

EQUIVALENT CIRCUIT IMPROVEMENT METHOD FOR INDUCTION MOTOR EFFICIENCY

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Abstract. *The parameters of the equivalent Γ -circuit, which is proposed for this improvement method, may be derived by data from the no-load test and locked-rotor test at reduced frequency, but direct calculations, i.e. simpler than parameters of the conventional T -circuit. And as for the simpler Γ -circuit, the operating characteristics (losses, output power, efficiency and power factor) are calculated with no loss of accuracy. The accuracy of the proposed procedure was verified on the basis of results of comparative calculations for motor given in the literature.*

Key words: *equivalent T -circuit, Γ -circuit, induction motors, parameters estimation, efficiency*

1. INTRODUCTION

The methods for determination of three-phase induction motors efficiency, given in standards IEEE 112 1996 [1] and IEC 60034- 4 [2], include five methods. These may be divided into two parts [3].

(1) **Input–output methods** with direct measurement of input (electrical) power and direct measurement of output (mechanical) power:

Method A (brake method) with direct measurement of input (P_{inp}) and output (P_{out}) power, and efficiency (η) are calculated, $\eta = P_{\text{out}} / P_{\text{inp}}$;

Method B with direct measurement of input (P_{inp}) and output (P_{out}) power, with direct measurement of the stator winding (I^2R_s), rotor winding (I^2R_r), core losses (P_{fe}) and windage-friction losses (P_{fw}), and indirect measurement of the stray-load loss;

Method C (duplicate machines method), [1], and it is called a pump-back test because the power taken from the line (P_{inp1}) is returned again to the same system (P_{out2}) except for the losses in the test motor and the generator ($P_{\text{los2}} = P_{\text{los1}}$). Both power measurements, P_{inp1} and P_{out2} , are electrical power, and calculated values losses $P_{\text{los1}} + P_{\text{los2}} = P_{\text{inp1}} - P_{\text{inp2}}$ which are more accurate than the corresponding values calculated as the difference between the electrically measured power and mechanically measured power.

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(2) Methods with **determination of losses without output (mechanical) power measurement:**

Method E [1] - Input measurement method with direct measurement of all losses: I^2R_s , I^2R_r , P_{fe} , P_{fw} , from no-load test and from load test (full-load test, or reduced load at reduced voltage); and

Method F [1] uses the exact equivalent circuit (T- circuit, Fig 1) with parameters determined from locked-rotor test (impedance tests) and no-load tests.

For the last methods (Method E, Method F), stray load losses can be measured directly: using the removed-rotor and reverse-rotation tests to directly measure the fundamental-frequency and high-frequency stray load-losses. It is true these methods are too invasive and user unfriendly. For these reasons, stray-load loss standardized values can be used (Method E1 and Method F1, [1], [2]).

In addition to the efficiency and power factor at rated load, the efficiency and power factor at other load points are also usually needed. When using method B or Method E [1], the operating characteristics at various load can be determined by repeating the test at six load points ranging from 1/4 to 6/4 times rated load and plotting a curve, $\eta(P/P_n)$. However, this is costly and time-consuming, particularly for larger machines. The advantage of the equivalent circuit method, Method F or F1, is that the performance of the motor can be predicted at any load when the impedance values are known. The six impedances of equivalent circuit (see Fig. 1):

- stator resistance, R_s ,
- stator leakage reactance, $X_{\sigma s}$,
- magnetizing reactance, X_m ,
- iron-loss resistance, R_{fe}
- rotor leakage reactance, $X_{\sigma r}$, and
- rotor resistance, R_r .

