# Rule-Based Optimization of AND-XOR Expressions 

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#### Abstract

The problem of finding a minimum AND-XOR expression for a given Boolean function is known to be very hard. In this paper we investigate whether a rule-based approach can help minimizing AND-XOR expressions for functions which are too large to be handled by algorithmic-based approaches. We apply a simple greedy search based on a set of local transformations to the positive polarity ReedMuller expression of Boolean functions. Our experiments on large functions show surprisingly good results. We achieve $23 \%$ reduction in the number of literals on average. We believe that much better results can be achieved if a more sophisticated non-greedy search is used. The purpose of this paper is to motivate more research in this direction.


Keywords: Reed-Muller expression; AND-XOR expression; local transformation; greedy search.

## 1 Introduction

Logic optimization approaches can be divided into algorithmic-based (or global transformation) methods and rule-based (or local transformation) methods [1].

Rule-based methods use a set of rules which are applied when certain patterns are found. A rule transforms a pattern for a local sub-expression, or a sub-circuit, into another equivalent one. Since rules need to be described, and hence the type available of operations/gates must be known, the rule-based approach usually requires that the description of the logic is confined to a limited number of operation/gate types such as AND, OR, XOR, NOT etc. In addition, the transformations

[^0]have limited optimization capability since they are local in nature. Examples of rule-based systems include LSS [2] and LORES/EX [3].

Algorithmic methods use global transformations such as decomposition or factorization, and therefore they are much more powerful compared to the rule-based methods. However, general Boolean methods, including don't care optimization, do not scale well for large functions. Algebraic methods are fast and robust, but they are not complete and thus often give lower quality results. For this reasons, industrial logic synthesis systems normally use algebraic restructuring methods in a combination with rule-based methods.

For the case of AND-XOR optimization, many two- and multi-level algorithmic approaches have been proposed (see Chapter 2 of [4] for an excellent overview).

Most two-level minimization algorithms focus on finding the best polarity for input variables, e.g. [5, 6, 7]. Very good results have been achieved in multi-level minimization, especially for arithmetic functions [8].

We presented an approach to boolean function AND-XOR minimization based on binary trees expression of Reed-Muller form. We applied simple local transformations to the positive polarity Reed-Muller expression of Boolean functions. We tested our algorithm on large functions and we got results about $23 \%$ minimization.

For some functions, the improvement is over $50 \%$.
We also made experiments to different NLFSR minimization and we achieved $25 \%$ reduction of area and power after synthesis in RTLCompiler (Cadence).

The paper is organized as follows. Section 2 describes main notions and definitions used in the sequel. Section 3 presents the main idea of the proposed rule-based approach. Section 4 describes the algorithm. Section 5 summarizes experimental results. Section 6 concludes the paper and discusses open problems.

## 2 Preliminaries

In 1954 Reed [9] and Muller [10] observed that any Boolean function can be expressed as an expansion using AND and XOR operations. Their work leads to better implementations of some practical Boolean functions using AND-XOR arrays rather than AND-OR arrays [11].

An $n$-variable Boolean function has $2^{n}$ canonical fixed-polarity Reed-Muller expressions of type:

$$
f\left(x_{1}, x_{2}, \ldots, x_{n}\right)=c_{0} \oplus c_{1} \dot{x}_{1} \cdot \ldots c_{n} \dot{x}_{n} \oplus c_{2^{n}-1} \dot{x}_{1} \dot{x}_{2} \ldots \dot{x}_{n}
$$

where $\dot{x}_{i}$ is either $x_{i}$ or a complement of $x_{i}$, according to the polarity vector, and
$c_{j} \in\{0,1\}, j \in\left\{0,1, \ldots, 2^{n}-1\right\}$ are constants.
If all variables appear in the above expression uncomplemented, it is called positive polarity Reed-Muller expression. The positive polarity Reed-Muller expression is also known as Algebraic Normal Form (ANF) in cryptographic community, where it is used for representing feedback functions of Linear and Non-Linear Feedback Shift Registers (FSRs) [12, 13]. By minimizing the number of literals in a positive polarity Reed-Muller expression, we can reduce the area of the combinational logic implementing the feedback functions of an FSR. This is important for FSR-based stream ciphers such as Achterbahn [14], Grain [15], Dragon [16], Trivium [17], VEST [18], and [19]. At present, FSR-based stream ciphers are the most promising candidates for cryptographic primitives for advanced contactless technologies like RFID because they have the smallest hardware footprint of all existing cryptographic systems [20]. Since low-cost RFID tags which cannot afford more than a few hundreds of gates for security functionality [21], minimizing the area of a stream cipher is very important. See [22] for examples of positive polarity Reed-Muller expressions used in existing stream ciphers.

