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HIGH COMPRESSION RATE AND ERROR FREE IMAGES CODING METHOD USING MORPHOLOGICAL "TWO STEPS" SKELETON-SKELETON AND STRUCTURE-SKELETON

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Abstract: This paper addresses the representation of grayscale images by means of binary mathematical morphology, a relatively new nonlinear theory for image processing, based on set theory. The new image representation described in this work, called "two steps" skeleton representation, is a extension of the morphological binary skeleton. This article will present the theoretical background of the morphological image representation and will show an application for grayscale images. At the end, the possibility to generalize the "two steps" skeleton representation for multi-dimensional images will be analyzed, to extend the scope of its algebraic characteristics as much as possible. The application of the "two steps" skeleton representation is illustrated by computer simulations.

Key words: Image coding, grayscale image, skeleton, morfology, multi dimensional image.

1. Introduction-Image Representation via Mathematical Morphology

Image Representation is a key component in many tasks in computer vision and image processing. It generally consists of presenting an image in a form, different from the original one, in which the desired characteristics of the image are emphasized and can be easily accessed. In the following

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pages we will consider some morphological methods for binary and grayscale image representation.

Those methods are based on mathematical morphology, which is a relatively new, and rapidly growing, nonlinear theory for image processing.

Mathematical morphology is part of set theory, and it has a strong geometric orientation. For binary images, mathematical morphology provides a well-founded theory for analysis and processing.

Therefore, binary morphological representations can be developed and analyzed. Mathematical Morphology involves a study of different ways in which a structuring element interacts with a given set, modifies its shape, and extracts the resultant set.

Mathematical Morphology is a general theory that studies decompositions of operators between sets in terms of a family of simple operators: dilations, erosions, anti- dilations and anti-erosions.

Nowadays, this theory is largely used in Computer Vision to extract information from images.

The fundamental operations are erosion and dilation. Based on those opening and closing operations are defined. The morphological operations have been successfully used in many applications including object recognition, image enhancement, texture analysis, and industrial inspection.

Mathematical Morphology can be used in various areas of image processing, such as image compression, pattern recognition, object recognition, image enhancing, etc. For binary images, Mathematical Morphology provides a well- founded theory for analysis and processing. Therefore, Binary Morphological Representations can be developed and analyzed.

The new morphological image representation presented in this article is the "two steps" skeleton.

The "two steps" skeleton (which will be denoted: "2S"), is a natural extension of the morphological skeleton. It consists of first calculating the morphological skeleton or structure using a structuring element, and then reiterating the above procedure (the skeleton of the skeleton or structure of binary image) using different standard structuring elements (especially lines).

The original shape can be perfectly reconstructed.

2. Morphological Skeleton - An Overview

The skeleton is one of the main operators in mathematical morphology and it can be calculated entirely using the basic morphological operators.

Dilation and erosion are the fundamental operators of the Mathematical Morphology. The key process in the dilation and erosion operators is the local comparison of a shape, called structuring element, with the object to be transformed.

The structuring element is a predefined shape, which is used for morphological processing of the images. The most common shapes used as structuring elements are horizontal and vertical lines, squares, digital discs, crosses, etc.

The fundamental morphological operators are based on the operation of translation. Let B be a set contained in the Euclidean space E, and let x be a point in E. The translation of the set B by the point x, denoted B_x, is defined as follows

$$B_x = \{b + x | b \in B\} \tag{1}$$

The dilation of the image X by the structuring element B, denoted, is defined by

$$X \oplus B = \bigcup_{x \in X} B_x \tag{2}$$

For dilation: when the structuring element is positioned at a given point and it touches the object, this point will appear in the result of the transformation, otherwise it will not.

The erosion of X by the structuring element B, denoted $X\$B$, is defined in the following way:

$$X\$B \bigcap_{x \in X} X_{-b} \tag{3}$$

For erosion: when positioned at a given point, if the structuring element is included in the object, then this point will appear in the result of the transformation, otherwise will not.

Based on those fundamental operators, two morphological operators are developed. These are the opening and closing operators. They are dual operators.

The opening operator, denoted "$\circ$", can be expressed as a composition of erosion followed by dilation, both by the same input structuring element

$$X \circ B = (X\$B) \oplus B \tag{4}$$

The closing operator, denoted "$\bullet$", can be expressed as composition of dilation followed by erosion by the same input structuring element

$$X \bullet B = (X \oplus B)\$B \tag{5}$$

Lantuejoul proved that the skeleton $S(X)$ of a topologically open shape X in Z^2 can be calculated by means of binary morphological operations, in the following way

$$\begin{aligned} S(X) &= \bigcup_{n>0} S_n(X) \\ &= \bigcup_{n>0} \{X!nB - (X!nB) \circ B\} \end{aligned} \tag{6}$$

where B is a structuring element. The skeleton can be computed using as structuring element any kind of geometrical figure.

In Figure 1 is shown a binary image. Skeletons are obtained using rhombs (Figure 2) and squares (Figure 3) as structuring elements.

