. In this paper, a detailed analysis of attenuation and insertion loss in the bandwidth and around the stop-band cut-off frequency, ω_{cs}, are carried out using 3D plots and using examples of the effect of finite tolerance of quality factor module, Q, of critical conjugate-complex poles of considered RC active filter functions.*

Keywords: Christoffel-Darboux formula, orthogonal Gegenbauer polynomials, RC active Gegenbauer filter, frequency response analysis

1. INTRODUCTION

CHRISTOFFEL-DARBOUX formula, [1,2], can be efficiently implemented in filter function synthesis in continuous domain, as described by analytical method in works [3-6]. Related results in application of the Christoffel-Darboux formula in design of one-dimensional z domain multiplierless linear-phase FIR filters and 2D FIR filter have also been reported in works [7-12]. Popular one dimensional multiplierless CIC filters are described in literature [13,14]. Syntheses of the RC active filters are described in literature [15,16].

Accepted January 22, 2013
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Acknowledgement: The work presented here was supported by the Serbian Ministry of Education and Science (Projects TR32032 and TR32052).

Due to the effect of element tolerances and the limited gain-bandwidth product of operational amplifiers used in passive and active filter realizations, the pass-band response of the filter departs from the theoretical optimum much more in the frequency range near the band edge frequency than in the rest of the pass-band. An established technique to increase the element tolerances in the design of selective passive filters or to decrease the adverse effect of limited gain-bandwidth product of operational amplifiers uses transfer function with a special type of magnitude response for which the pass-band ripple amplitude decreases with the frequency increment, [17,18]. Some designs of low-sensitivity RC active filters are considered in literature [19-23]. Helpful literature for filter theory is given in literature [24-30].

In this paper it is given illustration of the examples of the filter, of recently introduced by analytical method of approximation filter function in work [4], for specific numerical values of the real free parameter, v, and even filter orders, $n = 6$, $n = 8$ and $n = 10$.

These filter functions depend on one variable parameter v, enabling the last attenuation maximum in the pass-band to be adjusted by numerical computation, as shown in 3D plots frequency response analysis examples. The RC ladder nature of the resulting filter circuits as described in the work [4], reduced the sensitivity to component tolerances sufficiently to eliminate the need for tuning. Approximation filter functions constructed in this work are such that have small pass-band magnitude ripples. Consequently, the sensitivity performance of the resulting network has advantage of improving the sensitivity performance. To avoid the need for filter tuning, filters of medium to low selectivity and low sensitivity to component tolerances are requisite. Sensitivity of the filter to finite tolerances of the critical module and the critical quality factor of the normalized complex conjugate poles are analyzed in this paper.

2. MATHEMATICAL BACKGROUND

Mathematical background for continual filter function with one free real parameter, v, and order of the filter function, n, is considered in this part of the paper. Set of orthogonal Gegenbauer polynomials, $C_n^{\nu}(\omega)$, is orthogonal on segment, $-1 \le \omega \le +1$, in respect to weighting function, $w(\omega) \ge 0$,

$$w(\nu, \omega) = (1-\omega^2)^{\nu - \frac{1}{2}}. \quad (1)$$

Certain values of free real parameter, ν, which are listed in Table 1, defines

Table 1 The hierarchy of classical orthogonal polynomials.

ν	$w(\nu, \omega)$	Type of polynomials
ν	$(1-\omega^2)^{\nu-1/2}$	Gegenbauer
$1/2$	1	Legendre
0	$1/\sqrt{(1-\omega^2)}$	Chebyshev First kind
1	$\sqrt{(1-\omega^2)}$	Chebyshev Second kind

Gegenbauer, Legendre and Chebyshev first and second kind orthogonal polynomials.

Norm of the r-th order of the classical Gegenbauer orthogonal polynomials has, for $r = 0,1,2,...$, the form:

$$h(r,\nu)=\int_{-1}^{+1}(1-\omega^2)^{\nu-\frac{1}{2}}C_r^{\nu}(\omega)C_r^{\nu}(\omega)d(\omega)\,. \tag{2}$$

Directly applying the Christoffel-Darboux formula for the Gegenbauer orthogonal polynomials, we derive the characteristic function, $A(n,\nu,\omega)$, of the proposed filter function.

$$A(n=2R,\omega)=\frac{\sum_{r=1}^{n=R}(C(r,\nu,\omega)^2-C(r,\nu,0)^2)/h(r,\nu)}{\sum_{r=1}^{n=R}(C(r,\nu,1)^2-C(r,\nu,0)^2)/h(r,\nu)}\,, \tag{3}$$

where, $C_r^{\nu}(\omega) = C(r,\nu,\omega)$, are Gegenbauer orthogonal polynomials of r-th order, $r = 1,2,...R$, is number of cascade section of second order, $h(r,\nu)$ is the norm of orthogonal polynomials and ν is free real parameter.

