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# H.F. YAGI ANTENNAS ABOVE A CONDUCTING HALF-SPACE

*This paper is dedicated to Professor dr Jovan V. Surutka on the occasion of his 80th birthday*

**Predrag D. Rančić**

**Abstract:** The unknown current distributions (UCDs) and input impedances/ admittances of H.F. Yagi antennas placed horizontally above real earth, are determined in this paper. The UCDs are determined by numerical solving of the system of integral equations of Hallen's type (SIE-H) using polynomial approximation for the currents. The simplified expressions for potentials (the Hertz's vector potential and electric scalar potential), obtained with the assumption that the module of refractive index ground/air is much greater than one, $n_{10} \gg 1$, are used for establishing the SIE-H. Thus, the problem of numerical evaluation of so-called Sommerfeld's integrals is partially avoided. The paper shows that very accurate results for the current distributions and input impedances of the antennas are obtained even if $n_{10} > 3$. Fortunately, the last one is satisfied in most of the practical cases.

**Key words:** Yagi antenna, conducting half-space, current distribution, input impedance.

## 1. Introduction

When the author obtained the call for paper from the editor (Prof. dr Vidosav S. Stojanović) and the editorial board of the Journal Facta Universitatis, Series: Electronics and Energetics, to submit a paper for special issue dedicated to Prof. dr Jovan V. Surutka on the occasion of his 80th birthday, he had no doubt about answering the call. The author also had no doubt about the subject of the paper, i.e. the paper should treat problems which are associated to the influence of real earth on the characteristics of linear

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antennas. Namely, the author received Ph.D. degree in this field, under the supervisory of Prof. dr Jovan V. Surutka. Considering that the author also had several common papers in this field with Prof. dr Jovan V. Surutka, the only doubt was: to make a review paper based on the published papers, or to make the paper based on the non-published results? The author chose the second variant, i.e. the influence of real earth on the characteristics of the H.F. Yagi antennas, whose conductors are placed horizontally above the ground surface, to be the subject of this paper.

Strict analyses of the influence of finite conductivity of the earth on the wire antenna characteristics were started by the pioneer work of Sommerfeld (1909, [1]). Up to this day, many methods and mathematical models were developed for this purpose. These methods and models differ regarding efficiency, simplicity, accuracy and proposed limitations in their practical applications. Listing, analyzing and classification of known and available methods (and models), because of very rich bibliography in this field, require much more time and space than the ones usually used for presenting papers of this kind (e.g. [2]-[20]).

For understandable reasons, regarding to the occasion for writing of this paper, the author will pay attention only to the papers of Serbian authors who dedicated their researches to modeling of the influence of finite conductivity of the earth on the wire antennas characteristics, or in general, on the EM field characteristics of wire structures.

Leading researches in this field were started under the supervisory of Prof. dr Jovan V. Surutka. These researches were conducted by Dragutin N. Mitić, and the results were presented in his Ph.D. thesis named "Influence of finite conductivity of the earth on the characteristics of horizontal linear dipole antennas" (1980, [22]). For feeding of the dipole antennas, the author used Dirac's $\delta$-generator and the model of two–wire line ([21]). The author obtained the integral equation of Hallen's type for unknown current distribution on dipole antenna, which was numerically solved using the point matching method with polynomial current approximation. The influence of finite conductivity of homogeneous earth is described with strict Sommerfeld's formulation of integral kernel, i.e. so–called Sommerfeld's integrals. For their numerical solving, Mitić has developed a number of complex, but efficient and satisfactory accurate mathematical models, so he obtained considerable results for current distribution and input impedance. Those results were obtained using IBM 1130 computer.

The researches at the Faculty of Electronic Engineering of Niš, under the supervisory of Prof. dr Jovan V. Surutka, were continued. The author of this paper starts his researches in the year of 1978/79. Initial motive for these theoretical researches was previously made decision for building of the Radio Titiograd (today Radio Podgorica) M.W. transmitting antenna on the wide region of the Skadar lake's coast. Because of the possible presence of water flood layer at the spot chosen for building the antenna, the feeding had to be lifted above the ground at height of 4.3 m. Thus, the model had to be theoretically solved considering vertical monopole antenna with elevated feeding in presence of water flood layer. Part of these researches were finished in 1983 and presented in Ph.D. thesis entitled "Vertical monopole antenna with elevated feeding in presence of water flood layer" ([23]). At first in the thesis, as theoretical background, a common structure of the Hertz's vector which originates from arbitrary placed Hertz's dipole in the presence of stratified multilayer media, was evaluated. This problem was solved using the two - pair Fourier's integral transformations. At the same time the experimental studies of these problems were also preformed ([25]). After a short break, the investigations in this field are continued and the new results were presented by the author at the national conferences (ETRAN, ΠEC), international conferences (ISEF, ISTET, EMC-Roma, COMPUMAG), and also in leading international journals (IEEE Trans. on Mag., AEU, El. Letters, [26]-[43]).

Also under the supervisory of Prof. dr Jovan V. Surutka, colleague Hildegard Božilović has worked on her Ph.D. thesis at the Faculty of Electronic Engineering of Niš, analyzing the influence of finite conductivity of the earth on the characteristics of vertical mast antenna with counterpoise made of radial wires. Whole system is placed above the homogeneous ground surface. The results were presented in her Ph.D. thesis entitled "Analysis of vertical monopole antenna with radial counterpoise above a plane conducting ground". Thesis defense was done at the Electrotechnical Faculty of Belgrade (1989, [44]). The author continued the researches in this field and has published a number of papers so far (ETRAN, AEÜ, e.g. [45]).