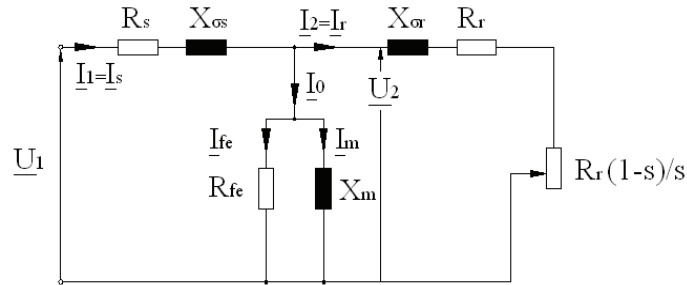


Fig. 1. Induction machine, T-model with equivalent iron loss resistor

Although this method is expected to be quite accurate, it is still considered to be too intrusive for routine field use [4]. To my mind, there are two reasons for this. Firstly, the impedance values can change a great deal when the motor speed varies between standstill and no-load. Namely, with increasing motor slip (s), rotor resistance (R_r) increases and rotor leakage reactance ($X_{\sigma r}$) decreases. Due to that, the author would like to note that it is

necessary to include a calculated frequency correction of the tested resistance and reactance (R_r and $X_{\sigma r}$), by extrapolation, even when the impedance test is performed at $f=0.25f$, particularly for the great power motor [4].

The other reason is that the complicated iterative procedure is used to determine the parameters of equivalent circuit in Fig. 1. The iterative procedure requires one to either know the design value of the ratio $X_{\sigma s}/X_{\sigma r}$ or to use the standard NEMA design values. Contrary to that, the parameters of the equivalent circuit in Fig. 2 (Γ -circuit) may be derived directly from the no-load test and locked-rotor test at reduced frequency [5], i.e. simpler than parameters of the conventional T-circuit (see Fig. 1) – recalculated by equation (A1)-(A3) in Appendix, whereas $X_{\sigma s}$ and X_m deviate less than 0.1 % from the values of the preceding step. And in simple Γ -models operating characteristics (losses, efficiency, power factor, torque, etc.) are calculated with no loss of accuracy [6].

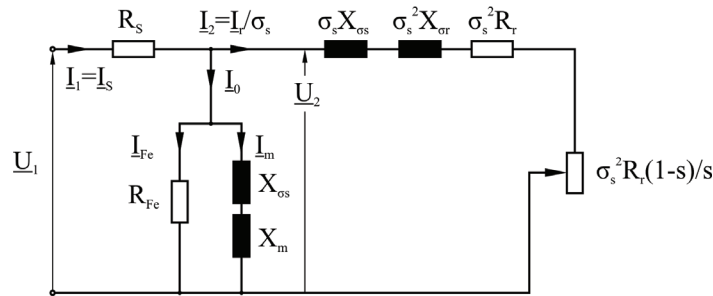


Fig. 2. Induction machine, Γ -circuit with equivalent iron loss resistor

2. EQUIVALENT CIRCUIT IMPROVEMENT METHOD

2.1. Equivalent Γ -circuit

The T-circuit (see Fig. 1) can be transformed into simpler models Γ -model, (see Fig. 1) with no loss of information or accuracy [5], [6]. The parameters of equivalent Γ -circuit (see Fig. 2), [5] may be derived directly from the no-load test and locked-rotor test at reduced frequency [5], [6], i.e. simpler than parameters of conventional T-circuit (see Fig. 1). Equivalent Γ -circuit differs from T-circuit in that the magnetizing arm is connected immediately after the resistance stator, R_s (see Fig. 2).

The reactance of the magnetizing arm in the Γ -circuit is taken equal to $(X_{\sigma s} + X_{\sigma r})$, and it is assumed to carry a fictitious magnetizing current (I_0) which is the same as the actual magnetizing current (I_0) in the T – circuit under ideal no-load condition, when slip $s=0$ and $R_{fe} = \infty$. The current I_2 in the right-hand arm of the Γ -circuit (the rotor arm) may be found subject to the requirement that the primary current $I_1 = I_s$ should remain unchanged in the Γ -circuit. The parameters of Γ -circuit (see Fig. 2) may be expressed by the parameters of T-circuit (see Fig. 1), i.e.:

- the reactance in magnetizing branch ($X_{\sigma_s} + X_m$)

$$(X_{\sigma_s} + X_m) = X_{\sigma_s} + X_m \quad (1)$$

- the reactance in the load branch ($\sigma_s X_{\sigma_s} + \sigma_s^2 X_{\sigma_r}$)

$$(\sigma_s X_{\sigma_s} + \sigma_s^2 X_{\sigma_r}) = \sigma_s X_{\sigma_s} + \sigma_s^2 X_m \quad (2)$$

