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DESIGN OF COMPANDOR QUANTIZER FOR LAPLACIAN SOURCE FOR MEDIUM BIT RATE USING SPLINE APPROXIMATIONS *

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Abstract. *In this paper, the approximation of the optimal compressor function using spline function of the first and second degree is done. On the basis of the obtained approximate spline functions, a quantizer designing is done. The achieved SQNR is very close to that of the model of quantizer based on the nonlinear compressor function. Also, it is shown that for the medium bit rate a higher value of SQNR is achieved by using quantizer model proposed in this paper for the spline of degree 2 than by optimal compandor.*

Key words: *spline approximations, compressor functions, scalar compandor*

1. INTRODUCTION

With the scalar quantization process, the current value of continual input signal rounds up to the nearest allowed value from the finite set of discrete amplitude levels. A scalar quantizer is unambiguously determined with the set of allowable output amplitude levels, called reproduction levels, and with division of the values of input range on cells or quantization intervals. A quantizer can be uniform (all the quantization intervals are of the same width) and nonuniform (different width of the quantization intervals) [1]. Uniform quantizers are suitable for signals that have approximately uniform distribution. As most of the signals do not have a uniform distribution (usually small current values are more likely than the large ones), there is a need for using nonuniform quantizers. One of the most used methods for the realization of the nonuniform quantizer is companding technique, in which a specific compressor function is applied on an input signal. The most often used compressor functions are optimal compressor function (which gives the maximum signal to quantization noise ratio (SQNR) for the reference variance of the input signal) and a logarithmic A-law and μ-law compression functions, by which maximum

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SQNR cannot be achieved, but that provides a constant SQNR in wide range of an input signal variance [1]. These compressor functions are very complicated to be realized practically. Therefore, in order to achieve easier practical realization, approximation of the optimal compressor function and linearization of the optimal compressor function is performed. Thus linearization of A- law and μ-law, done by defining a well known segment A-law and segment μ-law compression functions [1], where support region of quantizer is divided into segments and inside each segment a linear compressor function is used, i.e. uniform segment division on cells is done. This way the piecewise uniform scalar quantizer (PUSQ) is obtained. The PUSQ is analyzed in [2]. By the algorithm realization for the speech signal [2], not only is the higher quality signal obtained than a quality defined by standard G.711, but also the bit-rate reduces for about 1bit/samples. Also, a comprehensive analysis of SQNR behavior in a wide range of variances for the PUSQ designed for a Laplacian source according to the piecewise linear approximation to the optimal compressor law is reported in [2].

Unlike the PUSQ described in [2], the number of cell per segments of PUSQ has been optimized in [3]. This contributes to the SQNR increase. The linearization of the optimal compressor function is done in [4], [5]. In [4], the linearization with unequal number of cells per segment is done, i.e. for each segment, the optimization of the number of cells is done. The demerit of this method is a high complexity of quantizer, the complexity of coding and decoding, and the impossibility to apply hierarchical coding. The analysis of compressor function for Laplacian source is shown in [5]. In this paper scalar quantization of Laplacian memoryless source is done. In order to reduce complexity and maintenance of a reasonably good performance of quantizer for different amplitude of input signal, or use in this case useful robustness of uniform quantization on variations amplitude of the input signal, we develop a new method of construction quantizer which introduces different number of cells per segments. The number of cells per segments is determined on the basis of approximate spline functions to approximate the optimal compressor function. In the reference [6], the robustness conditions of the PUSQ based on a piecewise uniform approximation to the optimal compressor law are analyzed.

In this paper, the approximation of the optimal compressor function using spline function of the first and second degree, for Laplacian probability density function (PDF) is done. The support region of quantizer is divided into $2L$=4 equal segments, each of which has an unequal number of cells. By designing quantizer based on approximate spline functions of the first and second degree, SQNR is obtained that is close to that of the model of quantizer based on the nonlinear optimal compressor function.

The rest of the paper is organized as follows: In section 2 the detailed description of nonlinear optimal compressor function is given. The procedure for determining approximate spline function of the first and second degree is described in section 3. The design of quantizer based on spline functions is described in section 4. Finally, section 5 presents numerical results and discusses their implications.

