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MULTI BAND ELIMINATION ACTIVE RC FILTER

This paper is dedicated to Professor Jovan Surutka
on the occasion of his 80th birthday

Vidosav Stojanović and Siniša Minić

Abstract: In this paper an approximation and realization of multi band elimination active RC filter has been described. Since two Q-factors of the filter are relatively low (less than ten), the fourth order filter with one operational amplifier should be used for cascaded form realization. Sections can be arranged so that the fourth order section is the first. This filter is very suitable for data extraction which are transmitted over power line, because dynamic range is big and the fourth order section more save operational amplifier due to power line tension.

1. Introduction

The power line frequency components are unwanted signal for data transmission through power line [1] and their suppression is an important task in both, analog and digital signal processing. It is necessary to remove these components before analogue to digital conversion because their amplitudes are very high. For example, the third, the fifth and the seventh harmonic component of power line voltage have amplitudes more than 20 V_{max} and suppression only the base component is not enough. Except suppression of power line frequency components, active filter must have passive higher order RC network, between power line and operational amplifier, in order to protect the operational amplifier from the power line tension. This protection made with one simple voltage divider, which has attenuation about 40 dB, is not recommended because it attenuates both, power line frequency and wanted signal, and the sensitivity of receiver is also reduced to 40 dB.

The main objective of this paper is design of active RC filter which suppresses power line frequency and the third, the fifth and the seventh

The authors are with School of Electronic Engineering, Beogradska 14, Niš, Serbia (e-mail: vitko@elfak.ni.ac.yu).

harmonic. In the first part of the paper the amplitude characteristic for multi band elimination filter is derived, and in the second part of the paper, synthesis of the four pole active RC filter with one operational amplifier is proposed. Finally, sensitivity coefficients of the fourth order filters with one operational amplifier to the element change are computed. This filter is suitable for data extraction which are transmitted through power line, because it increases sensitivity of receiver about one power of ten.

2. Approximation

Let

$$\omega_{\infty 1} < \omega_{o2} < \omega_{\infty 3} < \omega_{o4} < \cdots < \omega_{om} < \infty$$

where ω_{oi} is the center frequency of the pass band and $\omega_{\infty i}$ is the center frequency of the elimination band. Then the squared amplitude characteristic is expressed in the form

$$H^2(\omega^2) = \frac{1}{1+\varepsilon^2 \prod_{i=1}^{m} \frac{(\omega^2 - \omega_{02i}^2)^2}{(\omega^2 - \omega_{\infty(2i-1)}^2)^2}} \tag{1}$$

where ε is real positive constant less than unity and m is an even integer number. In this case the number of pass bands is equal with the number of elimination bands. Let note that for high pass multi band elimination filter $\omega_{\infty 1} = 0$.

In order to design the high pass filter having amplitude characteristic given with equation (1), it is necessary to determine the location of the poles on the s plane. To make this let $\omega^2 = -s^2$, where $s = j\omega$, so that s lies on the imaginary axe. On the s-plane, the poles of the equation (1) will be obtain as the roots of the equation

$$s^4 \prod_{i=2}^{m} (-s^2 + \omega_{\infty(2i-1)}^2)^2 + \varepsilon^2 \prod_{i=1}^{m} (-s^2 + \omega_{o2i}^2)^2 = 0 \tag{2}$$

After determination the pole position on the s-plane we choose those that lie on the left half plane. The transfer function zeros lie on the imaginary axe and occur at $\pm j\omega_{\infty(2i-1)}$.

For example, if $m = 8$, $\varepsilon = 0.5$, $\omega_{\infty 1} = 0$ rad/s, $\omega_{\infty 3} = 2\pi 1.5$ rad/s, $\omega_{\infty 5} = 2\pi 2.5$ rad/s, $\omega_{\infty 7} = 2\pi 3.5$ rad/s, $\omega_{o2} = 2\pi 2$ rad/s, $\omega_{o4} = 2\pi 2.85$

rad/s, $\omega_{o6} = 2\pi 4.2$ rad/s and $\omega_{o8} = 2\pi 2$ rad/s then corresponding transfer function of multi band elimination filter will be

$$H(s) = m\frac{s^2(s^2+\omega_{\infty 3}^2)(s^2+\omega_{\infty 5}^2)(s^2+\omega_{\infty 7}^2)}{s^8+A_7s^7+A_6s^6+\cdots+A_1s+A_0} \tag{3}$$

where are

$$\begin{aligned} A_0 &=2.490360\times10^9 & A_4 &=3.996345\times10^5\\ A_1 &=2.548328\times10^8 & A_5 &=1.583853\times10^4\\ A_2 &=5.592134\times10^7 & A_6 &=1.105938\times10^3\\ A_3 &=3.767912\times10^6 & A_8 &=1.918428\times10 \end{aligned}$$

The amplitude response curves of this filter for two different value of ε are given in Fig. 1. Higher attenuation in the stop band is corresponding to the greater value of parameter ε.