- the resistance in the load arm ($\sigma_s^2 R_r$)

$$(\sigma_s^2 R_r) = \sigma_s^2 R_r \quad (3)$$

where σ_s – is the real coefficient, which is calculated by equation

$$\sigma_s = 1 + \frac{X_{\sigma_s}}{X_m} \quad (4)$$

The T- circuit (see Fig. 1) is actually more complex than necessary. This Γ -model is nearly the same as the "Γ-model" (approximate equivalent circuit, which has frequently been presented in traditional literature, [7]-[10], in which the magnetizing branch ($X_{\sigma_s} + X_m$) is moved out to the stator terminal (see Fig. 2).

2.2. The Γ -equivalent circuit parameters determination

The T-circuit (see Fig. 1) as said, the parameters of Γ -circuit, ($X_{\sigma_s} + X_m$), ($\sigma_s X_{\sigma_s} + \sigma_s^2 X_{\sigma_r}$), R_{fe} and ($\sigma_s^2 R_r$), may be derived directly from the no-load test and locked-rotor test at reduced frequency. Reactances of Γ -circuit are determined by calculating on the basis of Fig. 2:

- The reactance in magnetizing branch ($X_{\sigma_s} + X_m$) is calculated by no-load test date, i.e.

$$X_{\sigma_s} + X_m = \frac{U_0}{I_0} \quad (5)$$

because the no-load voltage drops on the stator resistance is negligible.

- The reactance in load branch ($\sigma_s X_{\sigma_s} + \sigma_s^2 X_{\sigma_r}$) is determined by the locked-rotor test. First the value ($\sigma_s X_{\sigma_s} + \sigma_s^2 X_{\sigma_r}$)_{lr} is calculated from test data recorded during the locked-rotor test at reduced frequency (Subscript "lr"-quantities locked-rotor), i.e.:

$$(\sigma_s X_{\sigma_s} + \sigma_s^2 X_{\sigma_r})_{lr} = \frac{Q_{lr} \cdot (X_{\sigma_s} + X_m) \cdot (f_{lr} / f_n)}{3I_{lr}^2 (X_{\sigma_s} + X_m) \cdot (f_{lr} / f_n) - Q_{lr}} \quad (6)$$

and, the reactance in load branch ($\sigma_s X_{\sigma_s} + \sigma_s^2 X_{\sigma_r}$), for the line frequency (f_n), is:

$$(\sigma_s X_{\sigma_s} + \sigma_s^2 X_{\sigma_r}) = (f_n / f_{lr}) \cdot (\sigma_s X_{\sigma_s} + \sigma_s^2 X_{\sigma_r})_{lr} \quad (7)$$

Values of corresponding reactances may be calculated directly, successively, by expressions (5), (6) and (7). Contrary to classical procedure (Appendix 1), one should assume neither ratio X_{σ_s}/X_m value nor values of stator leakage reactance (X_{σ_s}) and magnetizing reactance (X_m).

Resistance in the load branch, referred to the stator side, is calculated by the following expression

$$\sigma_s^2 R_{r,lr} = \frac{P_{lr} - 3R_s I_{lr}^2}{3I_{lr}^2} \cdot \left(1 + \frac{(\sigma_s X_{os} + \sigma_s^2 X_{or})_{lr}}{X_{os} + X_m}\right)^2 \quad (8)$$

In deriving equation (9), the iron losses (P_{fe}) are neglected ($P_{fe} \approx 0$), as they are very small at locked-rotor test at reduced frequency ($f_{lr} \approx 0.25f_n$) and reduced voltage ($<0.20U_n$), in comparison with electromagnetic power ($P_{lr} - 3R_s I_{lr}^2$).

For calculating the currents, losses and reactive load it is not necessary to know the motor parameters which are denoted in Fig. 1. But, if this is not desired, the values of the stator leakage reactance (X_{os}), rotor leakage reactance (X_{or}) and magnetizing reactance (X_m), would be determined, on the basis of the calculated values of the reactances in magnetizing branch ($X_{os} + X_m$), in the load branch ($\sigma_s X_{os} + \sigma_s^2 X_{or}$) and ratio X_{os}/X_{sr} value for the given (design motors, A, B, C or D) machine.