2. Optimal Nonlinear Compressor Function

The scalar quantizer is determined by the reproduction levels $\{y_1,\ldots,y_{max}\}$ and decision thresholds $\{x_0, x_1,\ldots,x_{max}\}$. The input range of the quantizer is divided into N

cells or quantization intervals $\alpha_j = [x_{j-1}, x_j)$, j=1,2,...,N. During the quantization, the quantization error, expressed with distorsion, is made. The total distortion can be found as a sum of the granular D_g and the overload D_o distortion [1]:

$$D = D_g + D_o , \tag{1}$$

that is determined as follows [1]:

$$D_g = \sum_{j=1}^{N} \int_{x_{j-1}}^{x_j} (x - y_j)^2 p(x) dx , \tag{2}$$

$$D_o = 2 \int_{x_{\max}}^{\infty} (x - y_{\max})^2 p(x) dx , \tag{3}$$

where $p(x)$ is Laplacian PDF which is defined as follows [1]:

$$p(x) = \frac{1}{\sqrt{2}\sigma} e^{-\frac{|x|\sqrt{2}}{\sigma}} . \tag{4}$$

Since $p(x)$ is an even function, the quantizer will be symmetrical. The decision thresholds and reproduction levels in the negative section of the real axis will be symmetrical to those in the positive section of the real axis. Therefore, only the positive section of the real axis will be considered. One of the methods of the realization of the nonuniform quantization is companding technique. Nonuniform quantization can be achieved by compressing the signal x using a nonuniform compressor characteristic $c(\cdot)$, by quantizing the compressed signal $c(x)$ employing a uniform quantizer, and by expanding the quantized version of the compressed signal using a nonuniform transfer characteristic $c^{-1}(\cdot)$ that is inverse to that of the compressor. The overall structure of a nonuniform quantizer consisting of a compressor, a uniform quantizer, and expandor in cascade is called compandor [1], (see Fig. 1).

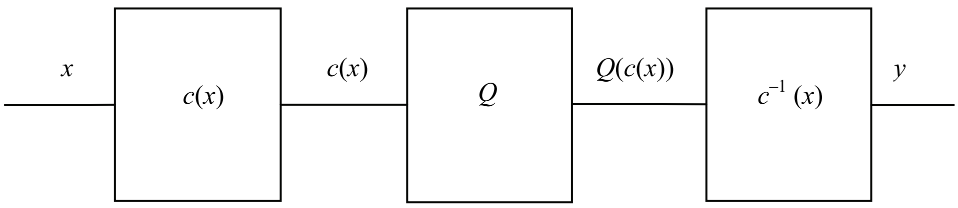

Fig. 1. Block diagram of the companding technique

The granular distortion for the companding quantizer is determined by Bennett's integral [1]:

$$D_g = \frac{1}{12N^2} \int_{-x_{\max}}^{x_{\max}} \frac{p(x)}{[\lambda(x)]^2} dx , \tag{5}$$

where $\lambda(x)$ is the density of the reproduction level and is defined as:

$$\lambda(x) = \frac{1}{N\Delta_i}, x \in (x_{i-1}, x_i], \tag{6}$$

where $\Delta_i = x_i - x_{i-1}$ denotes the i-th cell width. By substituting (6) in (5), Bennett's integral obtains the form:

$$D_g = \frac{1}{12} \int_{-x_{\max}}^{x_{\max}} \Delta_i^{\,2} p(x) dx \,. \tag{7}$$

The minimum of Bennett's integral (5) is also a minimum of the distortion of the nonuniform scalar quantizer, which means that by using the companding technique, optimal scalar quantization can be realized. By determination of the first derivative of the compressor function (for positive characteristic's section):

$$c^{'}(x) = \frac{2x_{\max}}{N\Delta_i} \approx c^{'}(y_i)\,, i = N/2+1,\ldots,N\,, \tag{8}$$

an expression for the density of reproduction level is achieved, and with substitution in (7), a known form of Bennett's integral is obtained:

$$D_g = \frac{x_{\max}^{\;2}}{3N^2} \int_{-x_{\max}}^{x_{\max}} \frac{p(x)}{[c^{'}(x)]^2} dx \,. \tag{9}$$