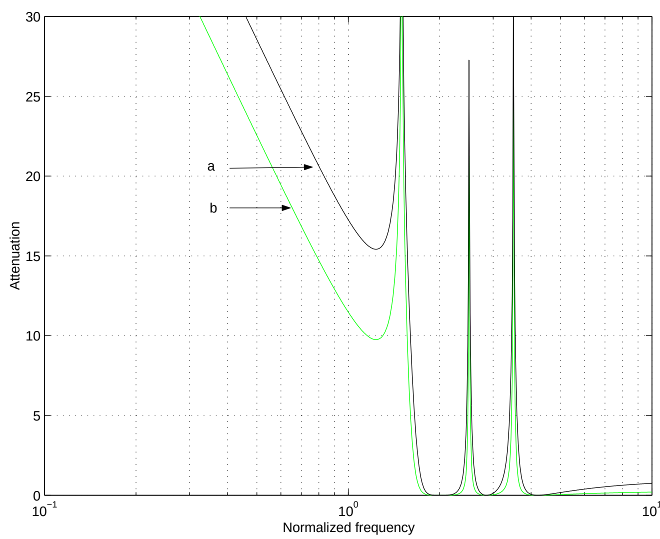


Fig. 1. Amplitude response curves of eight order filter for two different values of ε*: a)* $\varepsilon = 0.5$ *and b)* $\varepsilon = 0.25$

Poles and zeros locations of the filter transfer function in the s-plane for $\varepsilon = 0.5$, which transfer function is given by Eq. (3), are displayed on the Fig. 2. Poles lie on the left half plane and zeros lie on the imaginary axe.

There are also two zeros at the origin resulting in an equal number of poles and zeros. Poles Q-factors and poles magnitude have following values

$$Q_1 = 0.934557 \qquad \omega_1 = 13.599490$$
$$Q_2 = 5.944543 \qquad \omega_2 = 10.568190$$
$$Q_3 = 11.541171 \qquad \omega_3 = 22.089846$$
$$Q_4 = 16.710919 \qquad \omega_4 = 15.717589$$

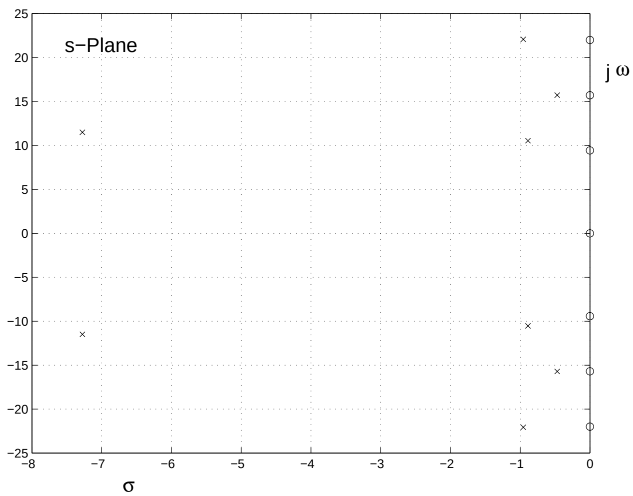


Fig.2. Poles and zeros of filter transfer function (3).

Now we divided transfer function (3) into three groups in order to synthesize the eight order filter by cascading fourth- and second-order filters. The first section will be designed by four-pole-one-amplifier technique proposed in this paper and the other two sections can be designed with well known two-pole one or more amplifier technique [2], [3]. These three cascaded sections have the following form

$$H_1(s) = m_1 \frac{s^2(s^2+\omega_{\infty 3}^2)}{s^4+\alpha_3 s^3+\alpha_2 s^2+\alpha_1 s+\alpha_0}, \tag{4}$$

where are $\alpha_0 = \omega_1^2\omega_2^2$, $\alpha_1 = \omega_1\omega_2^2/Q_1+\omega_1^2\omega_2/Q_2$, $\alpha_2 = \omega_1^2+\omega_2^2+\omega_1\omega_2/Q_1Q_2$, $\alpha_3 = \omega_1/Q_1+\omega_2/Q_2$,

$$H_2(s) = m_2 \frac{s^2 + \omega_{\infty 5}^2}{s^2 + \dfrac{\omega_3}{Q_3} s + \omega_3^2} \qquad \text{and} \tag{5}$$

$$H_3(s) = m_3 \frac{s^2 + \omega_{\infty 7}^2}{s^2 + \dfrac{\omega_4^2}{Q_4} s + \omega_4^2}. \tag{6}$$