It is not necessary to know the value of the stator winding leakage coefficient (σ_s), which is calculated by equation (4), for calculation by this proposed method. Symbol " σ_s " is marked in Fig. 2 in the compound denoting certain reactances (i.e. $\sigma_s X_{os}$, $\sigma_s^2 X_{or}$), in order to show the mutual dependence of circuits parameters in Fig. 1 and Fig. 2. For calculation of the currents, losses and reactive loads, it is sufficient to know the motor (compound) parameters which are determined by equations from (5)-(9) and stator winding resistance (R_s), which is identical for both schemes.

Resistance R_{fe} , which is represented to the iron losses (P_{fe}) at rated voltage, is calculated by equation

$$R_{fe} = \frac{3U_0^2}{P_{fe}} \quad (9)$$

where, P_{fe} is determined according to standard no-load test [1, 2]. Calculated values by equation (9) are greater by 3-5 % than the corresponding values calculated by equation (A5), but it is not reflected on increased iron losses for some particular regime (Example 1 in Section IV). In addition to that, it is essentially more correct to connect resistance R_{fe} immediately after the stator resistance (Fig. 1) as the iron losses also arose from the leakage flux of the stator coil.

If the resistance R_{ad} which is represented to the stray load losses to insert in the equivalent circuit, it is logical to insert R_{ad} in the load arm on the Fig. 1 and Fig. 2, [11]-[13].

3. COMPARISON OF PARAMETERS VALUES FOR T-CIRCUIT AND Γ -CIRCUIT

Methods Official method application based on the T-scheme is illustrated by the example from literature [3]. For that same example, parameters of proposed Γ -scheme are determined. In both cases, parameters are estimated on the basis of data from the same no-load test and same locked-rotor test [3] at reduced frequency ($f_{lr} \approx 0.25f_n = 15$ Hz, line frequency $f_n = 60$ Hz).

3.1. Parameters determination of the Γ -equivalent circuit

A three-phase, 2300 V, 60 Hz, four-pole, delta-connected induction motor, design dates are [3]:

- 800 HP (1HP=0.746 kW), 2300 V (Δ),
- $n_s=1800\text{r/min}$ ($f_n=60\text{ Hz}$), $I_n=178\text{ A}$ ($I_{ph\Delta}=102.8\text{ A}$),
- stator winding resistance $R_s=0.22005\ \Omega$, at measured ambient temperature $T_a=25\ ^\circ\text{C}$.

No-load test data [3]:

- $U_0=2300\text{ V}$ (Δ), $I_{0n}=42.8\text{ A}$, $P_{0n}=8.800\text{ kW}$,
- stator winding resistance $R_s=0.234000\ \Omega$, at measured temperature $T_a=41.5\ ^\circ\text{C}$.
- No-load test data for the 3 lowest voltage values
 - $U_0=460\text{ V}$ (Δ), $P_{0n}=3.920\text{ kW}$,
 - $U_0=300\text{ V}$ (Δ), $P_{0n}=3.760\text{ kW}$,
 - $U_0=260\text{ V}$ (Δ), $P_{0n}=3.720\text{ kW}$,

Based on these data, i.e. by $P_0(U_0^2)$ curve, using official method [1, 2, 3], are determined:

- Friction and windage losses value, $P_{fw}=3.642\text{ kW}$,
- Iron losses at rated voltage, $P_{fe0}=4.730\text{ kW}$.

Corresponding resistance value $R_{fe}=3181.976\ \Omega$, is calculated by equation (A5), for $X_m=90.767\ \Omega$ and starting value $X_{\sigma s}/X_{\sigma r}=1$, by official method (Appendix A.2).

Locked-rotor test at reduced frequency ($f_{lr} \approx 0.25f_n=15\text{ Hz}$) data [3]:

- $U_{lr}=145.00\text{ V}$ (Δ), $I_{lr}=180.00\text{ A}$, $P_{lr}=18.450\text{ kW}$,
- Stator winding resistance $R_s=0.226834\ \Omega$, at measured temperature $T_s=33.0\ ^\circ\text{C}$.