As an example, by characteristic function, (3) we generated pole locations of the normalized Gegenbauer filter function for $n = 10$, $\rho = 0.15$, $\nu = 0.777$, listed in Table 2.

Table 2 Pole locations of the proposed polynomial Gegenbauer filter function for $n = 10$, $\nu = 0.777$ and $\rho = 0.15$.

$s_r=\sigma_r \pm j\omega_r,\ r=1,2,...,10\,.$
- 0.593968973174199 $\pm$ j 0.16832288030512205
- 0.533395626654318 $\pm$ j 0.48950142374949457
- 0.41956141180743767 $\pm$ j 0.7654403062083845
- 0.26630210136050475 $\pm$ j 0.9694821178496846
- 0.09137276918104686 $\pm$ j 1.0786183282370532

The magnitude characteristics of the filter functions, $a(n,\nu,\omega)$ [dB], are defined as the logarithm of the square module of the characteristic function, $A(n,\nu,\omega)$,

$$a(n,\nu,\omega)=20\log\left|A(n,\nu,\omega)\right|[\mathrm{dB}]\,. \tag{4}$$

From the magnitude approximation in continuous domain by Christoffel-Darboux formula for Gegenbauer orthogonal polynomials defined by (3), we derive the 3D plot of normalized magnitude characteristics of the filter function in the pass-band, $\omega \in (0,1)$, for band of the free real parameter, $\nu \in (0.9,\ 1.2)$, and for order, $n = 8$, and parameter, $\rho = 0.15$, corresponding to in-band attenuation, $a_{max} = 0.1$dB, shown in Figure 1.

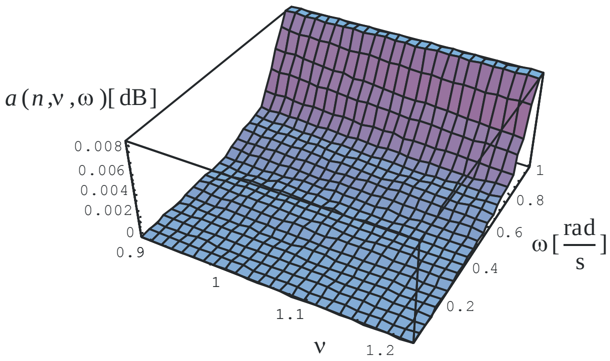

Fig. 1 3D plot of normalized magnitude characteristics of the filter function, in the pass-band for n = 8, ρ = 0.15, $v \in$ (0.9, 1.2) and $\omega \in$ (0,1).

3D plot, in Figure 1, of normalized magnitude characteristics of the filter function shows transition feature of Gegenbauer filter function between the traditional filter functions derived for special values of free real parameter, v , as particular solutions.

3. Particular Solutions of the Proposed Filter Function Derived for Classical Orthogonal Gegenbauer Polynomials

Particular solutions of filter function (3), are given for values of free real parameter, v , defined in Table 1.

3.1. First of particular solution of proposed filter function using Legendre orthogonal polynomials

For v = 1/2 we give weighting function (1) of value w (v = 0.5, ω) =1 and derive filter function for Legendre orthogonal polynomials,

$$A(n=2R,\omega)=\frac{\sum_{r=1}^{R}(P(r,\omega)^2-P(r,0)^2)\,(\frac{2r+1}{2})}{\sum_{r=1}^{R}(P(r,1)^2-P(r,0)^2)\,(\frac{2r+1}{2})}\,. \tag{5}$$

where, P (r,ω), represent the Legendre orthogonal polynomials.

As example, by characteristic function (5) we generated pole locations of the normalized filter function generated by described analytical method for Legendre orthogonal polynomials for n = 10, ρ = 0.15, v = 0.5, listed in Table 3.

Table 3 Pole locations of the proposed filter function for Legendre polynomials derived for order n = 10, v = 0.5 and ρ = 0.15.

$s_r = \sigma_r \pm j\omega_r ,\ r=1,2,\dots,10$.
- 0.49194779879928286418909 52 ± j 0.16549241025439987856053 76
- 0.44160944663089458311904 36 ± j 0.48115827317432078561256 94
- 0.34702123177841572659993 71 ± j 0.75209574386184678851500 38
- 0.21976376999094128624941 05 ± j 0.95226328391819900171841 44
- 0.07514667209247199446342382 ± j 1.05978855920565365899607 4

3.2 Particular solution of proposed filter function using Chebyshev first kind orthogonal polynomials

For v = 0 we give weighting function of value $w(\nu=0,\ \omega)=1/\sqrt{(1-\omega^2)}$ and derived Chebyshev first kind filter function,

$$A(n=2R,\omega)=\frac{\sum_{r=1}^{R}\left(T(r,\omega)^2-T(r,0)^2\right)}{\sum_{r=1}^{R}\left(T(r,1)^2-T(r,0)^2\right)} . \tag{6}$$

where, $T(r,\omega)$, represent the Chebyshev orthogonal polynomials of the first kind.