The first section $H_1(s)$ is high pass filter with one transmission zero, and the other two sections $H_2(s)$ and $H_3(s)$ are well known notch filters. The coefficients m_1, m_2 and m_3 are arbitrary constant: $m_1 m_2 m_3 = m$.

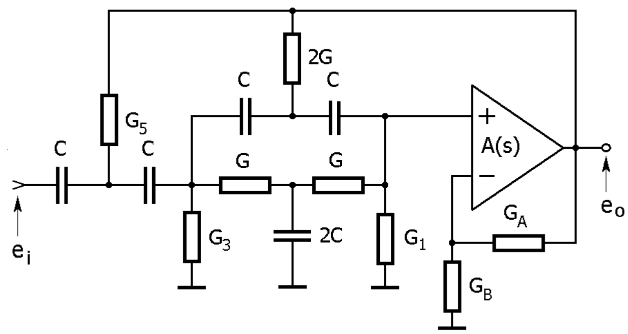
The pole-zero groupings are not unique, i.e., the other combination can work in the same way. It is well known, that the sensitivity of coefficients of the filter transfer function depends of the pole Q-factor and the order of filter. The sensitivity rises when Q-factor and filter order rises. Because of this fact, the first section has two pole with the lowest Q-factors.

3. Design of Fourth-order High-pass Filter with One Operational Amplifier

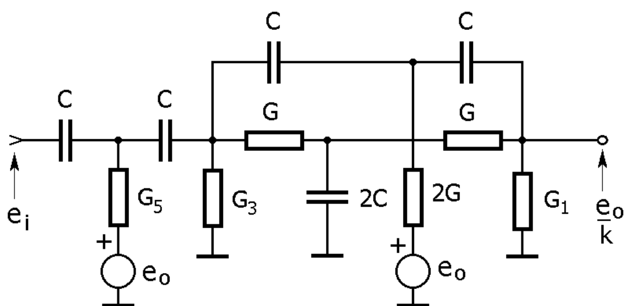
The network configuration of the fourth-order high-pass active RC filter, with one transmission zero, is shown in Fig. 3(a). It contains of an active symmetrical twin-T network and loading admittance G_1 [4] which is expanded in the fourth-order network with time constants C/G_3 and C/G_5. All capacitors have the equal value except the capacitor in the twin-T network which has double value. Notice that network has five capacitors and it is clear that it will realize a fifth order function. However, the elements of the twin T-network are chosen as it is shown in Figure 3(a) and a pole-zero cancellation occurs, resulting in four order function with zeros on the imaginary axis, as it is desired.

Let consider the circuit model in Fig. 3(b). Straightforward analysis of the circuit will give us the following set of five equations in matrix form

$$\begin{bmatrix} sE_i \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = [Y_{55}] \begin{bmatrix} E_5 \\ E_4 \\ E_3 \\ E_2 \\ E_o \end{bmatrix}, \tag{7}$$



(a)



(b)

Fig. 3. a) Fourth order high pass filter, with one pass band transmission zero. b) Circuit model.

where E_o and $E_j, j = 2, \dots, 5$ are corresponding node voltages and $[Y_{55}]$ is 5×5 admittance matrix of the passive network

$$[Y_{55}] = \begin{bmatrix} \frac{G_5}{C} + 2s & -s & 0 & 0 & -\frac{G_5}{C} \\ -s & \frac{G+G_3}{C} + 2s & -\frac{G}{C} & -s & 0 \\ 0 & -\frac{G}{C} & 2(\frac{G}{C} + s) & 0 & -\frac{G}{Ck} \\ 0 & -s & 0 & 2(\frac{G}{C} + s) & -\frac{2G}{C} - \frac{1}{k}s \\ 0 & 0 & -\frac{k}{2} & -s & \frac{G+G_1}{Ck} + \frac{1}{k}s \end{bmatrix},$$

in which $k = 1 + G_b/G_a$ is amplifier gain.

In terms of the determinant method of solution, one can write for output voltage E_o with respect to the exitation voltage generator E_i

$$E_o = \frac{\Delta_5}{\Delta} E_i,$$

where Δ is determinant of equation (7) and Δ_5 is its cofactor.