At the Faculty of Electronic Engineering of Niš, under the supervisory of Prof. dr Dragutin M. Veličković, Dušan Djurdjević has worked on the problem of vertical mast antenna with counterpois made of radial wires. The counterpoise is shallow buried in the ground which is considered homogeneous and isotropic conducting medium. The results are presented in his M.Sc. thesis entitled "Numerical solving of integral equations of linear antennas positioned above a ground plane" (1992, [46]). Djurdjević continues researches in this field at the Electrotechnical Faculty of Belgrade and, under the supervisory of Prof. dr Branko D. Popović, published the final results in Ph.D. thesis entitled "Analysis of wire antennas in the presence of dielectric multilayer media" (1996, [47]).

At the same time, Vladimir V. Petrović, also under the supervisory of Prof. dr Branko D. Popović, worked on modeling of wire antennas located above homogenous, semiconducting ground. In his Ph.D. thesis entitled "Analysis of wire antennas in the presence of real ground using method of images", Petrović replaced the influence of real earth with finite number of fictitious sources (1996, [48]).

At the Technical Faculty of Čačak, under the supervisory of Prof. dr Dragutin M. Veličković, Jasna J. Radulović in her Ph.D. thesis analyzed the influence of frequency and finite ground conductivity on the EM field characteristics in the vicinity of overhead power lines (2000, [49]).

It should be pointed out that earlier mentioned authors have published a number of papers in leading international journals, and that they have presented results at the international and national conferences. Emphasize should be put on the fact that the researches in this field were continued both at the Faculty of Electronic Engineering of Niš and the Electrotechnical Faculty of Belgrade, and are conducted by mentioned authors and a number of younger associates, so new contributions should be expected very soon.

Finally, since the author has chosen the non–standard form of introduction, he asked Prof. dr Jovan V. Surutka to point out, in this part of the introduction so it would be noted, few details (scientifique, or technical, or engineering, ...) associated with investigations in this field.

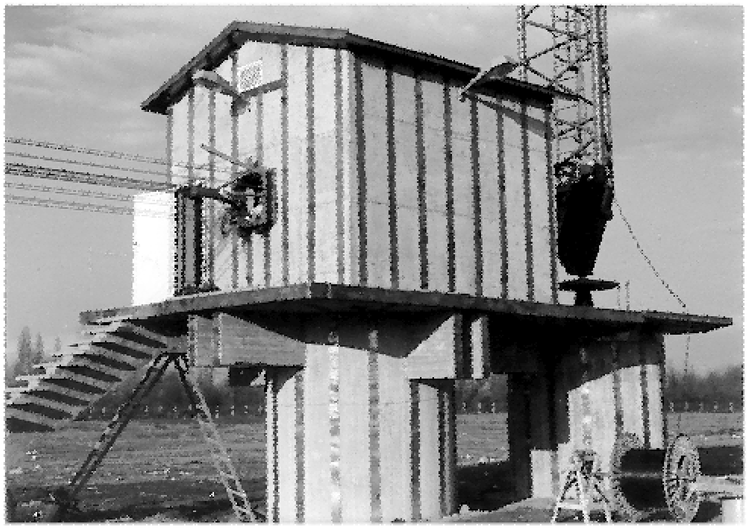
* * *

Professor dr Jovan Surutka, in the conversation with the author, emphasizes investigations carried out with his co-workers during projecting and constructing of building-ground of the MF broadcast antenna of Radio Podgorica (1987). The antenna base is placed immediately on the coast of Scadar's Lake, that is a floodable terrain with the seasonal change of water level.

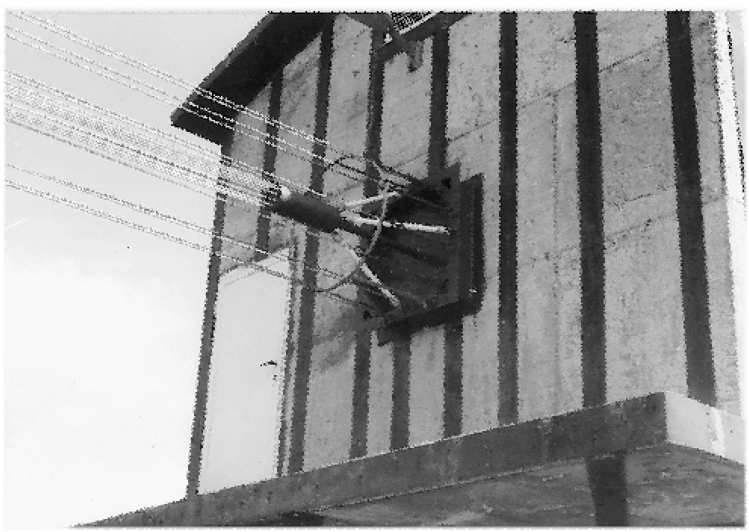
There are three main results that have been obtained during vast investigation process: Novel approach to the design of insulators in the antenna stays ([57]); Exhaustive theoretical as well as experimental studies of the influence of the change of water level on antenna impedance ([23], [24], [25]) were carried out. Photographs of the realized antenna are best representatives of these results, Pictures 1 - 3.

* * *

*Picture 1. MF broadcast antenna of Radio Podgorica. The antenna feeder line is in front plane. The building-ground height is about 185 m, and elevated antenna base height is 3.3 m. The mast has four levels of stays which are arranged in the three vertical planes symmetrically placed at* 120° *angles. The antenna operating frequency is 882 kHz, and unmodulated carrier power is 600 kW.*



*Picture 2. A detail of elevated antenna base.*



*Picture 3. A detail of the feeder line end.*

Many of the important H.F. (Short Wave) transmitting antennas have complex structure containing horizontal (and vertical) wires and radiating elements, positioned at different heights above (real) lossy earth ([2], [51], [52], [53]).