The performances of the quantizer is determined by SQNR which is defined as follows [1]:

$$SQNR = 10\log_{10}\left(\frac{\sigma^2}{D}\right) [\text{dB}]. \tag{10}$$

The optimal compressor function $c(x)$ by which the maximum SQNR is achieved for the reference variance of an input signal is defined as [1]:

$$c(x) = \begin{cases} x_{\max} \dfrac{\int_0^x p^{1/3}(x)dx}{\int_0^{x_{\max}} p^{1/3}(x)dx}\,, & 0 \le x \le x_{\max} \\[2ex] -x_{\max} \dfrac{\int_x^0 p^{1/3}(x)dx}{\int_{-x_{\max}}^0 p^{1/3}(x)dx}\,, & -x_{\max} \le x \le 0 \end{cases} \tag{11}$$

Without diminishing the generality, the quantizer design will be done for the reference input variance of $\sigma_{ref}^2 = 1$ and $x \ge 0$.

3. APPROXIMATIONS USING SPLINE FUNCTIONS

The theory of approximation is the area of numerical mathematics that deals with problems of replacing one function with another. In this paper, the approximation of the optimal compressor function using spline function of the first and second degree is done. The spline function is a function that consists of polynomial pieces joined together with certain smoothness conditions. A simple example is the polygonal function (or spline of degree 1), whose pieces are linear polynomials joined together to achieve continuity [7]. The points x_0, x_1,..., x_L at which the function changes its character are termed knots in the theory of splines. Such a function appears somewhat complicated when defined in explicit terms.We are forced to write [7]:

$$S(x)=\begin{cases} S_0(x), & x\in[x_0,x_1] \\ S_1(x), & x\in[x_1,x_2] \\ \vdots & \\ S_{L-1}(x), & x\in[x_{L-1},x_L]\ , \end{cases} \tag{12}$$

where

$$S_i(x)=a_i x+b_i\,. \tag{13}$$

because each piece of $S(x)$ is a linear polynomial. Such a function $S(x)$ is piecewise linear. If the knots x_0, x_1,..., x_L were given and if the coefficients a_0, b_0, a_1, b_1,..., a_{L-1}, b_{L-1} were all known, then the evaluation of $S(x)$ at a specific x would proceed by first determining the interval that contains x and then using the appropriate linear function for that interval. If the function S defined by equation (12) is continuous, we call it a first-degree spline. It is characterized by the following three properties [7].

Definition 1. A function S is called a spline of degree 1 if:

1. The domain of S is an interval $[a, b]$.
2. S is continuous on $[a, b]$.
3. There is a partitioning of the interval $a = x_0 < x_1 <_{...} < x_L = b$ such that S is a linear polynomial on each subinterval $[x_i, x_{i+1}]$.

Outside the interval $[a, b]$, $S(x)$ is usually defined to be the same function on the left of a as it is on the leftmost subinterval $[x_0, x_1]$ and the same on the right of b as it is on the rightmost subinterval $[x_{L-1}, x_L]$, namely, $S(x) = S_0(x)$ when $x < a$ and $S(x) = S_{L-1}(x)$ when $x > b$. Continuity of a function f at a point s can be defined by the condition

$$\lim_{x\to s^+} f(x)=\lim_{x\to s^-} f(x)=f(s) \tag{14}$$

Here, $\lim_{x\to s^+}$ means that the limit is taken over x values that converge to s from above s; that is, $(x-s)$ is positive for all x values. Similarly, $\lim_{x\to s^-}$ means that the x values converge to s from below.

First-degree spline, also called the polygonal function, is consisted of line segments that are connected so that the given function is continuous. The points where the function changes its shape are called knotes [7]. Approximate function $g^{s1}(x)$, by which a nonlinear compressed $c(x)$ function is approximated in this paper, for the number of segments $2L = 4$, has the following form:

$$g^{s1}(x) = \begin{cases} c(x_1) + m_1(x - x_1), & x \in [0,\ x_1] \\ \\ c(x_2) + m_2(x - x_2), & x \in [x_1,\ x_2], \end{cases} \tag{15}$$

where m_1 and m_2 are the coefficient of direction of the line given by the formula:

$$m_i = \frac{c(x_i) - c(x_{i-1})}{x_i - x_{i-1}},\ i = 1, \ldots, L. \tag{16}$$

When approximating a function, the goal is to get the approximate function that will not only be continuous but its derivative will also be continuous functions. Therefore, the approximate function is required to be smooth and without sudden changes when passing through knotes. Although the first-degree spline has a certain usage, it also has some deficiencies that make it unsuitable for significant application. Therefore, the second-degree spline is defined. We now take up the quadratic splines [7]. Let's use the letter Q to remind ourselves that we are considering piecewise quadratic functions. A function Q is a second-degree spline if it has the following properties [7].

