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TLM Z-TRANSFORM METHOD MODELLING OF LOSSY GRIN MTM WITH DIFFERENT REFRACTIVE INDEX PROFILES *

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Abstract. *In this paper, a numerical three-dimensional (3D) model of electromagnetic left-handed metamaterials (LH MTM) is applied for the modelling of composite right-left handed (RH/LH) structures with a graded refractive index profile commonly named GRIN MTM. The core of this model is Transmission Line Matrix (TLM) Z-transform method that incorporates the Drude function to account for dispersive LH MTM properties in the time-domain. Lossy GRIN slabs with abrupt, hyperbolic tangent, cosine, and linear refractive index profiles have been considered. The accuracy, efficiency and stability of the proposed approach are verified using analytical solution.*

Key words: *metamaterials, 3D TLM method, Drude model, graded refractive index*

1. INTRODUCTION

Numerous solutions based on metamaterials (MTM), artificial electromagnetic (EM) materials with extreme values of effective permittivity and permeability, have been proposed for the realisation of many different types of microwave components which have advanced characteristics and a small size. Despite a few challenges regarding practical implementation of these proposed solutions, such as fabrication problems, prohibitive loss, bandwidth restriction, a large number of the MTM applications have been developed. There are zero-order resonators, innovative filters, high-directivity antenna arrays, distributed negative refractive index (NRI) lenses, to name but a few [1-2].

The negative refraction phenomenon displayed by left-handed metamaterials (LH MTM) has sparked interest among the research community. Recent theoretical and experimental studies imply promising solutions based on composite right-left handed (RH/LH) structures with graded refractive index profile (GRIN MTM) [3-9]. The unique

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ability of MTM to manipulate the EM radiation in ways that are not possible in natural materials along with the transformation optics principles represents the fundamentals of GRIN structure applications. Unlike in conventional media, in GRIN MTM media there is possibility to simultaneously change the values of permittivity and permeability. Thus, the main advantage of these composite GRIN MTM structures is the additional degree of freedom in the design of specified component features. Another advantage is the effortless impedance matching in free space and possibly improved performance at microwave and optical frequencies. However, practical realisation of GRIN MTM structures could be challenging at high frequencies since standard fabrication methods are feasible for planar structures with a limited number of layers. Only moderate refractive index changes can be achieved. Hence, the effects predicted by analytical solutions derived for some profiles [6-7,9-10] are limited. Therefore, numerical characterisation of GRIN MTM is very important since it enables investigation of profiles that are possible to manufacture as well as possibly useful profiles without a derived analytical solution.

In order to capture the unusual features of MTM both in the frequency-domain and time-domain, a few differential and integral numerical techniques have been modified for LH MTM analysis. In most of the cases, these enhanced numerical techniques enable much faster analyses than the implementation of the MTM transmission line networks using circuit simulators. The most used differential numerical techniques in the time-domain are the Finite-Difference Time-Domain (FD-TD) method [11] and the Transmission-Line Matrix (TLM) method [12]. Incorporation of MTM properties into these approaches allows the time-harmonic and transient simulation of LH MTM structures for direct analysis of their dispersive behaviour.

In contrast to the FDTD method where a number of different techniques, described in [10], have been developed to incorporate the frequency dispersion of MTM, the TLM model of MTM proposed in [13] has been of limited applicability since it only allows the specification of lossless MTM properties only at particular design frequency. Therefore, a variation of the TLM method based on Z-transform, naturally suited to the description of arbitrary time-dependent responses of general frequency-dependent isotropic, bi-isotropic, anisotropic and nonlinear natural materials [14], has been extended in [15,16] for the direct time-domain modelling of lossy LH MTM. The Drude dispersive model has been adopted to describe the frequency-dependent properties of the LH MTM, specified by the electric and magnetic susceptibilities or conductivities, over a wide frequency range. The bilinear Z-transform has been then used to transfer this dependence in the discrete time-domain and to develop the numerical procedure for incorporation into the one-dimensional (1D) [15] and three-dimensional (3D) [16] TLM mesh. Model ability to accurately describe MTM structures and especially GRIN MTM has been illustrated in [17].