Using this data, reactive power value is calculated (Q_{lr})

$$Q_{lr} = \sqrt{(1.73U_{lr}I_{lr})^2 - P_{lr}^2} = 41.270\text{ k var} .$$

Calculations:

The T-scheme parameters values, first, are calculated based on the equation from (A1)-(A7), and the obtained results are given in Table 1, column "Official method". Stable values of X_m and $X_{\sigma s}$ (within 0.1 %) are obtained in 4th iteration.

According to the proposed method, Γ -scheme (Fig. 2), parameters values ($X_{\sigma s}+X_m$), $(\sigma X_{\sigma s}+\sigma^2 X_{\sigma r})$, $(\sigma^2 R_r)$, R_{fe} and P_{fe} , also, are calculated according to the equation from (5)-(9) and (17)-(19), and the obtained results are given in Table 1, column "New Γ -circuit".

Table 1. Differences (%) in the obtained results by official method and new method

| Parameters | Equation Number | Official T-circuit | New Γ -circuit | Differences % |
|----------------------------------------------------------------|---------------------------|--------------------|-----------------------|---------------|
| 1. R_s Fig. 1 and Fig. 2 | | 0.27941 | 0.27941 | |
| 2. X_m Fig.1 $\downarrow\uparrow$ - Iteration | (A1) $\downarrow\uparrow$ | 90.7677 | | |
| 3. $X_{\sigma s}$ Fig.1 $\downarrow\uparrow$ -Iteration | (A2) $\downarrow\uparrow$ | 2.4399 | | |
| 4. $X_m + X_{\sigma s}$ | | 93.2076 | | |
| 5. $(X_{\sigma s} + X_m)$ Fig. 2 | (5) | | 93.0750 | |
| 6. $6=5-4$ (%) | | | | -0.1422 % |
| 7. $X_{\sigma r}$ Fig. 1 | (A4) | 2.7702 | | |
| 8. $\sigma_s = 1 + X_{\sigma s}/X_m$ | (4) | 1.02688 | | |
| 9. $\sigma_s X_{\sigma s} + \sigma_s^2 X_{\sigma r}$ Fig. 1 | | 5.42661 | | |
| 10. $(\sigma_s X_{\sigma s} + \sigma_s^2 X_{\sigma r})$ Fig. 2 | (6) and (7) | | 5.42345 | |
| 11. $11=10-9$ (%) | | | | -0.0582 % |
| 12. R_r Fig. 1 | (A8) | 0.363692 | | |
| 13. $\sigma_s^2 \cdot R_r$ | | 0.383507 | | |
| 14. $(\sigma_s^2 R_r)$ Fig. 2 | (8) | | 0.38347 | |
| 15. $16=15-14$ (%) | | | | -0.00975 % |
| 16. R_{fe} Fig. 1 | (A5) | 3181.976 | | |
| 17. R_{fe} Fig. 2 | (9) | | 3355.174 | |
| 18. $18=17-16$ (%) | | | | 5.4431 % |
| 19. U_{2T} | (A7) | 2205.65 | | |
| 20. $U_{2\Gamma}$ | (9.22) | | 2263.388 | |
| 21. $21=20-19$ (%) | | | | 2.6177 % |
| 22. P_{fe} Fig. 1 | (A5) | 4586.67 | | |
| 23. P_{fe} Fig. 2 | (19) | | 4580.72 | |
| 24. $18=17-16$ (%) | | | | -0.1297 % |

3.2. Comparison of parameters corresponding values calculated for the Γ -circuit and the T-circuit

The obtained values for T-circuit parameters are compared with the corresponding values Γ -circuit obtained by the new method, i.e.:

- Sum $X_{\sigma_s} + X_m = 2.4399 + 90.7677 = 93.2076 \Omega$ is greater only by 0.1422 %, in comparison with compound parameter value $(X_{\sigma_s} + X_m) = 93.0750 \Omega$, calculated by equation (5);
- Sum $\sigma_s X_{\sigma_s} + \sigma_s^2 X_{\sigma_r} = 1.027 \cdot 2.4399 + 1.027^2 \cdot 2.7702 = 5.4266 \Omega$ is greater only by 0.0582%, in comparison with compound parameter value $(\sigma_s X_{\sigma_s} + \sigma_s^2 X_{\sigma_r}) = 5.42345 \Omega$ calculated by equation (6), and
- Value $\sigma_s^2 \cdot R_r = 1.02688^2 \cdot 0.363692 = 0.383507 \Omega$ is greater only by 0.0097 %, in comparison with value $(\sigma_s^2 R_r) = 0.38347 \Omega$, calculated by equation (8), i.e. in within 1%.