As example, by characteristic function, (6), we generated pole locations of the normalized Chebyshev first kind filter function for n = 10, ρ = 0.15, v = 0, which are listed in Table 4.

Table 4 Pole locations of the proposed filter function for Chebyhev first kind polynomials derived for order n = 10, v = 0 and ρ = 0.15.

$s_r = \sigma_r \pm j\omega_r ,\ r=1,2,\dots,10$.
- 0.36115517356093959253849 8 ± j 0.16131339458822867763822 69
- 0.32404210090781497697926 77 ± j 0.46882252422220966453973 91
- 0.25428098759144786483078 88 ± j 0.73229128404252864319406 22
- 0.16042303139723103497395 62 ± j 0.92648650813489412691175 47
- 0.05418824935966591610568113 ± j 1.03117541361348147056487 4

3.3 Particular solution of the proposed filter function using Chebyshev second kind orthogonal polynomials

For v = 1 we give weighting function of value $w(\nu=1,\ \omega)=\sqrt{(1-\omega^2)}$ and derived Chebyshev second kind filter function,

$$A(n=2R,\omega)=\frac{\sum_{r=1}^{R}\left(U(r,\omega)^2-U(r,0)^2\right)}{\sum_{r=1}^{R}\left(U(r,1)^2-U(r,0)^2\right)} , \tag{7}$$

where, $U(r,\omega)$, represent the Chebyshev orthogonal polynomials of second kind.

As an example, by characteristic function (6), we generated pole locations of the normalized Chebyshev second kind filter function for $n = 10$, $\rho = 0.15$, $v = 1$, listed in Table 5.

Table 5 Pole locations of the proposed filter function for Chebyhev second kind polynomials derived for order $n = 10$, $v = 1$ and $\rho = 0.15$.

$s_r = \sigma_r \pm j\omega_r,\ r = 1, 2, \ldots, 10.$
- 0.361155173560939592538498 ± j 0.161313394588228677638226 9
- 0.324042100907814976979267 7 ± j 0.468822524222209664539739 1
- 0.254280987591447864830788 8 ± j 0.732291284042528643194062 2
- 0.160423031397231034973956 2 ± j 0.926486508134894126911754 7
- 0.054188249359665916105681 13 ± j 1.031175413613481470564874

Comparasion of normalized magnitude characteristics of the filter functions with pole locations listed in Tables 2 to 5 for $n = 10$, $\rho = 0.15$ and $v = 0.777$ for Gegenbauer filter function**,** $v = 0.5$ for Legendre filter function, $v = 0$ for Chebyshev first kind filter function, and $v = 1$ for Chebyshev second kind filter function, in the stop-band and the pass-band are shown in Figures 2 and 3, respectively.

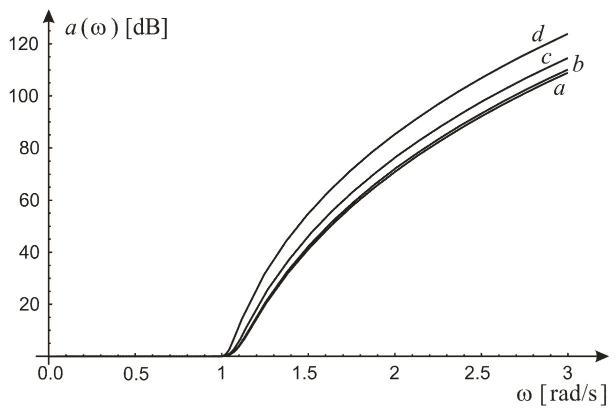

Fig. 2 Comparison of normalized magnitude characteristics in the stop-band of the filter functions, derived for $n = 10$, $\rho = 0.15$ and for: (a) $v = 0.5$, Legendre orthogonal polynomials, (b) $v = 0$, Chebyshev first kind orthogonal polynomials, (c) $v = 0.777$, Gegenbauer orthogonal polynomials and (d) $v = 1$, Chebyshev second kind orthogonal polynomials.