In this article, dispersive 3D TLM model of LH MTM is used to describe lossy GRIN MTM with different refractive index profiles. GRIN slabs with abrupt, hyperbolic tangent, cosine and linear variation of refractive index across the slab length have been considered. The obtained results of numerical simulation of EM wave interaction with GRIN MTM interface having different refractive index profiles are in close agreement with the analytic solutions. Therefore, the accuracy and efficiency of proposed model are verified and model applicability to GRIN MTM with arbitrary refractive index profiles is possible.

2. GRIN MTM

Recently, a subject of numerous MTM researches has been a unique EM phenomenon, which occurs at the boundary surface of the composite RH/LH structure with a graded refractive index profile. These composite structures exhibit low return loss, a significant decrease in the effect of geometric aberration, etc. Principally, GRIN MTM structure can be represented by the gradient change of real part of refractive index across its length, as illustrated in Fig. 1 and shown by Eq. 1.

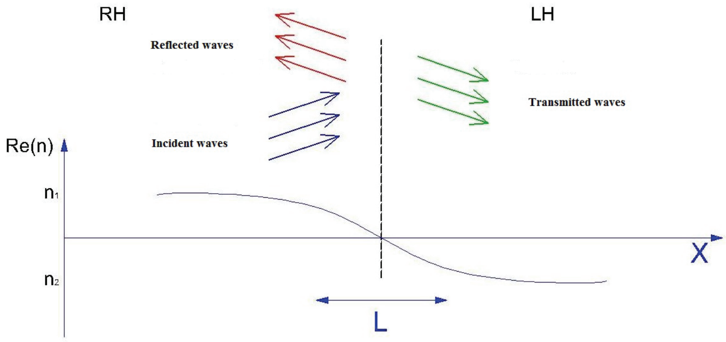

Fig. 1. GRIN MTM

$$\mathrm{Re}\{n(x)\} = \begin{cases} n_1 & x \le -\mathrm{L}/2 \\ f(x) & -\mathrm{L}/2 < x < \mathrm{L}/2 \\ n_2 & x \ge \mathrm{L}/2 \end{cases} \tag{1}$$

Different refractive index profiles, described by function $f(x)$ in Fig.1, and LH MTM properties are suited for numerous practical applications such as lenses. For instance, GRIN spherical lenses with radial profile were discussed and introduced in [3] while GRIN MTM structure of linear profile based on split ring resonator (SRR) with different substrate thickness was analysed in [4]. The concept of GRIN MTM with SSR was used in [5] for the practical realization of the invisibility cloak in microwave range (8-10) GHZ. The analysis of EM wave propagation through GRIN MTM structure revealed the phenomenon of EM field enhancements and improved resonant absorption [8]. This phenomenon occurs in the transit area of composite RH/LH structure with a gradient refractive index profile in the area where the refractive index is zero and for oblique incident angle. The phenomenon of EM field enhancements and improved resonant absorption is characteristic of plasma, but in GRIN MTM occurs for both polarizations (TE and TM).

The practical realisation of GRIN structures could be challenging particularly at high frequencies because standard fabrication methods such as photo or electron lithography fabrication allow only planar structures with a limited number of layers. Only moderate changes of refractive index can be achieved which limits the effects predicted by analytical solutions derived for some profiles [6-7,9-10]. Therefore, numerical GRIN MTM models become more important since they enable investigation of profiles feasible in practice.

3. DISPERSIVE 3D TLM Z-TRANSFORM MODEL OF LH MTM

The details of the Z-transform based TLM approach for the simulation of various types of conventional linear time-dependent materials are given in [14]. As described in those papers, for the description of the susceptibility or conductivity time-dependence an appropriate dispersion model has to be used. Depending on the bandwidth of interest, typical realistic LH MTM responses can be characterized by using either the Drude or Lorentz dispersion models. These models are different in that the real parts of the MTM properties are negative in different regions of the frequency band. In [15,16] the Drude model has been used for both the susceptibility and the conductivity models, enabling the specification of the EM parameters of lossy LH MTM in a wide frequency range.