Among these antennas it is worthwhile to mention the following ones: Horizontal dipole antennas; Yagi antennas; Arrays of horizontal dipoles arranged vertically (curtain antennas); Arrays of horizontal dipoles arranged horizontally (tropical antennas); Omnidirectional arrays of horizontal dipoles; Quadrant antennas; Crossed dipole antennas; Aperiodic screen reflector antennas; Tuned reflector antennas; Horizontal (and vertical) log-periodic antennas; Rhombic antennas, etc.

It is quite natural to expect greater or less influence of the conducting earth on the characteristics and performances of such antennas. This influence depends mainly on conductivity and permittivity of the ground, frequency, and, especially, on the heights of radiating elements placed above the earth surface.

It has to be noted that the degree of the practical importance of this influence often depends on the considered performance itself. For example, under some conditions, in evaluating the radiation pattern of the antenna, the losses in the earth are neglected. Likewise, in analysis of such antennas, the influence of the real ground on the current distribution in antenna elements is neglected, and sinusoidal current distribution in radiating elements is adopted (see: ITU-R, Recommendation No. BS.705, [51]).

In order to take in to account the influence of the finite ground conductivity, in a strict and correct manner, the analysis of the antenna performances must be based on so-called Sommerfeld's formulation ([1]). The system of integral equations, containing very complex Sommerfeld's integrals should be established, and finally numerically solved assuming suitable (e.g. polynomial) current distribution functions on the wire conductors.

In order to avoid the tedious evaluation of Sommerfeld's integrals, and to facilitate the solving of different problems connected to the influence of the earth, the different approximate procedures were developed in the analysis of such antennas.

A very illustrative example for that can be found in the ITU-R No. BS.705 in the recommendation for calculating of radiation patterns and gain. According to this recommendation, in the calculation of the radiation patterns and gain, the following assumptions have to be used: the antenna is situated over a flat homogeneous ground; antenna elements are thin linear wires; the current in the radiating elements is sinusoidally distributed.

Regarding the influence of the imperfect ground on the resulting radiated electromagnetic field of the antenna, the recommendation in the ITU-R No. BS.705 assumes that the far field at the observation point $P$ is the geometrical sum of the field components of the direct (incident) wave and the wave which is reflected from the ground. According to the recommendation, the incident radiation on the ground is assumed to have plane wave front. In other words, the technique of plane wave reflection coefficients has to be used. (Depending on the polarization of the incident radiation, the horizontal or vertical complex reflection coefficients should be used).

If the radiator's heights above the earth are large enough (comparing to the wave length), the last assumption can be treated as acceptable. But in the case of radiators which are close to the earth's surface (and near the virtual "reflection point"), the mentioned assumption is quite false: the radiated and reflected fields are neither plane non transversal waves. They have the typical character of a near field. In that case the usage of the Fresnel's reflection coefficients is meaningless, i.e. the reflection coefficient method (RCM) is not acceptable for the accurate UCD and near fields calculations.

In the previous papers ([26]-[43]), the author has been solving problems of this kind, where the influence of a conducting ground (homogeneous or multilayer) on the characteristics of wire structures was respected by numerical solving of Sommerfeld's integrals in its originate or appropriately transformed form. Very difficult problem of numerical calculation of these integrals has been partially avoided using the approximate expressions ([35], [36]). The last approach is applicable in the case when the refractive index ground/air is much greater than one ($n_{10} \gg 1$). Numerical experiments have shown that, even in the case $n_{10} > 3$, the proposed model of a vertical antenna ([32], [33], [34], [36], [38], [39]), along with simplicity, gives very accurate results for all characteristics of the vertical antenna structure.

In this paper the mathematical model proposed in [35] is improved and applied to the problem of H.F. Yagi antennas placed horizontally above a conducting half–space ([51], [52]). The earth is treated as a homogeneous, isotropic medium, having refractive index $n_{10} \gg 1$. Like in [36], in this paper is also shown that very accurate results for the UCDs and input admittance/impedance of horizontal antennas placed above a lossy half–space are obtained even when $n_{10} > 3$. Fortunately, this condition is satisfied in most of the practical cases. This improved mathematical model is already successfully applied for solving coupled dipole antennas positioned above the conducting half-space ([37]).

Considering the fact that the author chose the results obtained by Mitić given in [22], in order to compare his results with appropriate other ones, the author has used the SIE-H (proposed in [27] and improved models [35], [37], [42]) and polynomial current approximation ([54]-[56]) for solving this problem.

## 2. Short Theoretical Background of the Mathematical Model

In this mathematical model Yagi antenna made with one symmetrical dipole as an active element - No.1, and with $N_c - 1$ passive elements is considered (Fig 1.).