Definition 2. A function Q is called a spline of degree 2 if:
1. The domain of Q is an interval $[a, b]$.
2. Q and Q' are continuous on $[a, b]$.
3. There are points x_i (called knots) such that $a = x_0 < x_1 <_{\ldots} < x_L = b$ and Q is a polynomial of degree at most 2 on each subinterval $[x_i , x_{i+1}]$.

In brief, a quadratic spline is a continuously differentiable piecewise quadratic function, where quadratic includes all linear combinations of the basic functions $x \rightarrow 1, x, x^2$.

The second-degree spline, also called a quadratic spline, is consisted of parabola parts between two consecutive knotes, but elected to have the same tangent at knote. The approximate function $g^{s2}(x)$, which approximates a nonlinear compressed function $c(x)$, for the number of segments $2L = 4$, has the following form:

$$g^{s2}(x) = \begin{cases} a_1 + b_1 x + d_1 x^2, & x \in [0,\ x_1] \\ \\ a_2 + b_2 x + d_2 x^2, & x \in [x_1,\ x_2]. \end{cases} \tag{17}$$

To determine coefficents of the aproximate function $g^{s2}(x)$, it is necessary to meet the defined requirements for the second-degree spline function. The number of conditions set is equal to the number of coefficients that should be determined. In fact, as we have three knotes and two subintervals, and each second-degree polynomial has three coefficients, it means determinating six coefficients in total [7]. Solving the following system of equations, where the number of conditions set is equal to the number of coefficients to be determined [7]:

$$g^{s2}(0) = 0\ , \tag{18}$$

$$c(x_1) = a_1 + b_1 x_1 + d_1 x_1^2\ , \tag{19}$$

$$c(x_1) = a_2 + b_2 x_1 + `d_2 x_1^2 \,, \tag{20}$$

$$c(x_2) = a_2 + b_2 x_2 + d_2 x_2^2 \,, \tag{21}$$

$$\lim_{x \to x_1^-} g^{s2'}(x) = \lim_{x \to x_1^+} g^{s2'}(x) \Rightarrow b_1 + 2d_1 x_1 = b_2 + 2d_2 x_1 \,, \tag{22}$$

$$g^{s2'}(x_2) = 0 \Rightarrow b_2 + 2d_2 x_2 = 0 \,, \tag{23}$$

coefficents values $a_1, b_1, d_1, a_2, b_2, d_2$ are obtained.

4. Design of Quantizer Based on Spline Functions

This Section provides us with a detailed description of the quantizer whose performances are determined by the approximative spline function of the first and second degree. The support region of the quantizer is divided into the L segments in both quadrants, where each segment is divided into specific number of cells whose size differs from segment to segment as well as from cell to cell. The maximum amplitude of the quantizer, $x_{\max}$, is final, and in the paper [4] it is shown that then its optimal value is as follows:

$$x_{\max} = \frac{3}{\sqrt{2}} \ln(N+1) \,. \tag{24}$$

The total number of the reproduction levels per segments in the first quadrant is:

$$\sum_{i=1}^{L} N_i = \frac{N}{2} \,, \tag{25}$$

where the optimal number of reproduction levels per segments, N_i, is determined from the following condition:

$$N_i = \frac{N}{2} \frac{c_i(x_i) - c_{i-1}(x_{i-1})}{c_L(x_L)} \,, \; i = 1, \ldots, L \,, \tag{26}$$

Reproduction levels are determined as the solution of the approximate spline function as follows:

$$y_{i,j} = c_i^{-1}\left(\sum_{k=1}^{i-1} N_k + \left(\frac{2j-1}{2} \right) \Delta \right), \; i = 1, \ldots, L, \; j = 1, \ldots, N_i \,, \tag{27}$$

where for the $y_{i,j}$ is taken the solution that belongs to the spline function domain. Indexes i and j indicate the j-th reproduction levels within the i-th segment The step size is equal to:

$$\Delta = \frac{2x_{\max}}{N} \,. \tag{28}$$

Cells lengths per segments of the considered quantizer are equal to:

$$\Delta_{i,j} = \frac{\Delta}{c_i'(y_{i,j})} \,, i = 1, \ldots, L, \; j = 1, \ldots, N_i \,. \tag{29}$$