Such obtained differences are very small (around 1/100 %), i.e. they are within an error in a round digit especially in calculations by the standard iteration method. These numerical results have confirmed an evident presumption that a T-circuit (see Fig.1) can be transformed into a simpler Γ -circuit (Fig. 2) with no loss of information or accuracy [5], [6].

4. COMPARISON OF CURRENTS, LOSSES AND EFFICIENCY CALCULATED BY T-CIRCUIT AND Γ -CIRCUIT

The parameters of equivalent circuit in Fig.2 (equation from 5-9) may be found subject to the requirement that the stator current ($I_1 = I_s$) should remain unchanged in the Γ -circuit. The current in the load arm of Γ -circuit in Fig. 2 is

$$I_{2\Gamma} = \frac{I_r}{\sigma_s} \quad (10)$$

where I_r – rotor current.

Stator winding losses (P_{R_s}), calculated for T-circuit ($P_{R_s(T)}$) or Γ -circuit ($P_{R_s(\Gamma)}$), have equally values, i.e.:

$$P_{R_s(\Gamma)} = 3R_{R_s} I_s^2 = P_{R_s(T)} = P_{R_s} \quad (11)$$

where R_s – stator winding resistance obtained by measurement, and corrected to specified temperature.

Rotor winding losses (P_{R_r}), calculated for T-circuit ($P_{R_r(T)}$) or Γ -circuit ($P_{R_r(\Gamma)}$), have equally values, also, i.e.:

$$P_{R_r(\Gamma)} = 3 \left(\frac{I_r}{\sigma_s} \right)^2 \cdot (\sigma_s^2 R_r) = 3I_r^2 R_r = P_{R_r(T)} = P_{R_r} \quad (12)$$

Electromechanical (electromagnetic) power (P'_{meh}) developed by the electromagnetic torque, calculated for T-circuit ($P'_{\text{meh}(T)}$) or Γ -circuit ($P'_{\text{meh}(\Gamma)}$), have equally values, also, i.e.:

$$P'_{\text{meh}(\Gamma)} = 3 \left(\frac{I_r}{\sigma_s} \right)^2 \cdot (\sigma_s^2 R_r \frac{1-s}{s}) = 3I_r^2 R_r \frac{1-s}{s} = P'_{\text{meh}(T)} \quad (13)$$

The useful mechanical power (P_{meh}) transmitted by the shaft to the associated driven machine is less than mechanical power P'_{meh} applied to the rotor by an amount equal to

the friction and windage losses (P_{fw}) and the additional (or stray) load losses (P_{LL}) in windings and cores, associated with higher-harmonics currents and fields, i.e.:

$$P_{mech} = P'_{mech} - P_{fw} - P_{LL} \quad (14)$$

Calculated values R_{fe} , by equation (9) are greater by 3-5 % than the corresponding vales by equation (A5). But this difference in calculating the resistance values (R_{fe}) by equation (A5) and (14), does not practically carry any change in the iron losses, calculated for T-circuit ($P_{fe(T)}$) or Γ -circuit ($P_{fe(\Gamma)}$), i.e.:

$$P_{fe(T)} \approx P_{fe(\Gamma)} \approx P_{fe} \quad (15)$$

Physically observed, it is correct to connect resistance R_{fe} as in Fig. 2, because the iron losses (P_{fe}) originate from stator leakage flux. Namely, new standard [2] requires that the iron losses be determined by the no-load test, i.e. with curve $P_{fe}=f(U_0)$ for voltage value $U_0=U_2$, which is calculated by equation:

$$U_2 = U_{2\Gamma} = \sqrt{(U_0 - \frac{\sqrt{3}}{2} \cdot R_s I_1 \cos \varphi)^2 + (\frac{\sqrt{3}}{2} \cdot R_s I_1 \sin \varphi)^2} \quad (16)$$

or, approximative

$$U_{2\Gamma} \approx U_0 - (\sqrt{3}/2)R_s I_1 \cos \varphi \quad (17)$$

where

$$\cos \varphi = \frac{P_1}{\sqrt{3} \cdot U_1 I_1}; \quad \sin \varphi = \sqrt{1 - \cos^2 \varphi} \quad (18)$$

i.e. for voltage values after stator winding resistance (R_s) on the Γ - circuit (Fig.2). *Accuracy would be increased when it should calculate the decreased iron loss reduction due to decreased loaded machines saturation.*