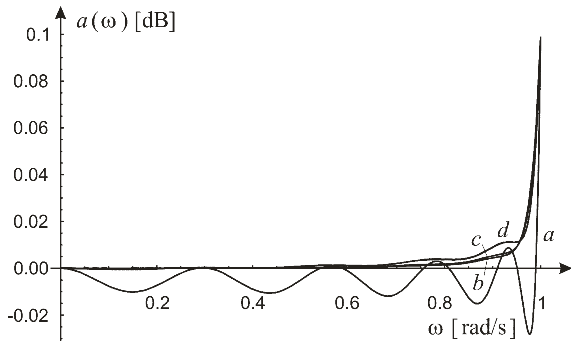

Fig. 3 Zoomed in the pass-band comparison of normalized magnitude characteristics from Fig. 2, derived for $n = 10$, $\rho = 0.15$ and for: (a) $v = 0.5$, Legendre orthogonal polynomials, (b) $v = 0$, C hebyshev first kind orthogonal polynomials, (c) $v = 0.777$, Gegenbauer orthogonal polynomials and (d) $v = 1$, Chebyshev second kind orthogonal polynomials.

4. Insertion Loss Analysis

The magnitude characteristics of the filter functions can be expressed by means of insertion loss characteristics (in decibels),

$$L_A = -10\log_{10}|s_{21}(\mathrm{j}\omega)|^2 \ [\mathrm{dB}]. \quad (8)$$

The proposed polynomial normalized prototype filter functions generated by the Christoffel-Darboux formula for the classical Gegenbauer orthogonal polynomials are determined by Eq. (3) for parameter values $v = 0.888$ and $\rho = 0.15$, and orders $n = 6$, $n = 8$ and $n = 10$. Pole locations, $s_{r,r+1} = \sigma_r \pm \mathrm{j}\omega r$, where $r = 1,2,..., R/2$ and $n = 2R$, of proposed filter functions of order $n = 6$, $n = 8$ and $n = 10$, are given in Tables 6, 7, and 8, respectively.

Table 6 Pole locations of the proposed filter function for Gegenbauer polynomials derived for order $n = 6$, $v = 0.888$ and $\rho = 0.15$.

$s_r = \sigma_r \pm j\omega_r,\ r = 1,2,...,6.$
- 0.8407318203193898 ± j 0.31241580541972985
- 0.6124720589625705 ± j 0.8563289133194147
- 0.22337564180560512 ± j 1.173878292324402

Table 7 Pole locations of the proposed filter function for Gegenbauer polynomials derived for order $n = 8$, $v = 0.888$ and $\rho = 0.15$.

$s_r = \sigma_r \pm j\omega_r,\ r = 1,2,...,8.$
- 0.6876481240806619 ± j 0.2178180922759596
- 0.5797626599071993 ± j 0.622201686102077
- 0.38359414776159495 ± j 0.9359537946631831
- 0.13378307736950462 ± j 1.1089521249640877

Table 8 Pole locations of the proposed filter function for Gegenbauer polynomials derived for order n = 10, v = 0.888 and ρ = 0.15.

$s_r = \sigma_r \pm j\omega_r ,\ r = 1, 2, \ldots , 10$.
- 0.5875688362574722 ± j 0.1674108697212237
- 0.527418267040304 ± j 0.48691652199092045
- 0.4144387083241363 ± j 0.7616228307010962
- 0.2625060737480009 ± j 0.965131202458954
- 0.08962753303437143 ± j 1.0746866509926576

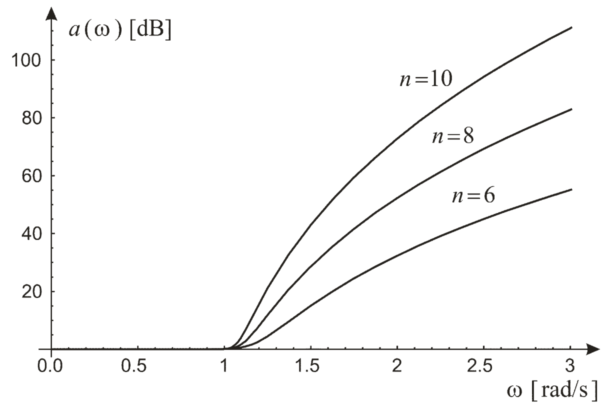

Fig. 4 Normalized magnitude characteristics in the stop-band of the Gegenbauer filter functions for v = 0.888 and ρ = 0.15, of order n = 6, n = 8 and n = 10.