After simple manipulation determinant $\Delta = |Y_{55}|$ can be written in the following form

$$\Delta = \frac{2}{k}(\frac{G}{C}+s)\begin{vmatrix} \frac{G_5}{C}+2s & -s & 0 & 0 & -\frac{G_5}{C} \\ -s & \frac{G+G_3}{C}+2s & -\frac{k}{2} & -s & 0 \\ 0 & -\frac{G}{C} & k & 0 & -\frac{G}{Ck} \\ 0 & -s & k & \frac{2G}{C}+2s & -\frac{2G}{C}-\frac{1}{k}s \\ 0 & 0 & -\frac{k}{2} & -s & \frac{G+G_1}{Ck}+\frac{1}{k}s \end{vmatrix}.$$

The cofactor can be expressed as

$$\Delta_5 = 2s^2(s^2+\frac{G^2}{C^2})(s+\frac{G}{C})$$

Also the transfer voltage ratio E_o/E_i of the network displayed in Figure 3(a), assuming that the amplifier is ideal, is given by

$$\frac{E_o(s)}{E_i(s)} = k\frac{s^2(s^2+\omega_z^2)}{s^4+a_3s^3+a_2s^2+a_1s+a_0} \tag{8}$$

where

$$\begin{aligned} a_0 =& \frac{G^2G_1G_5}{C^4}+\frac{G^2G_3G_5}{C^4}+\frac{2GG_1G_3G_5}{C^4} \\ a_1 =& \frac{2G^2G_1}{C^3}+\frac{2G^2G_3}{C^3}+\frac{4GG_1G_3}{C^3}+\frac{G^2G_5(5-3k)}{C^3} \\ &+\frac{6GG_1G_5}{C^3}+\frac{GG_3G_5(4-2k)}{C^3}+\frac{2G_1G_3G_5}{C^3} \\ a_2 =& \frac{G^2(9-4k)}{C^2}+\frac{10GG_1}{C^2}+\frac{GG_3(8-4k)}{C^2}+\frac{4G_1G_3}{C^2} \\ &+\frac{GG_5(8-4k)}{C^2}+\frac{3G_1G_5}{C^2}+\frac{G_3G_5}{C^2} \\ a_3 =& \frac{G(12-6k)}{C}+\frac{4G_1}{C}+\frac{2G_3}{C}+\frac{G_5(1-k)}{C} \\ \omega_z =& \frac{G}{C} \end{aligned}$$

If we equate the corresponding coefficients α_i in denominator of transfer function for fourth order section in equation (4) and coefficients a_i, $i = 0,1,2,3$ from eq. (8), we have $m_1 = k$ and set of four simultaneous nonlinear equations in terms of unknown variables (C, G, G_1, G_3, G_5 and k) and known filter parameters (α_0, α_1, α_2 and α_3). The technique for solution of our four equation requires four independent parameters. Since, we have six independent parameters, it is possible to fix two parameters in order to solve the equations. One simple choice for the fixed parameters is C and $G = C\omega_z$. Now, the system of nonlinear equations may be written in the matrix form

$$\boldsymbol{F}(\boldsymbol{x}) = 0, \tag{9}$$

where $\boldsymbol{F}(\boldsymbol{x}) = \boldsymbol{a}(\boldsymbol{x}) - \boldsymbol{\alpha}$, $\boldsymbol{a} = [a_0\ a_1\ a_2\ a_3]^T$ and $\boldsymbol{\alpha} = [\alpha_0\ \alpha_1\ \alpha_2\ \alpha_3]^T$ are coefficients vectors, and $\boldsymbol{x}[G_1\ G_3\ G_5\ k]$ is element vector. The system of nonlinear equations (9) can be solved using the Newton-Raphson iterative procedure [4]. The associate function $\boldsymbol{g}(\boldsymbol{x})$ for this procedure is

$$\boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{x} - \boldsymbol{K}(\boldsymbol{x})\boldsymbol{F}(\boldsymbol{x}), \tag{10}$$

where $\boldsymbol{K}(\boldsymbol{x})$, in our example, is 4×4 matrix function of $\boldsymbol{x}$ with property that $\boldsymbol{K}(\boldsymbol{x})$ is non-singular at any fixed point of $\boldsymbol{g}(\boldsymbol{x})$. A common choice of $\boldsymbol{K}(\boldsymbol{x})$ is

$$\boldsymbol{K}(\boldsymbol{x}) = J^{-1}(\boldsymbol{x}), \tag{11}$$

where $\boldsymbol{J}^{-1}(\boldsymbol{x})$ is the Jacobian matrix of $\boldsymbol{F}(\boldsymbol{x})$. In this case (10) reduce to

$$\boldsymbol{x}_{\nu+1} = \boldsymbol{x}_{\nu} - \lambda \boldsymbol{J}^{-1}(\boldsymbol{x})\boldsymbol{F}(\boldsymbol{x}), \tag{13}$$

where $\boldsymbol{x}_{\nu+1}$ is solution in step ν, and λ is parameter that control the rate of convergence. In this way, divergence of iteration sheme is absolutely prevented, but convergence is not necessary guaranted.