## 3 Intuitive Idea

Our goal is to further reduce the number of literals in a given positive polarity Reed-Muller expression of a Boolean function. We do it by subsequently applying three transformation rules: $x \cdot y \oplus x \cdot z=x \cdot(y \oplus z) . x \oplus 1=\bar{x}, x \cdot 1=x$. In our implementation of the algorithm, we represent the Reed-Muller expression by a binary tree with internal nodes labelled by AND and XOR operations, and rest nodes are labelled by variables or constants. For example, the circuit representing the expression $f\left(x_{1}, x_{2}, x_{3}\right)=1 \oplus x_{1} x_{2} \oplus x_{3} x_{1} x_{2}$ is shown in Figure 3.


Fig. 1. Example binary tree representation of RM-expression
The first rule, $x \cdot y \oplus x \cdot z=x \cdot(y \oplus z)$, factors out a literal common for two
product-terms.
If either $y$ or $z$ is a constant 1 , then the rule reduces to $x \cdot y \oplus x=x \cdot(y \oplus 1)$. So, the transformation $x \cdot y \oplus x=x \cdot \bar{y}$ is done in two steps. Applying the first rule allows us reduce one gate (see Figure 2a). The second rule switches from fixed-polarity RM-form to mixed-polarity RM-form (see Figure 2b). And the third rule as the first one reduces one gate (Figure 2c).

c)

Fig. 2. Rules base: a) the first rule, b) the second rule, c) the third rule.

The search is carried out starting from the root of the binary tree depth-first. We look for patterns corresponding to the transformation rules and apply them on firstfound basis. For example, if we have an expression $1 \oplus x_{1} x_{2} \oplus x_{2} x_{3}$, we identify that the literal $x_{2}$ is common for the product-terms $x_{1} x_{2}$ and $x_{2} x_{3}$ and factor it out as $1 \oplus x_{2}\left(x_{1} \oplus x_{3}\right)$. If we have an expression $1 \oplus x_{1} \oplus x_{1} x_{2}$, we identify that the literal $x_{1}$ is common for the product-terms $x_{1}$ and $x_{1} x_{2}$ and factor it out as $1 \oplus x_{1}\left(x_{2} \oplus 1\right)$. By applying the transformation $x_{2} \oplus 1=\overline{x_{2}}$ we then further reduce this expression to $1 \oplus x_{1} \overline{x_{2}}$.

## 4 Rule-Based Algorithm

The pseudo-code of the algorithm is shown in Algorithm 1. The input is a positive polarity Reed-Muller expression of a Boolean function.

The output is an AND-XOR expression of this Boolean function with a smaller number of literals.

```
Algorithm 1 Minimizes the number of literals in a given positive polarity Reed-Muller expression.
    \(C:=C_{0} ; / * C_{0}\) is the binary tree representing the Reed-Muller canonical form*/
    cost \(:=|C| ; / *|C|\) is the number of vertices in \(C^{* /}\)
    while \(|C|<\) cost do
        for every vertex \(v\) in \(C\) do
            flag \((v):=\) true;
            matchlist \((p):=\operatorname{MATCH}(v) ; / * \operatorname{MATCH}(v)\) returns the best match for \(v\) and -1 if there is no
            match*/
        end for
        for every vertex \(v\) in \(C\) do
            if \(\operatorname{flag}(v)=\) true then
                if \(\operatorname{matchlist}(v) \neq-1\) then
                    \(C:=\operatorname{APPLY}(v\), matchlist \((v))\);
                    \(\operatorname{MARKAFFECTED}(C) ; / * \operatorname{Sets} \operatorname{flag}(v)=\) false for all vertices \(v\) affected by the applied
                    local transformation*/
                end if
            end if
        end for
    end while
    Return C;
```

The input Reed-Muller expression is represented by a binary tree with internal nodes labelled by AND and XOR operations, and leaf nodes labeled by variables. The tree is traversed depth-first. The procedure $\operatorname{Match}(\mathrm{v})$ tries to match the subgraph rooted at vertex $v$ to any of the patterns for which a local transformation is defined. We use the following three transformations:

1. $x \cdot y \oplus x \cdot z=x \cdot(y \oplus z)$,
2. $x \oplus 1=\bar{x}$,
3. $x \cdot 1=x$.