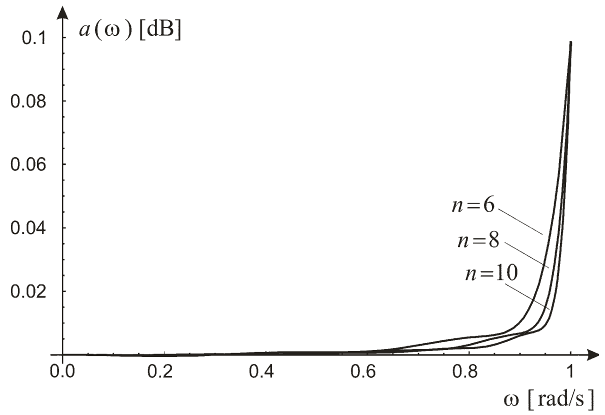

Fig. 5 Normalized magnitude characteristics from Fig. 4 zoomed in the pass-band of the Gegenbauer filter functions for v = 0.888 and ρ = 0.15, of order n = 10, n = 8 and n = 6.

5. FINITE TOLERANCE OF THE FILTER FUNCTION ANALYSIS

Analog continuous-time all-pole low-pass filter function can be expressed in the form:

$$H_n(s)=\prod_{r=1}^{R}\frac{\omega_{pr}^2}{s^2+s\dfrac{\omega_{pr}}{Q_{pr}}+\omega_{pr}^2}\,, \tag{9}$$

where, $n = 2R$ is order of filter function, R is a total number of biquadrate sections, ω_{pr}, is module of conjugate-complex poles, s_{pr},

$$\omega_{pr}^2=\mathrm{Re}\{s_{pr}\}^2+\mathrm{Im}\{s_{pr}\}^2\,,\quad r=1,2,\ldots,R\,. \tag{10}$$

Conjugate-complex poles in Figure 6 are defined as, $s_{pr}=\sigma_r\pm j\omega_r$, and its module as,

$$\omega_{pr}=\sqrt{(\sigma_r)^2+(\omega_r)^2}\,,\quad r=1,2,\ldots,R\,. \tag{11}$$

The critical module of conjugate-complex poles, ω_{pc}, is determined as,

$$\omega_{pc}=\max\{\omega_{p1},\omega_{p2},\ldots,\omega_{pR}\}\,. \tag{12}$$

Then the pole quality factor of r-th biquadrate section is expressed by,

$$Q_{pr}=\frac{\omega_{pr}}{2\,\mathrm{Re}\{s_{pr}\}}\,,\quad r=1,2,\ldots,R\,, \tag{13}$$

and critical pole quality factor, Q_{pc}, is determined as,

$$Q_{pc}=\max\{Q_{p1},Q_{p2},\ldots,Q_{pR}\}\,. \tag{14}$$

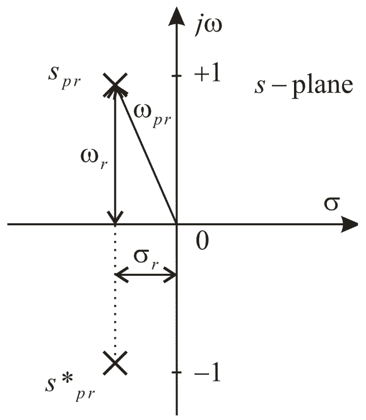

Fig. 6 Locations of pairs of complex conjugate poles with terms of definition quality factor, Q_{pr}, where, ω_{pr} is module of conjugate-complex poles, s_{pr}.

6. Analysis of Finite Tolerance of the Module of Critical Pole of Filter Function

The limited gain-bandwidth product has the effect to the each biquadrate sections pass-band response, $r = 1,2,..., R$, in the part of frequency range near the pass-band edge frequency of the RC active filter. The limited gain-bandwidth product has an equal impact in the part of frequency range near the band edge frequency and at the band edge frequency to the biquadrate section of critical complex conjugate poles which has maximum value of the pole quality factor, Q. In this part of paper, only the adverse impact to the biquadrate section of the critical quality factor is analyzed.

The sensitivity of analog continuous-time filter function, $H_n\,(s \to j\omega)$, to the critical module of the normalized complex conjugate poles is defined as,

$$\left|\frac{\partial H_n(s\to j\omega)}{\partial\omega_{pc}}\right| = \left|\frac{\partial}{\partial\omega_{pc}}\prod_{r=1}^{R}\frac{\omega_{pr}^2}{s^2+s\dfrac{\omega_{pr}}{Q_{pr}}+\omega_{pr}^2}\right| = \left|\left\{\frac{\partial}{\partial\omega_{pc}}\frac{\omega_{pc}^2}{s^2+s\dfrac{\omega_{pc}}{Q_{pc}}+\omega_{pc}^2}\right\}\cdot\left\{\prod_{r=1}^{R-1}\frac{\omega_{pr}^2}{s^2+s\dfrac{\omega_{pr}}{Q_{pr}}+\omega_{pr}^2}\right\}\right|. \quad (15)$$

Values of attenuation on frequency, $\omega = 1$, for set of tolerance values of critical module of complex conjugate poles, $\Delta\omega_{pc}$, of the normalized 10-th order filter function derived for Gegenbauer polynomials by equation (3) for order $n = 10$, and parameters $v = 0.888$ and $\rho = 0.15$, are listed in Table 9.