$$\varepsilon(\omega) = \varepsilon_0(\varepsilon_\infty - \frac{\omega_{pe}^2}{\omega^2 - j\omega\gamma_e}),\ \mu(\omega) = (\mu_\infty - \frac{\omega_{pm}^2}{\omega^2 - j\omega\gamma_m}) \quad (2)$$

$$\sigma_e(\omega) = \frac{\sigma_{e0}}{1 + j\omega\tau_e},\ \sigma_m(\omega) = \frac{\sigma_{m0}}{1 + j\omega\tau_m} \quad (3)$$

In these expressions, $\omega_{pe,m}$, $\gamma_{e,m}$ and $\sigma_{e,m0}$ are the electric or magnetic plasma frequencies and the corresponding collision frequencies and static conductivities, respectively. Electric and magnetic collision times can be expressed through corresponding collision frequencies as $\tau_{e,m} = 1/\gamma_{e,m}$. For a LH MTM which is matched to free-space, the static electric and magnetic conductivities are related by $\sigma_{m0} = \eta_0^2\sigma_{e0}$ where η_0 is the wave impedance of free-space. Both the susceptibility and the conductivity models give identical results as shown in [16]. Also, this approach can be easily modified for Lorentzian or higher-order material responses.

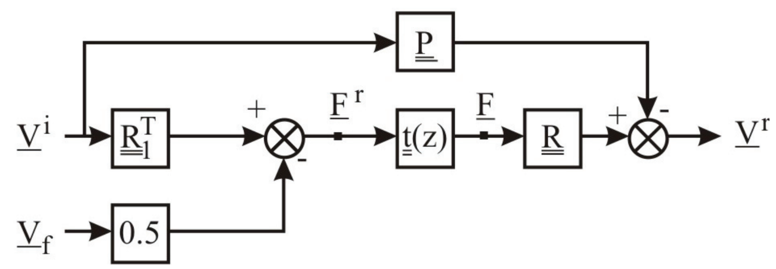

Fig. 2. Signal flow diagram of the general algorithm of TLM Z-transform method

The algorithm of Z-transform based TLM approach, which includes the scattering process in TLM nodes and the process of connecting the neighbouring nodes, can be described in general through a signal flow diagram shown in Fig. 2. Details of the inclusion of the Z-transform models into the 3D TLM method as well as the expressions for vectors and matrices from Fig. 2 can be found in [14].

The representation of frequency-dependant complex EM properties of LH MTM structures by using Drude dispersion model and the usage of bilinear Z-transform in order to transfer this dependence into the time-domain are performed in the block $\underline{t}(z)$. Fig. 3 illustrates the calculation of the y-component of the electric field and z-component of the magnetic field, performed in the block $\underline{t}(z)$ when Drude model for susceptibility is used; in the same way other components of the electric and magnetic field can be calculated.