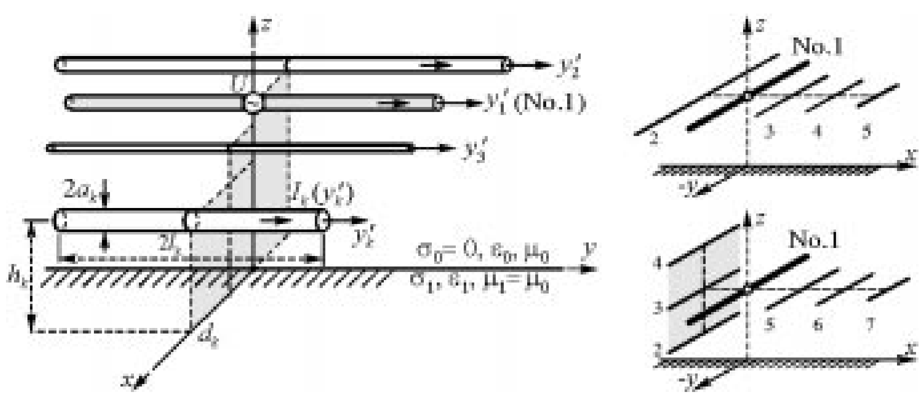



*Fig. 1. Illustrations of the Yagi antennas.*

$N_c$ is the total number of the Yagi antenna conductors. The antenna conductors, whose arm lengths are $l_k$, $k = 1, 2, \ldots, N_c$, and conductors cross–section radii $a_k$, $k = 1, 2, \ldots, N_c$, satisfy the conditions $a_k \ll l_k$ and $a_k \ll \lambda$ ($\lambda$: wavelength in the air), are placed in the air at heights $h_k$, $k = 1, 2, \ldots, N_c$, above a homogeneous and isotropic conducting medium with known electric parameters $\sigma_1, \varepsilon_1, \mu_1$ ($\sigma_1$: conductivity, $\varepsilon_1 = \varepsilon_{r1}\varepsilon_0$: permittivity, $\mu_1 = \mu_0$: permeability). In the text, following denotements are introduced which are related to the $i$-th medium ($i = 0$ for air, and $i = 1$ for ground): $\underline{\sigma}_i = \sigma_i + j\omega\varepsilon_i$ - complex conductivity, $\underline{\gamma}_i = \alpha_i + j\beta_i = (j\omega\mu_i\underline{\sigma}_i)^{1/2}$ - complex propagation constant, $\omega$ - angular frequency, and $\underline{n}_{10} = 1/\underline{n}_{01} =$

$\underline{\gamma}_1/\underline{\gamma}_0 = \underline{\varepsilon}_{r1}^{1/2} = (\varepsilon_{r1} - j60\sigma_1\lambda)^{1/2}$ - complex refractive index. The active antenna element is fed by ideal voltage generators (Dirac's $\delta$ -generators) having voltage $U$ and frequency $f = \omega/2\pi$. The UCDs are denoted by $I_k(y_k')$, $I_k(-y_k') = I_k(y_k')$, $0 \le y_k' \le l_k$, $k = 1, 2, \dots, N_c$.

For described model, the system of integral equations of Hallen's type, in which the part of the integral's kernel is transformed in an appropriate way (like this in [27]), has the following form

$$\begin{aligned}&\sum_{k=1}^{N_c} \int\limits_{y_k'=-l_k}^{l_k} I_k(y_k')\Big\{K_0(r_{1k}) - K_0(r_{2k}) + \underline{n}_{01}^2 \int\limits_{\alpha=0}^{\infty} \tilde{T}_{z10}(\alpha)\tilde{K}_0(\alpha, r_{2k})\mathrm{d}\alpha \\ &+ \underline{\gamma}_0 \int\limits_{s=0}^{y} \sinh[\underline{\gamma_0}(y-s)] \int\limits_{\alpha=0}^{\infty} [\underline{n}_{01}^2\tilde{T}_{z10}(\alpha) - \tilde{T}_{\eta 10}(\alpha)]\tilde{K}(\alpha, r_{2k})\mathrm{d}\alpha\mathrm{d}s\Big\}\mathrm{d}y_k' \\ =&C_n \cosh(\underline{\gamma}_0 y) - \delta_{n1}\frac{U}{60}\sinh(\underline{\gamma}_0 y), \\ &0 \le y \le l_n, \quad x = d_n \pm a_n, \quad z = h_n, \quad n = 1, 2, \dots, N_c,\end{aligned} \tag{1}$$

where $K(r_{ik}) = \exp(-\underline{\gamma}_0 r_{ik})/r_{ik}$ is standard form of kernel of potentials, $r_{ik}$ is the distance from the source, $i = 1$, and its image, $i = 2$, to the field point, respectively, $C_n$, $n = 1, 2, \dots, N_c$ are the unknown integration constants and $\delta_{n1}$ is the Kronecker's symbol. Integrals with the infinite upper bound are with the Sommerfeld's integral kernels (SIKs) and they are transformed (unlike in [36] and [27]) into integrals of the same form, i.e.

$$\begin{aligned}S_1 =&\underline{n}_{01}^2 \int\limits_{\alpha=0}^{\infty} \tilde{T}_{z10}(\alpha)\tilde{K}_0(\alpha, r_{2k})\mathrm{d}\alpha \\ &= \underline{n}_{01}^2 \int\limits_{\alpha=0}^{\infty} \frac{2u_0}{u_0 + \underline{n}_{01}^2 u_1} \frac{e^{-u_0(z+h_k)}}{u_0} \alpha \mathrm{J}_0(\alpha\rho_{2k})\mathrm{d}\alpha,\end{aligned} \tag{2a}$$

$$\begin{aligned}S_2 =& \int\limits_{\alpha=0}^{\infty} \tilde{T}_{\eta 10}(\alpha)\tilde{K}_0(\alpha, r_{2k})\mathrm{d}\alpha \\ &= \int\limits_{\alpha=0}^{\infty} \frac{2u_0}{u_0 + u_1} \frac{e^{-u_0(z+h_k)}}{u_0} \alpha \mathrm{J}_0(\alpha\rho_{2k})\mathrm{d}\alpha,\end{aligned} \tag{2b}$$