Table 9 Values of attenuation on, $\omega = 1$, for set of tolerance of critical pole module, $\Delta\omega_{pc}$, of normalized 10-th order filter function derived for Gegenbauer polynomials for order $n = 10$, $v = 0.888$ and $\rho = 0.15$.

$\Delta\omega_{pc}$	$a(\omega=1)$[dB]	$\Delta\omega_{pc}$	$a(\omega=1)$[dB]
+ 0.5 %	0.316371	- 0.5 %	2.14522
+ 1 %	0.533451	- 1 %	2.50501
+ 2 %	0.962716	- 2 %	3.22362
+ 3 %	1.38123	- 3 %	3.93541
+ 5 %	2.17430	- 5 %	5.32250

When the finite tolerance of critical module, $\Delta\omega_{pc}$, is limited to 0% and to –3%, normalized magnitude characteristics in the stop-band and zoomed in the pass-band and around stop-band cut-off frequency, ω_{cs}, as well normalized group delay characteristics of the 10-th order Gegenbauer filter function for $v = 0.888$ and $\rho = 0.15$, are analyzed in Figures 7, 8, 9, and 10, respectively.

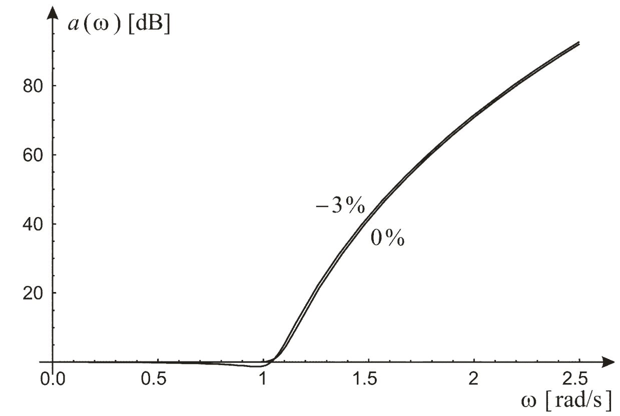

Fig. 7 Normalized magnitude characteristics of the Gegenbauer filter function for n = 10, ρ = 0.15, and v = 0.888, and finite tolerance of the critical module of 0% and –3%

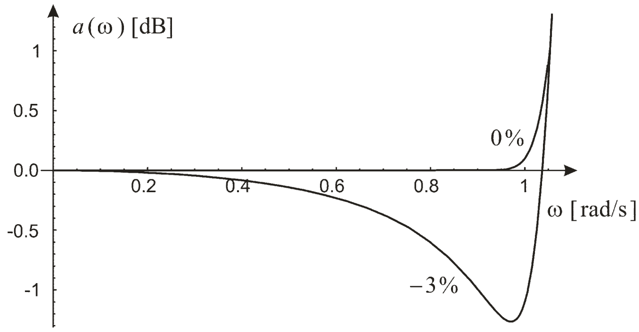

Fig. 8 Zoomed in the pass-band normalized magnitude characteristics from Fig. 7 for n = 10, ρ = 0.15 and v = 0.888, and finite tolerance of the critical module of 0% and –3%.

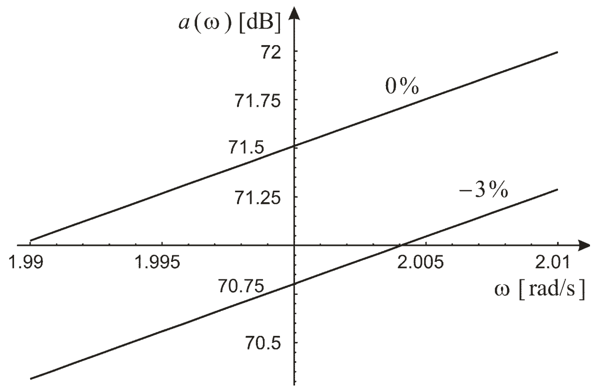

Fig. 9 Normalized magnitude characteristics from Fig. 7, for n = 10, ρ = 0.15 and v = 0.888, zoomed around the stop-band cut-off frequency, ω_{cs}, for 0% and –3% of finite tolerance of the critical module.