Denoted by $\Delta_{i,j}$ the j-th cells lengths within the i-th segment. The granular distortion for such a designed model, based on formula (9), is:

$$D_g = 2\frac{x_{\max}{}^2}{3N^2}\sum\frac{p(y_{i,j})}{[c_i^{'}(y_{i,j})]^2}\Delta_{i,j}\ , i=1,\ldots,L,\ j=1,\ldots,N_i\ . \quad (30)$$

The overload distortion D_o is defined by (3), where the y_N is determined from the condition:

$$y_{\max} = c_L^{-1}\left(x_{\max} - \frac{\Delta}{2}\right). \quad (31)$$

The c_L represents the approximate function of the last segment.

Determining the total distortion D, that is equal to the sum of the granular distortion D_g (30) and the overload distortion D_o (3), the SQNR of this quantizer is defined by (10). The total distortion of the optimal compandor, for case $c(x)$: $[-\infty,+\infty] \rightarrow [-1,1]$, is equal to:

$$D = \frac{9}{2N^2}, \quad (32)$$

while for the case $c(x)$: $[-x_{\max}, x_{\max}] \rightarrow [-x_{\max}, x_{\max}]$, the total distortion of the optimal compandor is:

$$D = \frac{9}{2N^2}\left(1-e^{-\frac{\sqrt{2}x_{\max}}{3}}\right)+\frac{e^{-\sqrt{2}x_{\max}}(N^2+\sqrt{2}Nx_{\max}+x_{\max}^2)}{N^2}. \quad (32)$$

5. Numerical Results and Conclusion

Numerical results presented in this section are obtained for the case when the number of segments is equal to $2L = 4$ and for different values of the number of levels N (from $N = 16$ to $N = 128$). This number of segments is proposed in order to design a quantizer as simple as possible. In this paper analysis of the performance of the proposed quantizer model for the medium bit rate is done. Table 1 shows values of the parameters for a different number of levels N, based on which an approximate spline of degree 1 is formed. Substituting the appropriate values of the parameters from Table 1 in equation (15), we obtain approximate spline of degree 1.

Table 1. The values of the parameters on which a spline of degree 1 is formed

N	x_1	x_2=$x_{\max}$	$c(x_1)$	$c(x_2)$	m_1	m_2
16	3.0051	6.0102	4.8371	6.0102	1.6096	0.3904
32	3.7086	7.4172	6.3175	7.4172	1.7035	0.2965
64	4.4276	8.8552	7.8781	8.8552	1.7793	0.2207
128	5.1546	10.3092	9.4749	10.3092	1.8382	0.1618

Table 2 shows the values of the coefficients which are determined by equations (18) - (23), for a different number of levels N, based on which an approximate spline of degree 2 is formed. Substituting the appropriate values of the coefficients from Table 2 in equation (17), we obtain the approximate spline of degree 2.

Table 2. The values of the coefficient on which a spline of degree 2 is formed

N	x_1	$x_2=x_{max}$	a_1	b_1	d_1	a_2	b_2	d_2
16	3.0051	6.0102	0	2.4385	-0.2785	1.3178	1.5615	-0.1299
32	3.7086	7.4172	0	2.8139	-0.2994	3.0184	1.1861	-0.0799
64	4.4276	8.8552	0	3.1173	-0.3022	4.9468	0.8827	-0.0498
128	5.1546	10.3092	0	3.3526	-0.2938	6.9723	0.6474	-0.0314