where $\tilde{T}_{\eta 10}$, $\tilde{T}_{z10}$, $\tilde{K}_0$ are transmission coefficients and the standard kernel of potential in the transformed domain of variable $\alpha$, $\alpha \in [0,\infty)$, respectively, $\mathrm{J}_0(\alpha\rho_{2k})$ is the zero-th Bessel's function of the first kind, $\rho_{2k} = \sqrt{a_n^2 + (d_n - d_k)^2 + (y - y_k')^2}$ is the radial distance and $u_i = (\alpha^2 + \underline{\gamma}_i^2)^{1/2}$, $i = 0, 1$. If the transmission coefficients ($\tilde{T}_{\eta 10}$ and $\tilde{T}_{z10}$), in the transformed domain and in the case $n_{10} \gg 1$, are expanded to series, then the integrations in (2.a) and (2.b) can be substituted with the following expressions

$$S_1 \simeq +\, 2\underline{n}_{01}^2 K_0(r_{2k}) - 2\underline{n}_{01}^3 \underline{\gamma}_0 \int\limits_{\nu_1 = z + h_k}^{\infty} K_0(r_{2k}) \mathrm{d}\nu_1 + \cdots , \tag{3a}$$

$$S_2 \simeq -\, \frac{2\underline{n}_{01}}{\underline{\gamma}_0} \frac{\partial K_0(r_{2k})}{\partial z} - \frac{2\underline{n}_{01}^2}{\underline{\gamma}_0^2} \frac{\partial^2 K_0(r_{2k})}{\partial z^2} - \cdots . \tag{3b}$$

Mathematical model was derived when the integrals $S_1$ and $S_2$ were approximated by first two terms from (3.a) and (3.b), and after that substituted into the SIE-H (1). Next, the SIE-H is numerically solved by point matching method ([54]) and by entire domain polynomial approximation for currents ([55], [56]). The input impedances/admitances is defined as $Z_a = Y_a^{-1} = R_a + jX_a = U/I_1(0^+)$.

*Notice 1:* Evaluation of series. If the refractive index is $n_{01} \gg 1$, and $u_1 \simeq \underline{\gamma}_1$, the transmission coefficients in (2.a) and (2.b) can be written in the following series forms

$$T_{z10} \simeq \frac{2}{1 + \dfrac{\underline{n}_{01}\underline{\gamma}_0}{u_0}} \simeq 2\Big(1 - \frac{\underline{n}_{01}\underline{\gamma}_0}{u_0} + \frac{\underline{n}_{01}^2\underline{\gamma}_0^2}{u_0^2} - \cdots\Big), \tag{4a}$$

$$T_{\eta 10} \simeq \frac{2\dfrac{\underline{n}_{01}}{\underline{\gamma}_0} u_0}{1 + \dfrac{\underline{n}_{01}}{\underline{\gamma}_0} u_0} \simeq 2\Big(\frac{\underline{n}_{01}}{\underline{\gamma}_0} u_0 - \frac{\underline{n}_{01}^2}{\underline{\gamma}_0^2} u_0^2 + \cdots\Big). \tag{4b}$$

Substituting (4a) and (4b) into (2a) and (2b), respectively, (3a) and (3b) are obtained.

*Notice 2:* The two terms approximation in (3.a) was used for the formulation of the vertical antenna model in [36]. Namely, the SIK signed in [36] as $L(z, z_k' = h_k)$ is only the second term in (3a), i.e.

$$L(z,h_k) = \int_{\alpha=0}^{\infty} \tilde{T}_{z01}(\alpha)\tilde{K}_0(\alpha,r_{2k})\mathrm{d}\alpha \simeq 2\underline{n}_{01}\underline{\gamma}_0 \int_{\nu_1=z+h_k}^{\infty} K_0(r_{2k})\mathrm{d}\nu_1$$
$$=2\underline{n}_{01}\underline{\gamma}_0\Big\{-\int_{\nu_1=0}^{z+h_k} K_0(r_{2k})\mathrm{d}\nu_1 - \frac{\pi}{2}\Big[\mathrm{N}_0(\beta_0\rho_{2k}) + j\mathrm{J}_0(\beta_0\rho_{2k})\Big]\Big\}, \tag{5}$$

where $\mathrm{N}_0$, $\mathrm{J}_0$ are the zero-th Neuman and Bessel functions of the first kind, respectively.

*Notice 3*: Using the system of integral equations of two potentials (SIE-TP, [29], [30]) and taking only the first terms from (3a) and (3b) into calculation of the potentials, the mathematical model for horizontal antenna analysis was proposed in [35].

## 3. Numerical Results

Based on the described mathematical model, the new software package for numerical calculation is formed as reliable tool for the design of Yagi antennas in the presence of conducting half-space.

Validity of proposed mathematical model is confirmed by numerous numerical experiments, where the results for input impedances of isolated hotizontal halfwave dipole situated above a ground plane are compared with those obtained from Mitić ([22]).

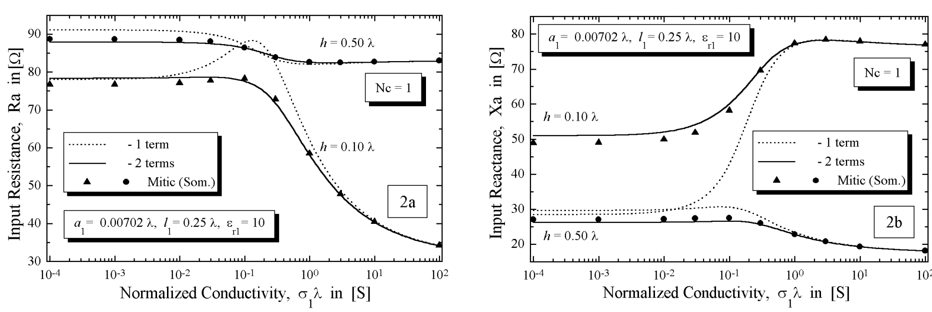



*Fig. 2. Input resistance and reactance of halfwave dipole situated above a conducting half-space as functions of normalized conductivity. The height $h_1 = h$ is parameter: $h = 0.1\lambda$ and $h = 0.5\lambda$.*