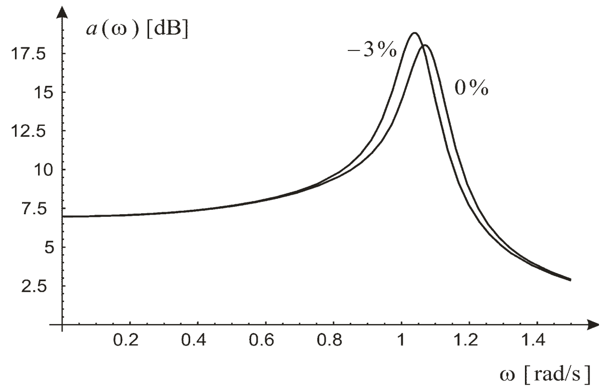

Fig. 10 Group delay of the normalized filter function, for $n = 6$, $\rho = 0.15$ and $v = 0.888$, for 0% and −3% of finite tolerance of the critical module.

7. Analysis of Finite Tolerance of the Critical Q Factor of Filter Function

The sensitivity of analog continuous-time filter function, $H_n(s \to j\omega)$, to the critical quality factor, Q_{pc}, of complex conjugate poles is defined as,

$$\left|\frac{\partial H_n(s\to j\omega)}{\partial Q_{pc}}\right| = \left|\frac{\partial}{\partial Q_{pc}}\prod_{r=1}^{R}\frac{\omega_{pr}^2}{s^2+s\dfrac{\omega_{pr}}{Q_{pr}}+\omega_{pr}^2}\right| = \left|\left\{\frac{\partial}{\partial Q_{pc}}\frac{\omega_{pc}^2}{s^2+s\dfrac{\omega_{pc}}{Q_{pc}}+\omega_{pc}^2}\right\}\cdot\left\{\prod_{r=1}^{R-1}\frac{\omega_{pr}^2}{s^2+s\dfrac{\omega_{pr}}{Q_{pc}}+\omega_{pr}^2}\right\}\right|. \quad (16)$$

Values of attenuation on frequency, $\omega = 1$, of normalized 10-th order filter function derived for Gegenbauer polynomials for $n = 10$, $v = 0.888$ and $\rho = 0.15$ are derived for set of the critical quality factor tolerance of complex conjugate poles, ΔQ_{pc}, and listed in Table 10.

When tolerance of the critical Q factor are limited to 0% and −3%, the normalized magnitude characteristics of the 10-th order filter function, in the stop-band and in the pass-band, are analyzed in the Figures 11 and 12. In Figure 13 are shown magnitude characteristics for same finite tolerance of the critical Q factor, zoomed in the stop-band around the stop-band cut-off frequency, ω_{cs}.

Table 10 Values of attenuation on, $\omega = 1$, for set of finite tolerances of critical quality factor, ΔQ_{pc}, of normalized 10-th order filter function derived for Gegenbauer polynomials for $v = 0.888$ and $\rho = 0.15$.

ΔQ_{pc}	$a(\omega=1)[\mathrm{dB}]$	ΔQ_{pc}	$a(\omega=1)[\mathrm{dB}]$
+ 0.5 %	0.0751704	- 0.5 %	2.30724
+ 1 %	0.0517322	- 1 %	2.26051
+ 2 %	0.0055166	- 2 %	2.16864
+ 3 %	0.039835	- 3 %	2.07883
+ 5 %	0.128026	- 5 %	1.90516

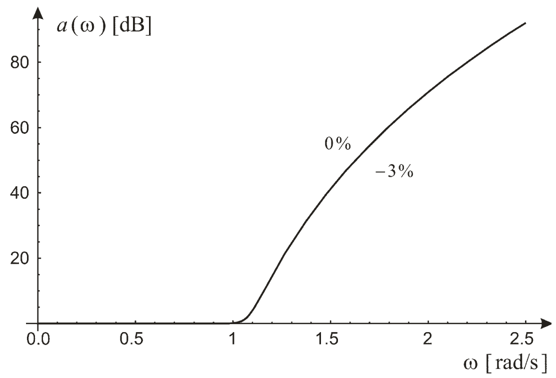

Fig. 11 Normalized magnitude characteristics in the stop-band of the Gegenbauer filter function for $n = 10$, $\rho = 0.15$, $v = 0.888$, and finite tolerance of critical quality factor of 0% and -3%.