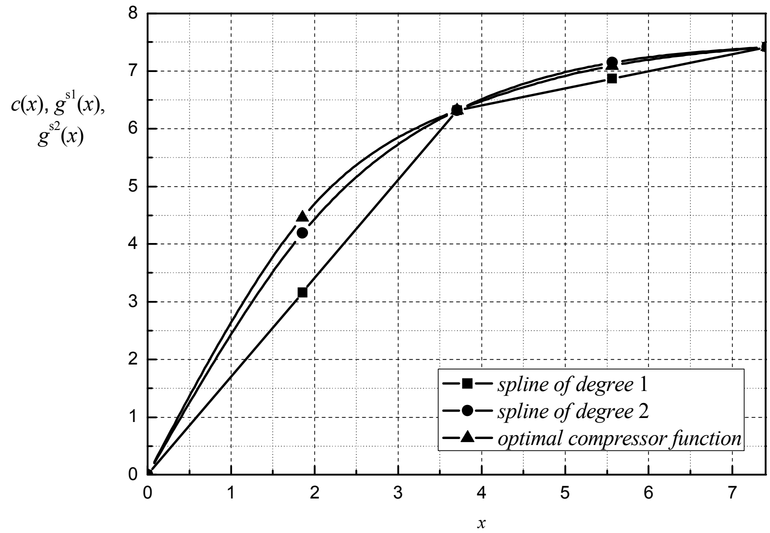

Fig. 2. Dependency of $c(x)$, $g^{s1}(x)$ and $g^{s2}(x)$ on x for N=32

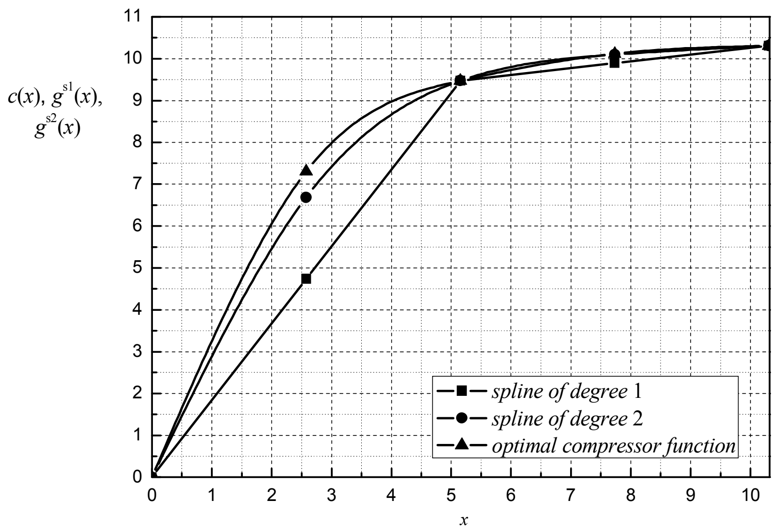

Fig. 3. Dependency of $c(x)$, $g^{s1}(x)$ and $g^{s2}(x)$ on x for N=128

In Figure 2 and Figure 3 the dependence of the optimal compression function , $c(x)$, an approximate spline of degree 1, $g^{s1}(x)$, and an approximate spline of degree 2 , $g^{s2}(x)$, on input signal x for the number of levels $N = 32$ and $N = 128$ is shown. Based on Figure 2 and Figure 3 it can be concluded that the spline of degree 2 better approximates the optimal compressor function than spline of degree 1. Also, by observation of Figure 2 and Figure 3 it can be concluded that the spline of degree 2 better approximates the optimal compressor function for the number of levels $N = 32$ than for the number of levels $N = 128$.

Table 3. The values of SQNR for the proposed quantizer model and the optimal compandor

N	SQNR^{s1} [dB]	SQNR^{s2} [dB]	SQNR^{inf} [dB]	SQNR^{opt} [dB]
16	16.4720	18.0205	17.5503	18.2404
32	21.9072	23.7028	23.5709	23.9303
64	26.3924	29.3821	29.5915	29.7752
128	31.6074	35.1126	35.6121	35.7050

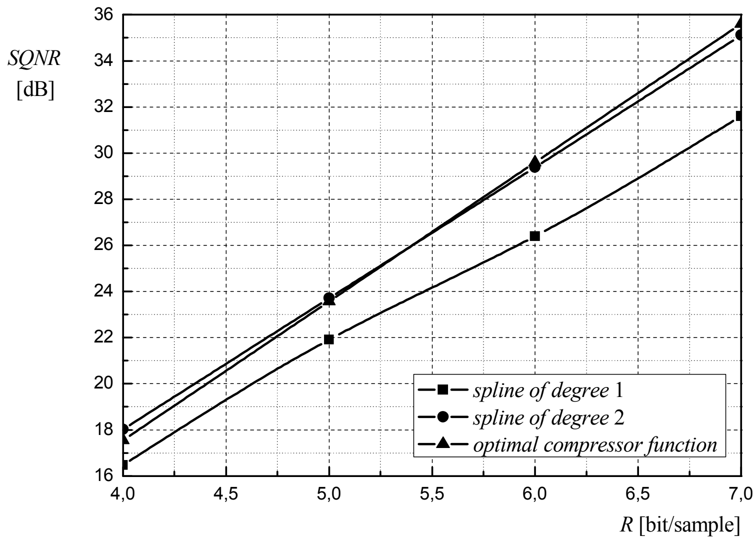

Fig. 4. Dependency of SQNR on the number bits per sample for the proposed quantizer model and the optimal compandor ($c(x)$: $[-\infty,+\infty] \to [-1,1]$)