The iron losses, for calculated U_2 value in given regime, may be calculated approximative and by equation

$$P_{fe,\Gamma} = \frac{3U_{2\Gamma}^2}{R_{fe,\Gamma}} \quad (19)$$

Friction and windage losses (P_{fw}) are determined by the no-load test [1], [2], i.e. by curve of loses against the voltage squared (U^2), curve ($P_O(U^2)$). Extrapolate a straight line to zero voltage. They intercept with the zero voltage axes is windage and friction losses (P_{fw}). These losses are considered to be independent of the load.

Stray-load losses (P_{LL}) are calculated from a value ($P_{LL,N}$) at rated load, either assigned value or measured values, i.e.:

$$P_{LL} = P_{LL,N} \cdot \frac{\sqrt{I_1^2 - I_0^2}}{\sqrt{I_{1n}^2 - I_0^2}} \quad (20)$$

The total losses ($P_{\Sigma\gamma}$), determined for T-circuit ($P_{\Sigma\gamma(T)}$) or Γ -circuit ($P_{\Sigma\gamma(\Gamma)}$), have equally values, i.e.:

$$P_{\Sigma\gamma} = P_{Rs} + P_{Rr} + P_{fe} + P_{fw} + P_{LL} \quad (21)$$

The efficiency (motoring operation) results from:

$$\eta = \frac{P_{\text{mech}}}{P_1} \quad (22)$$

And with simple Γ -models (Fig. 2) operating characteristics (losses, efficiency, power factor, torque, etc.) are calculated with no loss of accuracy [5].

5. CONCLUSION

When it is practically not possible to do the test under load, the energy characteristics (losses, output power, efficiency and power factor) are determined based on the equivalent T-scheme, [7]- [10]. The machine parameters in T-scheme are calculated on the basis of data from the no-load test and locked-rotor test at reduced frequency, but only by iteration calculations.

The parameters of the equivalent Γ -circuit, which is proposed for this method, may be derived, also, by data from the no-load test and locked-rotor test at reduced frequency, but direct calculations, i.e. simpler than parameters of the conventional T - circuit. And as for simpler Γ -models, the operating characteristics (losses, output power, efficiency and power factor) are calculated with no loss of accuracy. The proposed Γ -circuit is nearly the same as the "T-circuit", which has frequently been presented in literature, but the magnetizing arm ($X_{\text{cs}}+X_{\text{m}}$) is connected to the stator resistance.

New method is especially convenient for Test method E/F, Standard [1], with the equivalent circuit parameters calibrated by the load-point test, [11] and [12].

A. APPENDIX-EQUIVALENT CIRCUIT METHOD [1, 2, 3]

A.1. General

When the test under load is not done, operating characteristics (efficiency, power factor, torque, etc.) are calculated based on the conventional equivalent T-circuit shown in Figure 1 [1, 2], including an equivalent iron-loss resistor (R_{fe}) parallel to the main field reactance (X_{m}). The rotor side parameters and quantities are referred to the stator side. The machine parameters in equivalent circuits (X_{m} , X_{cs} , X_{cr} , R_{fe} and R_r) are derived from test data recorded during a no-load test and the locked-rotor test at reduced frequency. Accurate prediction of machine characteristics in the normal operating range will depend primarily on the closeness by which R_r represents the actual rotor resistance to currents of low frequency and, secondarily, on the closeness by which X_{cr} represents the actual rotor leakage reactance to currents of low frequency.