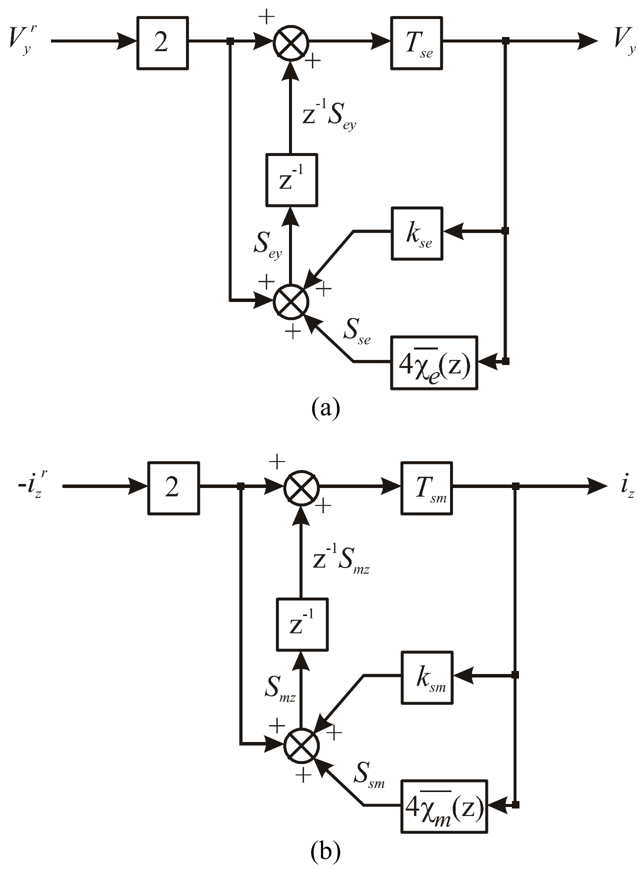

Fig. 3. Dispersive 3D TLM Z-transform model for LH MTM: (a) calculation of E_y in the time-domain, (b) calculation of H_z in the time-domain

The calculation of the accumulator $S_{se/sm}$ by using the block $4\overline{\chi_{e/m}}(z)$ in order to describe via partial fraction expansions the frequency dependence of EM properties of LH MTM as function of their values in previous time-steps is shown in Fig.4. The full description of dispersive 3D TLM Z-transform model of LH MTM, including the expressions for variables from Figs. 2-3, is given in [16].

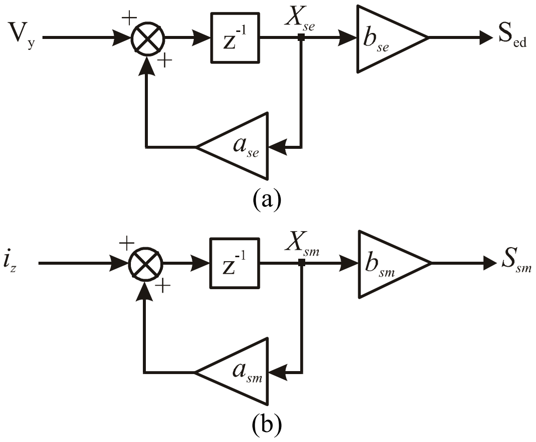

Fig. 4. Calculation of: (a) accumulator S_{se}, (b) accumulator S_{sm}.

4. GRIN MTM MODELLING EXAMPLES

The capability of the proposed dispersive 3D TLM Z-transform model to accurately describe GRIN MTM structure is illustrated for several variations of real part of refractive index across its length (from -30λ to 30λ). These profiles are shown in Fig. 5: so-called abrupt (thick solid line 1), hyperbolic tangent (dash line 2), cosine (dot line 3) and linear profile (thin solid line 4).

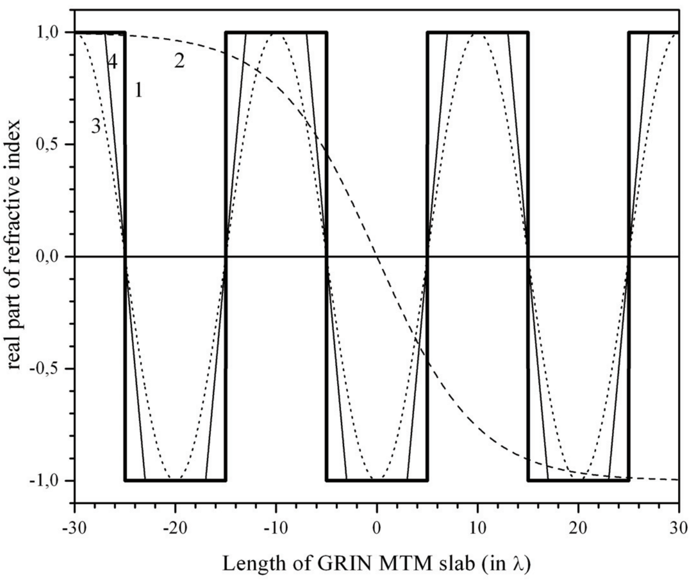

Fig. 5. Variation of real part of refractive index across the length of GRIN MTM slab for different profiles: a) abrupt (1), hyperbolic tangent (2), cosine (3) and linear (4)