The match which reduces the number of literals the most is returned. Only transformations which reduce the number of literals are considered. In case of multiple matches with equal cost, the first match is returned. In case of no match, -1 is returned.

The first rule is main in our rule base, let us consider how to apply it on binary trees:

1. Start from the root of tree,
2. Look for XOR node recursively,
3. If found then looking for common subgraph recursively (Figure 3a),
4. If found copy subgraph in temp (variable tmp), and replace it by 1 in tree (Figure 3b),
5. Add new AND node and connect it with common subgraph and XOR node (Figure 3c).


Fig. 3. An example of applying the first rule.

The optimization algorithm iteratively traverses the binary tree marking possible sub-graph transformations vertices and then, after the marking, replaces as many of them as possible. The procedure $\operatorname{Apply}(v)$ applies a given transformation to a vertex $v$. After the tree has been locally transformed by Apply, the flags assigned to vertices affected by the transformation are set to false. The procedure

Apply is only called for a vertex product, $v$, if its flag is true.
The algorithm stops iterating when no further minimization can be made. Full process of optimization for the function: $f\left(x_{1}, x_{2}, x_{3}\right)=1 \oplus x_{1} x_{2} \oplus x_{3} x_{1} x_{2}$ is shown in Figure 4.

## 5 Experimental Results

To evaluate the proposed approach, we applied it to more then 30 benchmark circuits with 15 and more inputs (we choosed it because it have a big number of inputs, it is important for NLFSR). The original benchmarks were in espresso (two-level AND-OR) format. We transformed them to the positive polarity Reed-Muller expression using our implementation of the Functional Decision Diagrams (FDD)based algorithm [23] in CUDD package [24].

The results are summarized in Table 1. Columns 1, 2 and 3 show the name of the benchmark, the number of primary inputs and primary outputs, respectively. Columns 4 and 5 show the number of literals in the original positive polarity ReedMuller expression and in the AND-XOR expression computed using the presented rule-based approach.


Fig. 4. An example of full process of optimization.

Our current implementation supports single output functions only. For multiple-output functions, we calculate the number of literals separately for each output. The numbers shown in columns 4 and 5 are the sums of literals for all outputs, without taking sharing into account.

In the last column, we show the run time of the presented approach (a total sum of run times for each individual output).

The difference between number of cells in PPRM-expression and the result of our algorithm for this benchamrk is shown on Figure 5


Fig. 5. Experimental results for some benchmark functions with 15 or more inputs

As we can see, on average, we achieve $23 \%$ reduction in the number of literals
using 30.6 min for all outputs (which is 1 min per output on average).
Table 1. Experimental results for 30 benchmark functions with 15 or more inputs.