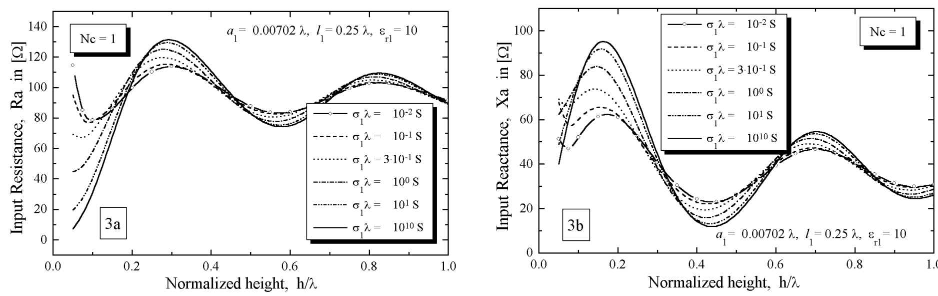



*Fig. 3. Input resistance and reactance of halfwave dipole situated above a conducting half-space as function of normalized height $h_1 = h$ and with $\sigma_1\lambda$ as parameter.*

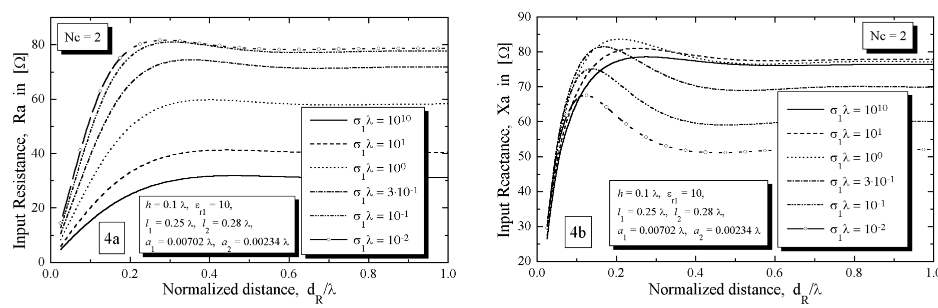



*Fig. 4. Input resistance and reactance of halfwave dipole with one parasite element ($N_c = 2$), situated above a conducting half-space at the height $h_1 = h_2 = h = 0.10\lambda$, as functions of normalized distance $d_2 = d_R$ and with $\sigma_1\lambda$ as parameter.*

The input resistance and reactance of horizontal halfwave dipole ($N_c = 1$) as functions of normalized conductivity of the earth are shown in Fig.2. The height $h_1 = h$ above ground is parameter: $h = 0.10\lambda$ and $h = 0.50\lambda$. The results obtained by one term approximation of the SIKs are given by dotted lines, and those by two terms by solid lines. In same figures corresponding results taken from [22] are marked by solid triangles and circles. The last ones are numerical results calculated by solving the SIE of Hallen's type with numerical integration of the Sommerfeld's integrals. It can be concluded that the proposed model has very good convergence, and that the two - terms SIKs approximations give very accurate results for input

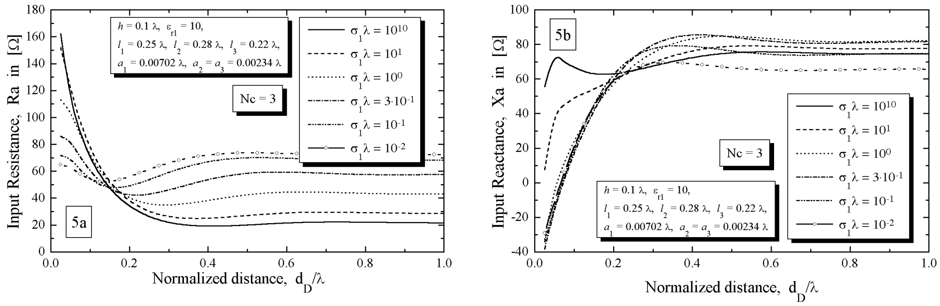



*Fig. 5. Input resistance and reactance of horizontal three elements Yagi antenna ($N_c = 3$: halfwave dipole, reflector, director), situated above a conducting half-space at the height $h_1 = h = 0.1\lambda$, as functions of normalized distance $d_3 = d_D$, for $d_2 = -0.16\lambda$ and with $\sigma_1\lambda$ as parameter.*

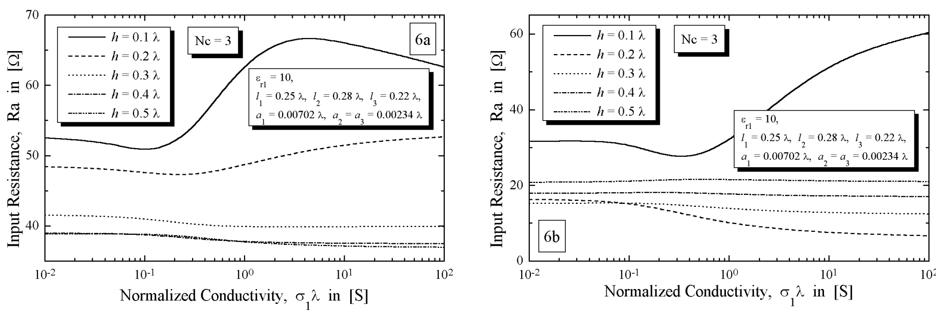



*Fig. 6. Input resistance and reactance of horizontal three elements Yagi antenna, situated above a conducting half-space as function of normalized conductivity, with normalized height $h_1 = h$ as parameter, and for following distances of parasite elements: $d_2 = -0.16\lambda$ and $d_3 = 0.12\lambda$.*

impedances even if $n_{10} > 3$.