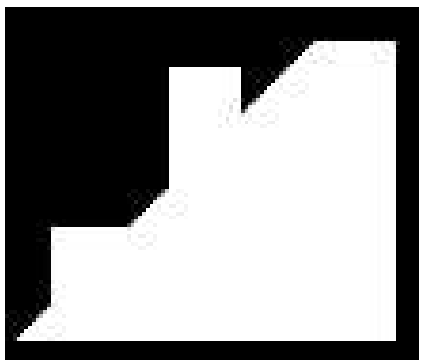


Fig. 1. The original image.

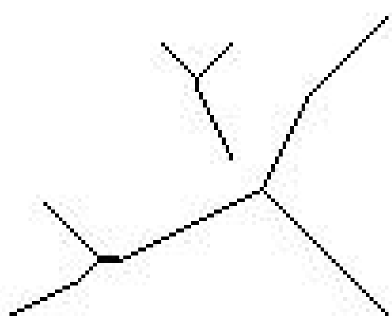


Fig. 2. Skeleton of original image using rhombs as structuring elements.

Fig. 3. Skeleton of original image using squares as structuring elements.

The compression rate for this example is about 4%. This means that, for the skeleton, we need 25 times less information in order to reconstruct the original image.

The reconstruction process needs additional information about the size of the structuring element for each point of the skeleton.

By adding (for skeleton obtained in Figure 2) the information about the structuring element to the skeleton, the resulting image can be considered as a grayscale image. In this case, the resulting image is shown in Figure 4.

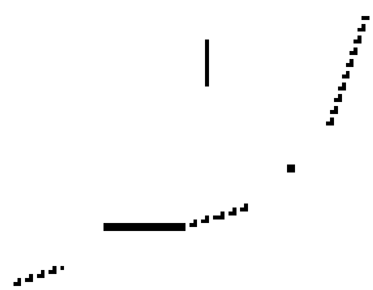


Fig. 4. The skeleton completed with structuring element information.

The biggest problem with the skeleton representation is the fact that it contains many redundant points.

These points are not needed for reconstruction, but appear in the skeleton. Several methods were proposed for reducing skeleton's redundancy, and the use of these methods reduces the number of redundant points in various degrees.

The image representations obtained from these methods are called reduced skeletons: RS.

From the collection of subsets $\{S_n(X)\}_{n>0}$ and knowing the radius n for each pixel, the original shape X can be perfectly reconstructed in the following way

$$X = \bigcup_{n \geq 0} S_n(X) \oplus nB \tag{7}$$

The Morphological Skeleton representation permits also partial reconstruction, yielding simplified versions of the original shape. This is obtained by eliminating from the skeleton the pixels with values smaller and equal to a given value k

$$X \circ kB = \bigcup_{n \geq k} S_n(X) \oplus nB \tag{8}$$

The same results are obtained using reduced skeleton

$$X = \bigcup_{n \geq 0} RS_n(X) \oplus nB \tag{9}$$

3. Morphological Structure - An Overview

The Morphological Binary Structure is a compact error free representation of images, a property useful for image compression.

The Binary Structure is a redundant representation, i.e., some of its points may be discarded without affecting its error free characteristic. In some applications, such as coding, no importance is attributed to the binary Structure shape or its connectivity, except for its ability to fully represent images in a compact way.

The Morphological Binary Structure representation of a binary image X, with a given binary structuringelement B, is a collection of sets $ST_n(X)$ called Binary Structure subsets of order n.

N is the maximum value defined by with

$$ST_n(X) = X \mathcal{B}\ NB - (X \mathcal{B}\ NB) \circ B = \emptyset, \qquad n > N \tag{10}$$

and

$$ST_n(X) = X \mathcal{B}\ NB - (X \mathcal{B}\ NB) \circ B \neq \emptyset \tag{11}$$

We can define iteratively for $n = N, N-1, \ldots, 1$

a) Initial condition

$$ST_n(X) = X \;\mathcal{B}\; NB - (X \;\mathcal{B}\; NB) \circ B \tag{12}$$

b) Recursive relations

$$X^{(n-1)} = X^{(n)} - ST_n \oplus nB \tag{13}$$

$$ST_{n-1} = X^{(n-1)} \;\mathcal{B}_{(n-1)}\; B - [X^{(n-1)} \;\mathcal{B}_{(n-1)}\; B] \circ B \tag{14}$$

Perfectly reconstruction is given by

$$X = \bigcup_{n \geq 0} ST_n(X) \oplus nB \tag{15}$$

Partial reconstruction is given by

$$\hat{X}_k = \bigcup_{n \geq k} ST_n(X) \oplus nB \tag{16}$$

Skeletons are obtained using rhombs (Figure 5) and squares (Figure 6) as structuring elements.

The compression rate for this example is about 4%. This means that, for the structure, we need 25 times less information in order to reconstruct the original image.

Fig. 5. Structure of original image using rhombs as structuring elements.

Fig. 6. Structure of original image using squares as structuring elements.

The reconstruction process needs additional information about the size of the structuring element for each point of the skeleton.

By adding the information about the structuring element to the skeleton, the resulting image can be considered as a grayscale image. In this case, the resulting image is shown in Figure 2.