There is an analytical solution for all considered profiles and it is valid for arbitrary losses and for fully arbitrary choice of the frequency-dependence of the EM properties [7-8,9-10]. In numerical 3D TLM simulations of EM wave propagation across the GRIN MTM, based on a dispersive model that incorporates the Drude function, the lossy gradient MTM slab is illuminated by a plane wave having a frequency of 300 THz (i.e. λ=1 μm). It is assumed that the imaginary part of relative permittivity and permeability is 0.02 at that frequency. The electric field distribution observed at 300 THz across the one part of GRIN MTM slab (from -10λ to 10λ) for abrupt, hyperbolic tangent, cosine and linear profiles is shown in Figs. 6, 7, 8 and 9, respectively. These results show close agreement between the analytical and dispersive 3D TLM Z-transform model results. From Figs. 7, 8 and 9 it can be seen that there is a change of the direction of the wave at the points of GRIN MTM slab where the real part of refractive index gradually changes sign (at 0 for hyperbolic tangent profile and at ± 5λ for cosine and linear profiles). Such change can not be observed for the abrupt profile in Fig.6. In addition, there is a moderate attenuation of the signal over the considered length of GRIN MTM slab for all considered profiles.

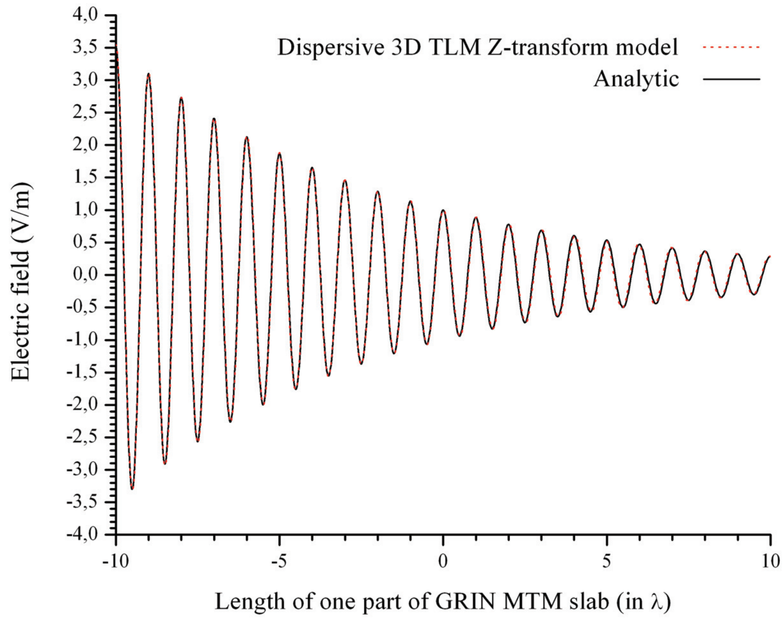

Fig. 6. Comparison of the analytical and numerical results for the lossy GRIN MTM slab with abrupt profile

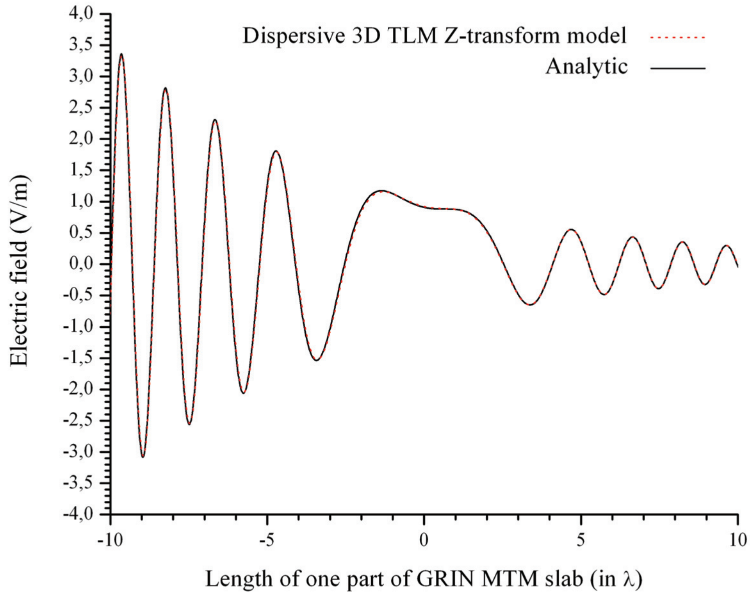

Fig. 7. Comparison of the analytical and numerical results for the lossy GRIN MTM slab with hyperbolic tangent profile