The input impedance (resistance and reactance), as an integral characteristic of wire antenna, is the best parameter for estimation of accuracy of applied method for determining the UCDs. For this reason, the input resistance and reactance versus one constructive parameter are only presented in this paragraph, taking the other parameters as constants. All details for understanding the illustration are given in the captions of the figures, i.e. the antenna geometry is completely described in the same figure and its caption (Figs. 3–9).

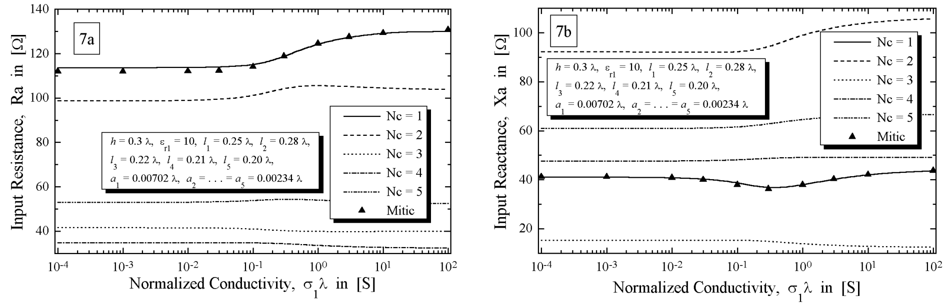



*Fig. 7. Input resistance and reactance of Yagi antenna, situated in horizontal plane above a conducting half-space at the height $h_1 = h = 0.30\lambda$, as functions of normalized conductivity, with the number of parasite elements $N_c$ as parameter, and their distances are: $d_2 = -0.16\lambda$, $d_3 = 0.12\lambda$, $d_4 = 0.23\lambda$, $d_5 = 0.33\lambda$.*

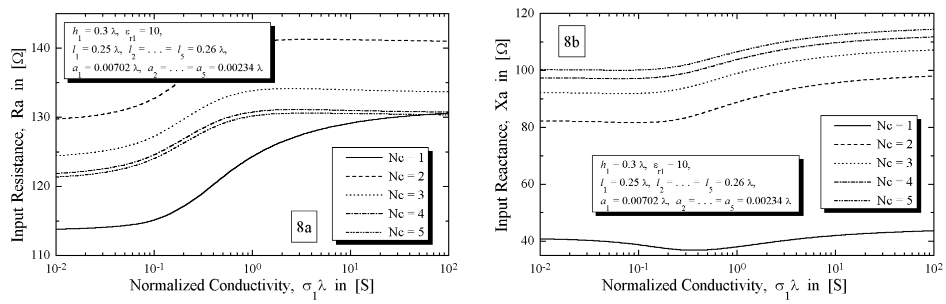



*Fig. 8. Input resistance and reactance of halfwave dipole with reflector, situated above a conducting half-space at the height $h_1 = 0.30\lambda$, as functions of normalized conductivity. All reflector elements are situated in vertical plane $d_R = -0.25\lambda$, and their number $N_c - 1$ is parameter.*

All the results presented in this paper are calculated by using the second degree in polynomial approximations of the UCDs, i.e. $M_1 = \ldots = M_{N_c} = 2$. In all presented examples, the horizontal halfwave dipole of radius $a_1 = 0.007022\lambda$ is used as an active element. The one is always positioned at height $h_1 = h$ in vertical plane $x = d_1 = 0$. The radii of the other antenna elements (reflectors or directors) are the same, i.e. $a_2 = \ldots = a_{N_c} = 0.00234\lambda$. Other constructing antenna parameters ($N_c$, $l_k$, $h_k$, $d_k$, $k = 2, 3, \ldots, N_c$) are different from one to another examples. Relative permittivity of ground in all the cases is ten, $\varepsilon_{r1} = 10$. Normalized conductivity, $\sigma_1\lambda$ in $[S]$, is parameter

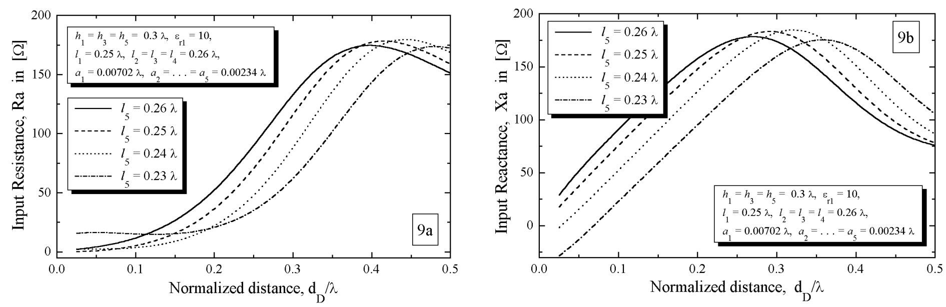



*Fig. 9. Input resistance and reactance of the five elements Yagi antenna ($N_c = 5$ halfwave dipole, three reflector elements positioned in vertical plane $d_R = -0.25\lambda$, and one director element) versus normalized distance of director, $d_5 = d_D$. Horizontal antenna plane is at height $h_1 = h_3 = h_5 = 0.30\lambda$ and heights of two reflector elements are $h_2 = 0.20\lambda$ and $h_4 = 0.40\lambda$. Normalized length of director arm is parameter, and normalized conductivity of ground is $\sigma_1\lambda = 10^{-1}$ S.*

or variable along the apices axes.