4. The "Two Steps" Skeleton

The method proposed in this paper is entirely based on the skeleton representation and it represents a generalisation of this method.

The main idea behind this method is to compute the skeleton twice.

First, the skeleton of the binary image is computed using Lantuejoul's formula.

The structural element used for this operation may be an open disk, a square or an elementary cross. The computation of the skeleton eliminates some of the redundant points from the original image.

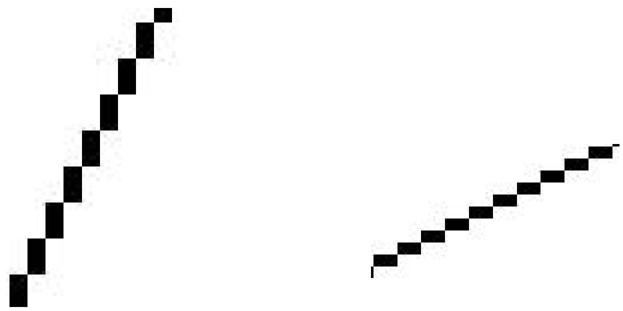
Still, the skeleton has a large number of redundant points that reduces the compression rate.

Because of these redundant points, the elements of the skeleton are highly connected.

For example, the skeleton in Figure 2 is made of ten straight lines. These lines are divided into two categories:

- The first category contains the lines that make with the horizontal angles that are multiple of 45 degrees;
- The second category contains all the other lines. If we take a closer look at these lines we notice that they are actually made of small

horizontal and vertical lines (Figure 7).



(a) (b)

Fig. 7. Details of the second category lines.

We will eliminate the redundant points that exist in the skeleton in the same manner, by computing the skeleton of the skeleton or structure.

$$\begin{aligned} 2SS(x) &= \bigcup_{n\geq 0} S_n(S_n(X)) \\ &= \bigcup_{n>0} \{S(X)!nL - \bigcup_{\Delta n>0} [S(X)!nL \circ \Delta nL]\} \end{aligned} \tag{17}$$

or

$$\begin{aligned} 2SST(x) &= \bigcup_{n\geq 0} S_n(ST_n(X)) \\ &= \bigcup_{n>0} \{ST(X)!nL - \bigcup_{\Delta n>0} [ST(X)!nL \circ \Delta nL]\} \end{aligned} \tag{18}$$

using a definition: $s = S$ or $s = ST$

$$\begin{aligned} 2SsT(x) &= \bigcup_{n\geq 0} S_n(s_n(X)) \\ &= \bigcup_{n>0} \{s(X)!nL - \bigcup_{\Delta n>0} [s(X)!nL \circ \Delta nL]\} \end{aligned} \tag{19}$$

Since the skeleton or structure is mainly composed of straight lines, the structuring elements used for the second skeleton computation will be:

a) Horizontal line;
b) Vertical line;
c) 45 degree inclined line;

d) 135 degree inclined line.

In equations (17), (18) and (19), L represents these structural elements used for $2SS$ computation.

In Figure 8 is shown the $2SS$ of the image from Figure 1. Is very easy to see that the number of points in $2SS$ is a lot smaller than the number of points in the skeleton.

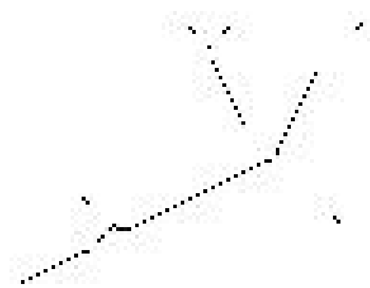


Fig. 8. The "Two Steps" Skeleton.

Figure 9 shows the redundant points that have been eliminated from the skeleton. For this example, these points represent about 2/3 of the number of points of the skeleton.

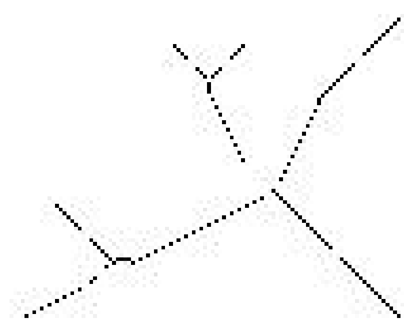


Fig. 9. The difference between the skeleton and the $2SS$.

The last step of the $2Ss$ method is to attach the information about the structuring elements to the points of the $2Ss$.

As it can be noticed, the "two steps" skeleton was computed from the skeleton without the structuring element information.

However, this information must be supplied if we want to reconstruct the original image from its $2Ss$. This information can be added in a similar way to the morphological skeleton method.

5. Conclusions

The method was generalised for greyscale images.

We just have to divide the greyscale images in binary images. The method obtains very good results.

A compression rate of 2-5% is usually (with perfectly reconstructions).

The "two steps" skeleton is a new and improved method for removing the redundant points from the morphological skeleton and structure of a binary image.

Because of these properties, the "two steps" skeleton is recommended for image compression in areas where image quality is essential, where a degradation of the image due to the compression process is not accepted.

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