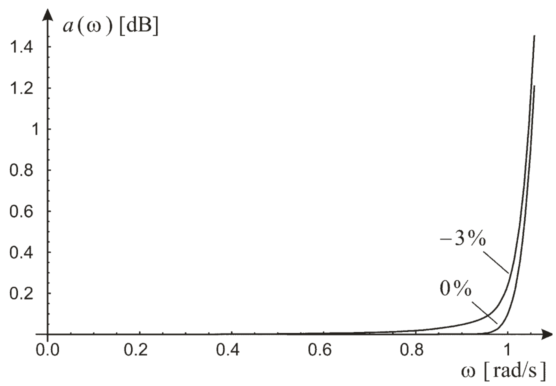

Fig. 12 In the pass-band zoomed normalized magnitude Gegenbauer characteristics from Fig. 11 for $n = 10$, $\rho = 0.15$, $v = 0.888$, and finite tolerance of critical quality factor of 0% and -3%.

When tolerances of the critical Q factor are limited to 0% and −3%, the normalized group delay characteristics of the 10 -th order filter function are analyzed in Figure 14.

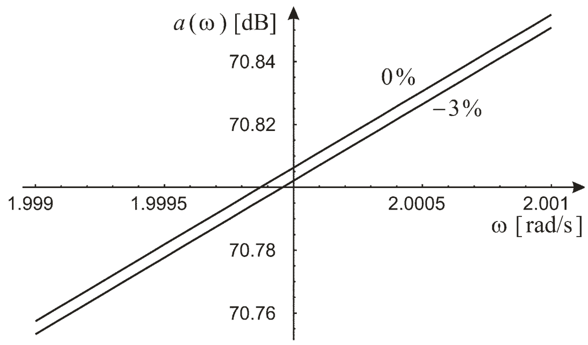

Fig. 13 Normalized magnitude characteristics of the Gegenbauer filter function for n = 10, ρ = 0.15, ν = 0.888, from Fig. 11, zoomed in the stop-band around the stop-band cut-off frequency, ω_{cs} , for finite tolerance of critical quality factor ΔQ_{pc} = 0% and ΔQ_{pc} = –3%.

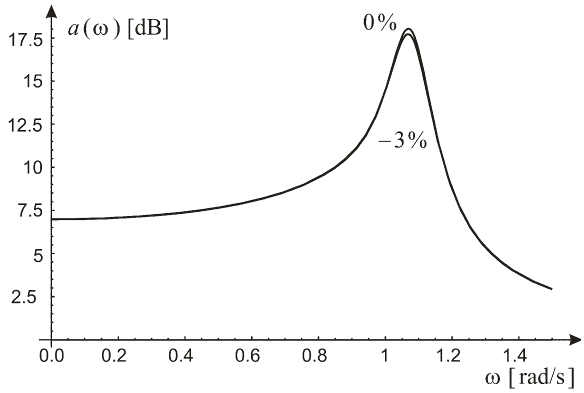

Fig. 14 Normalized group delay characteristics of the Gegenbauer filter function for n = 10, ρ = 0.15, ν = 0.888, and for 0% and –3% of finite tolerance critical quality factor, ΔQ_{pc}.

8. CONCLUSION

This paper presents 3D analysis and detailed sensitivity analysis to component tolerances in the pass-band and the stop-band of a new class of filter functions introduced in literature [4]. An analysis of determining the characteristic features of these filters applicable for the design of the RC active filter is presented. In literature [4] available technique for derivation of filter functions at the approximation stage of filter synthesis is entirely analytical. In addition, the design economy of selective filters can be considerably improved by analyzing 3D plots of the magnitude response and the magnitude of the in-band ripples as a function of the free parameter, v, the filter order, n, and the frequency for maximum in-band attenuation defined by the parameter, ρ, which defined the maximum value of reflection coefficient in the pass band. In this paper, an exact method has been presented to determine the explicit shape of the magnitude characteristic function for polynomial filters of even degree, for $n = 6$, $n = 8$ and $n = 10$. Amplitude characteristics are normalized by the maximum value of the attenuation in the pass-band of 0.1dB. Frequency characteristics of filter function for $n = 10$ and $v = 0.888$ are normalized to the maximum value of the attenuation in the pass-band of 0.1dB, and compared from point of view of finite tolerance of filter components.

The sensitivity of analog continuous-time filter function derived in literature [4] for Gegenbaure orthogonal polynomials, to finite tolerances of critical quality factor, ΔQ_{pc}, and to finite tolerances of critical module, $\Delta\omega_{pc}$, of complex conjugate poles are analyzed in this paper. Limited gain-bandwidth product has dominant impact to the biquadrate section of critical poles in the part of frequency range near the pass-band edge frequency and at the band edge frequency. Foregoing effect is inconsiderable on the filter insertion loss. From the point of view of finite pole tolerance in this paper it is analyzed the proposed new design method suitable for the design of filters that may satisfy different in-band attenuation specifications. The analysis of analytical approximation for design procedure of all-pole, RC active, low selectivity low-pass prototype filters, with low sensitivity to component tolerances is presented.

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