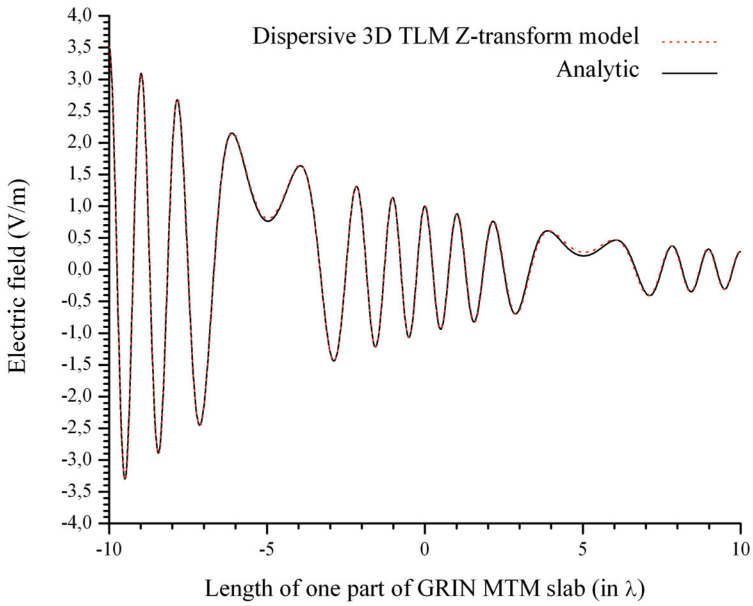

Fig. 8. Comparison of the analytical and numerical results for the lossy GRIN MTM slab with cosine profile

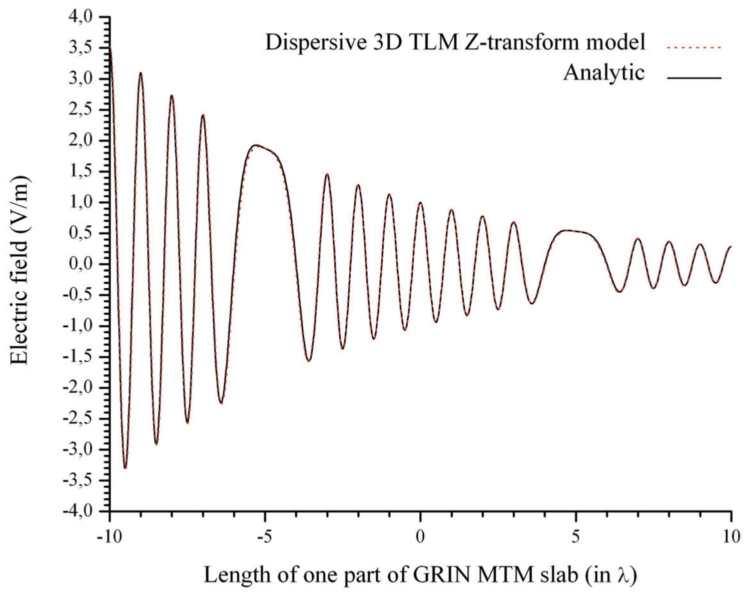

Fig. 9. Comparison of the analytical and numerical results for the lossy GRIN MTM slab with linear profile

5. Conclusion

In this paper, the enhanced Z-transform based TLM method which enables the direct time-domain modelling of metamaterials has been used to describe GRIN MTM. Composite RH/LH slabs with abrupt, hyperbolic tangent, cosine and linear variation of refractive index across the slab length have been considered. The numerical results confirmed theoretically predicted behaviour of EM wave interaction with lossy interface having some of these refractive index profiles and illustrating stability, accuracy and applicability of the proposed approach. As only one value of losses has been considered in this paper, a detailed study regarding the impact of losses on the characteristics of components based on GRIN MTM in a wide frequency range will be conducted in future research by using the presented dispersive 3D TLM Z-transform model.

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