A.2. Equivalent circuit parameters

The method is based on the T-model per-phase circuit (Figure 1). Unless otherwise indicated in the following, all voltages, currents and impedances are per phase for a three-phase machine in Y-connection; powers and reactive powers are per complete machine. The equivalent circuit parameters are determined in the following steps [1, 2]:

- Magnetizing reactance (X_m) is calculated from the no-load test

$$X_m = \frac{3U_0^2}{Q_0 - 3I_0^2 X_{cs}} \cdot \frac{1}{(1 + X_{cs} / X_m)^2} \quad (A1)$$

- Stator leakage reactance is determined (X_{cs}) from the locked-rotor test at 25% rated frequency ($f_{lr} \approx 0.25f_n$, f_n -line frequency). First ii calculated $X_{cs,lr}$ value from test date recorded during the locked-rotor test at reduced frequency (Subscript "lr"-quantities locked-rotor), i.e.:

$$X_{cs,lr} = \frac{Q_{lr}}{3I_{lr}^2 \left(1 + \frac{X_{cs}}{X_{or}} + \frac{X_{cs}}{X_m}\right)} \cdot \left(\frac{X_{cs}}{X_{or}} + \frac{X_{cs}}{X_m}\right) \quad (A2)$$

and, stator leakage reactance (X_{cs}), for the line frequency (f_n) is:

$$X_{cs} = (f_n / f_{lr}) \cdot X_{cs,lr} \quad (A3)$$

Application of the method to cage induction machines requires the designed values to be available. Calculate using following designed values as starting values

1. X_{cs}/X_m ratio (i.e. relationship between X_{cs} and X_m is to be assumed). When design details are available, use the calculated ratio X_{cs}/X_m . Otherwise, use

- $X_{cs}/X_m=1$, for design B, design D, and wound -rotor motors;
- $X_{cs}/X_m=0.67$, for design B motors, and
- $X_{cs}/X_m=0.43$, for design C motors.

2. Values of stator leakage reactance (X_{cs}) and magnetizing reactance (X_m) must be assumed for determination of X_m and X_{cs} values 1st iterations, by expressions (A1) and (A2);

3. Solve Eq 3 for X_{cs} (A3), in 1st iterations;

4. X_m and X_{cs} values 2nd iterations are determined from the X_{cs} and X_{cs}/X_m values 1st iterations;

5. Recalculate until X_{cs} and X_m deviate less than 0.1 % from the values of the preceding step.

The rotor leakage reactance (X_{or}) is

$$X_{or} = \frac{X_{cs}}{X_{cs} / X_{or}} \quad (A4)$$

Determine the resistance per phase equivalent to the iron losses (P_{fe}) at rated voltage from equation [1, 2]

$$R_{fe,T} = \frac{3U_0^2}{P_{fe}} \cdot \frac{1}{(1 + X_{cs} / X_m)^2} \quad (A5)$$

The iron losses, for calculated U_{2T} value in given regime, may be calculated approximative and by equation

$$P_{fe,T} = \frac{3U_{2T}^2}{R_{fe,T}} \quad (A6)$$

where

$$U_{2T} \approx (U_1 - 0.5\sqrt{3}R_s I_1 \cos \varphi) \cdot \frac{1}{(1 + X_{cs} / X_m)} \quad (A7)$$

Determine the uncorrected rotor resistance from the locked-rotor test at 25% rated frequency ($f_{lr} \approx 0.25f_n$, f_n -line frequency) by equation

$$R_{r,lr} = \left(\frac{P_{lr}}{3I_{lr}^2} - R_s \right) \cdot \left(1 + \frac{X_{\sigma r}}{X_m} \right)^2 - \left(\frac{X_{\sigma r}}{X_{os}} \right)^2 \cdot \frac{X_{os,lr}^2}{R_{fe}} \quad (A8)$$

Test Code [1, 2] includes alternate method for determining motor parameters:

"Three-phase locked-rotor impedance test at rated frequency, at approximately 50% of rated frequency, and at a maximum of 25% of rated current. Curves shall be developed from these three test points and used to determine the values of total reactance and rotor resistance at required frequency ($f_r \approx sf_i$). See 6.7.1.2.1 [1] for details." Sample examples and analysis using equivalent circuit method (IEEE Methods F, E/E) are given in [3].

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