Finally, regarding the input impedance, the presented results illustrate a part of design process of the H.F. Yagi antenna situated above a ground plane.

## 4. Conclusion

This paper is dedicated to Prof. dr Jovan V. Surutka on the occasion of his 80th birthday. In this paper a number of non-published results, obtained by the original mathematical model, are noted down so that they would not fall into oblivion.

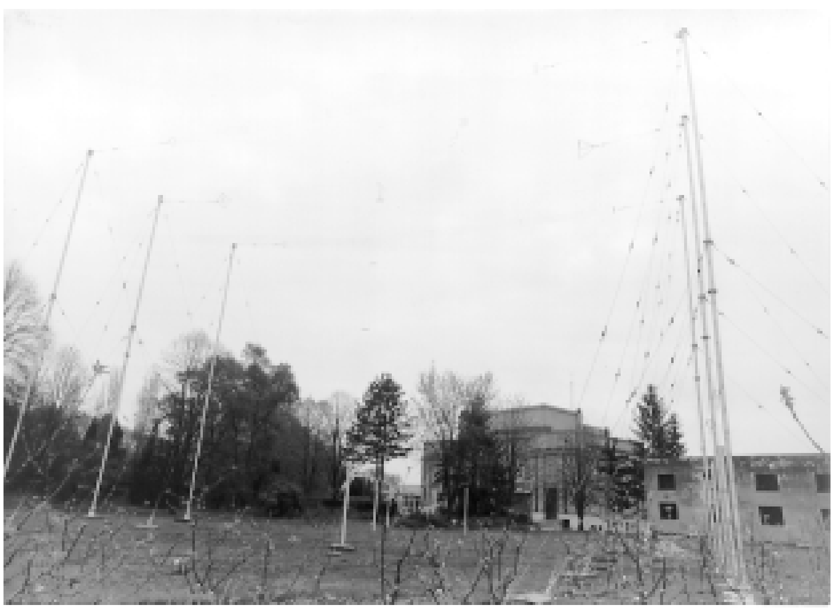
In this paper, as given in [35] and [37], the same or similar conclusions can be also evaluated for the proposed model and the presented results:

- Having regard to the presented theoretical explanations, as well as to the reported numerical results, one can conclude that this paper offers simple and sufficiently accurate mathematical model for the analysis of the horizontal wire antennas situated above a homogeneous conducting half–space;
- Conceptually, this model is exactly alike to the ones presented in [35], [36], [37]. Numerical experiments confirmed that this model gives sufficiently accurate results for impedances even if the modulus of refractive index is $n_{10} > 3$, i.e. like for the vertical antenna model in [36]. For-

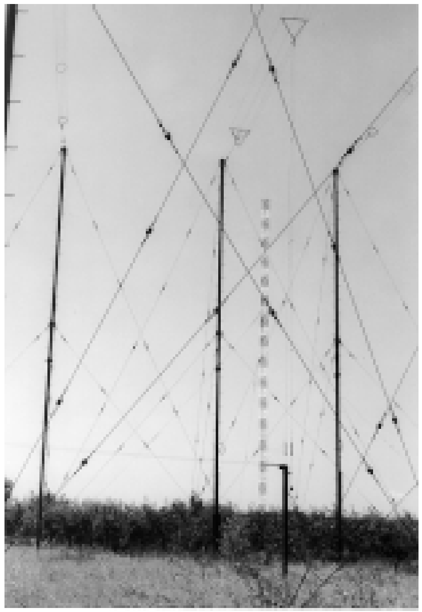
tunately, the condition $n_{10} > 3$ is satisfied in the most of the practical cases;

- Opposite to the vertical antenna model ([32], [34], [36], [38], [39]), which has no additional conditions (e.g., vertical Hertz dipole and field point can be both placed in the plane $z = 0$), the model of horizontal antenna gives sufficiently accurate results if the height of the horizontal dipole is greater than $0.025\lambda$. As the results shown in Fig.2 are concerned, the last condition can be alleviated using next terms in approximations of the SIK in (3a) and (3b). Of course, this procedure makes the mathematical model more complicated.
- Reliable tools for designing of linear horizontal antennas in the presence of real earth are given in this paper.

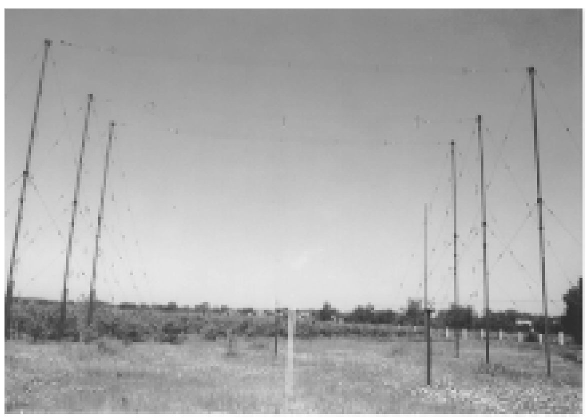
Finally, it should be noted that Prof. dr Jovan V. Surutka was leading the projection and construction of three-element Yagi antenna for broadcasting the First Channel of Radio Belgrade on SW (7.2 MHz, 50 kW) in the European area. The antenna is placed at the height 16.6 m ($0.4\lambda$) above a ground plane (1992, [52]). The antenna is installed in Emission Center Obrenovac, as shown in Pictures 4 - 6.



*Picture 4.*



*Picture 5.*



*Picture 6.*

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