Table 3 shows the values of SQNR of the proposed quantizer based on approximate spline of degree 1, (SQNR^{s1}), the values of SQNR of the proposed quantizer based on approximate spline of degree 2, (SQNR^{s2}), the values of SQNR of quantizer based on the optimal compression function $c(x)$, (SQNR^{inf}), $c(x)$: $[-\infty,+\infty] \to [-1,1]$ and the values of SQNR of quantizer based on the optimal compression function $c(x)$, (SQNR^{opt}), $c(x)$: $[-x_{max}, x_{max}] \to [-x_{max}, x_{max}]$, for a different number of levels N. In Figure 4 dependency of SQNR on the number bits per sample for the proposed quantizer model based on approximate spline function of the first and second degree and the optimal compandor, for the case $c(x)$: $[-\infty,+\infty] \to [-1,1]$ is shown.

Analyzing the results shown in Table 3 and Figure 4, one can notice that the design of quantizer based on approximate spline of degree 2 for the number of levels $N = 16$ and $N = 32$ achieves higher SQNR than the model of quantizer based on the nonlinear compressor function, while for the number of levels $N = 64$ and $N = 128$ achieved SQNR very close to that of the model of quantizer based on the nonlinear compressor function, for the case $c(x)$: $[-\infty,+\infty] \rightarrow [-1,1]$. Based on these results it can be concluded that for the medium bit rate a higher value of SQNR is achieved by using quantizer model proposed in this paper for the spline of degree 2 than by optimal compandor [1]. Figure 5 presents dependency of SQNR on the number bits per sample for the proposed quantizer model based on approximate spline function of the first and second degree and the optimal compandor, for the case $c(x)$: $[-x_{\max}, x_{\max}] \rightarrow [-x_{\max}, x_{\max}]$.

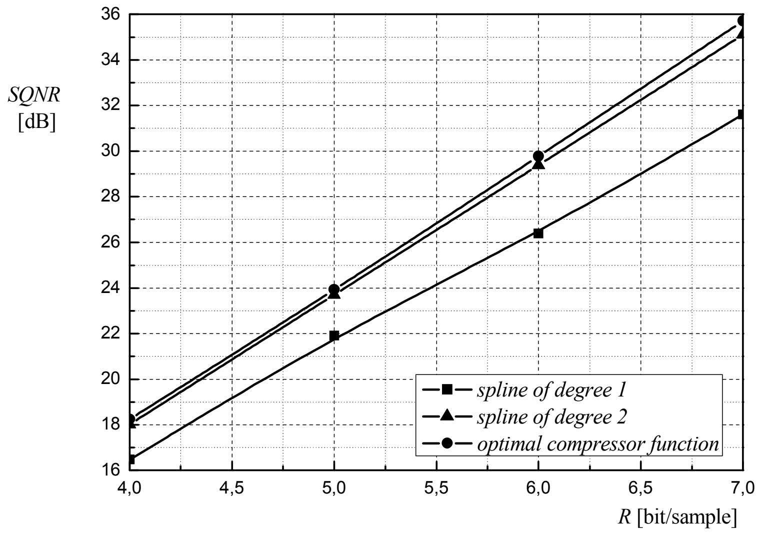

Fig. 5. Dependency of SQNR on the number bits per sample for the proposed quantizer model and the optimal compandor ($c(x)$: $c(x)$: $[-x_{\max}, x_{\max}] \rightarrow [-x_{\max}, x_{\max}]$)

Analyzing the results shown in Figure 5 and Table 3 it can be noticed that the design of quantizer based on approximate spline of degree 2 achieved SQNR very close to that of the model of quantizer based on the nonlinear compressor function. Comparing the performance of the proposed quantizer model with the optimal compandor [1], it can be concluded that the proposed model is a very effective solution because a simple quantizer model achieves a high quality signal.

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