For our example, Jacobian matrix is

$$\boldsymbol{J} = \begin{bmatrix} \frac{\partial a_0}{\partial G_1} & \frac{\partial a_0}{\partial G_3} & \frac{\partial a_0}{\partial G_5} & \frac{\partial a_0}{\partial k} \\ \frac{\partial a_1}{\partial G_1} & \frac{\partial a_1}{\partial G_3} & \frac{\partial a_1}{\partial G_5} & \frac{\partial a_1}{\partial k} \\ \frac{\partial a_2}{\partial G_1} & \frac{\partial a_2}{\partial G_3} & \frac{\partial a_2}{\partial G_5} & \frac{\partial a_2}{\partial k} \\ \frac{\partial a_3}{\partial G_1} & \frac{\partial a_3}{\partial G_3} & \frac{\partial a_3}{\partial G_5} & \frac{\partial a_3}{\partial k} \end{bmatrix} \tag{14}$$

Thus, one set of element values that will realize the given transfer function is: $C = 22\ \mu$F, $R = 48.228$ kΩ, $R_1 = 1.1.331$ kΩ, $R_3 = 29.39$ kΩ, $R_5 = 33.08$ kΩ and k=2.0317.

4. Sensitivity Considerations

In this section the sensitivity of the filter coefficients of the fourth order network which were introduced above, to the element change, is considered. As it is known, practical elements deviate from their nominal values due to the manufacturing tolerances, temperature and humidity changes. The relative sensitivity of coefficient a_i to variations of a variable x is defined as

$$S_x^{a_i} = \frac{x}{a_i}\frac{\partial a_i}{\partial x} \tag{15}$$

where x is anyone of the passive elements G_i, C_i or gain k. The coefficients a_i are functions of the G conductors, the C capacitors and the gain k. Thus with (7) the coefficient variations can be expressed in the form

$$\frac{\Delta a_i}{a_i} = \sum_{j=1}^{n} S_{G_j}^{a_i}\frac{\Delta G_j}{G_j} + \sum_{j=1}^{n} S_{C_j}^{a_i}\frac{\Delta C_j}{C_j} + S_k^{a_i}\frac{\Delta k}{k} \tag{16}$$

In general the individual conductor G_μ, capacitors C_ν, and gain k will be characterized by their mean μ_x, and standard deviation σ_x. The coefficient variations $\Delta a_i/a_i$ will then be random variables whose statistical behavior is a function of the components on which they depend.

To simplify the consideration, the deviations in conductivity is only considered and the operational amplifier is being assume to be ideal. For the point of the practical design, we may chose capacitors with nominal value and small tolerances.

Using (6) one gets from (7) results which are given in Table 1. The coefficients sensitivity to the conductivity are low. However, the variation in the amplifier gain k causes the coefficient change up to 9.17395 % if variabiliti of conductivity is 1 %. It is therefore often desirable to make final correction of transfer function using k, which alone can be adjusted in increasing or decreasing direction.

Table 1

$S_{x_i}^{a_i}$	G_1	G_3	G_5	k
a_0	0.08638	0.97983	1.00000	0
a_1	0.34295	1.51707	1.33474	−7.97015
a_2	0.20894	0.66701	0.65154	−9.17395
a_3	0.08365	1.89419	−0.86803	−8.74505

5. Conclusion

We have provided design equations for the multi band elimination active RC filter. We are first proposed an analytic approach for designing of multiband filter. The formulas can be derived for the computing the coefficients of multi band filter. We have also proposed the synthese of fourth pole active RC filter with one operational amplifier.

The desired multi band filter can be realized by cascade of circuit blocks, where one block realizes the biquadratic function, except the first section which realized the fourth order function with one operational amplifier. In order to prevent overdriving and save operational amplifier we select the first stage as the fourth order bloc.

This filter is very suitable for data extraction which are transmitted over power line.

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