| Name | Inputs | Outputs | Number of Literals |  | $\frac{L_{1}-L_{2}}{L_{1}}$ | Time, sec |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $L_{1}$, Original PPRM | $L_{2}$, Optimized |  |  |
| alcom | 15 | 38 | 590 | 283 | 0.52 | 151 |
| bc0 | 26 | 11 | 102807 | 61719 | 0.4 | 1209 |
| cps | 24 | 109 | 1254725 | 842617 | 0.33 | 8224 |
| ex4 | 128 | 28 | 492009 | 438561 | 0.11 | 1057 |
| soar | 83 | 94 | 1851018 | 1775561 | 0.04 | 4381 |
| spla | 16 | 46 | 1516116 | 1087487 | 0.28 | 4754 |
| pdc | 16 | 40 | 1306776 | 1006523 | 0.23 | 4681 |
| t1 | 21 | 23 | 4544 | 3040 | 0.33 | 906 |
| ti | 47 | 72 | 140494 | 99330 | 0.29 | 4084 |
| ts10 | 22 | 16 | 5616 | 4080 | 0.27 | 2114 |
| x9dn | 27 | 7 | 21775743 | 21730592 | 0.002 | 1084 |
| in4 | 32 | 20 | 3069036 | 2965936 | 0.03 | 2417 |
| in6 | 33 | 23 | 49866 | 38399 | 0.23 | 758 |
| in7 | 26 | 10 | 1269250 | 1164289 | 0.08 | 529 |
| jbp | 36 | 57 | 84808 | 53610 | 0.37 | 607 |
| mark1 | 20 | 31 | 7315436 | 6726301 | 0.08 | 4091 |
| misg | 56 | 23 | 823 | 542 | 0.34 | 302 |
| mish | 94 | 43 | 245 | 177 | 0.28 | 151 |
| misj | 35 | 14 | 115 | 76 | 0.34 | 2 |
| opa | 17 | 69 | 55489 | 34037 | 0.39 | 2114 |
| b3 | 32 | 20 | 2753233 | 2660698 | 0.03 | 2267 |
| b4 | 33 | 23 | 49831 | 38389 | 0.23 | 758 |
| chkn | 29 | 7 | 769518 | 735422 | 0.04 | 917 |
| ibm | 48 | 17 | 20128 | 12422 | 0.38 | 604 |
| in2 | 19 | 10 | 116031 | 74972 | 0.35 | 1060 |
| in3 | 35 | 29 | 612307 | 489596 | 0.2 | 1371 |
| in5 | 24 | 14 | 32676 | 23403 | 0.28 | 1057 |
| vg2 | 25 | 8 | 6996204 | 6948513 | 0.01 | 482 |
| x6dn | 39 | 5 | 124964 | 81280 | 0.35 | 755 |
| x7dn | 66 | 15 | 442103 | 350452 | 0.21 | 2265 |
| average | 37.47 | 30.73 | 1740416.7 | 1648276.9 | 0.23 | 1838.4 |

Also we evaluated our Algorithm on random NLFSRs as well as on NLFSRs of existing stream ciphers (just boolean functions without flip-flops). We compared numbers of cells and area of scheme of NLFSR. All characteristics are achieved by RTLCompiler. We tested 40 different NLFSRs.

The results of comapring between RM-exprerssion of NLFSR and expression in our algoriyhm before syntesis in RTLComlier are showed in Table 2. We also compared area and number of cells in PPRM-expression and our algorithm expression
after synthesis in RTLCompiler. The results showed in Table 3. This experiment showes ability of our algorithm to improve sinthesis work of RTLComplier about $10 \%$. The results of our algorithm work dependen on number of common literals in RM-from of Boolean function.

Let us look at one of the most popular NLFSR - grain128 and compare its expression in RM-form and our algorithm form:
$x_{62} \oplus x_{60} \oplus x_{52} \oplus x_{45} \oplus x_{37} \oplus x_{33} \oplus x_{28} \oplus x_{21} \oplus x_{14} \oplus x_{9} \oplus x_{0} \oplus x_{63} x_{60} \oplus$ $x_{37} x_{33} \oplus x_{15} x_{9} \oplus x_{60} x_{52} x_{45} \oplus x_{33} x_{28} x_{21} \oplus x_{63} x_{45} x_{28} x_{9} \oplus x_{60} x_{52} x_{37} x_{33} \oplus x_{63} x_{60} x_{21} x_{15} \oplus$ $x_{63} x_{60} x_{52} x_{45} x_{37} \oplus x_{33} x_{28} x_{21} x_{15} x_{9} \oplus x_{52} x_{45} x_{37} x_{33} x_{28} x_{21}$ - 50 nodes

After perfoming the algorithm we obtained

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    x _ { 6 2 } \oplus x _ { 6 0 } \oplus x _ { 5 2 } \oplus x _ { 4 5 } \oplus x _ { 3 7 } \oplus x _ { 3 3 } \oplus x _ { 2 8 } \oplus x _ { 2 1 } \oplus x _ { 1 4 } \oplus x _ { 9 } \oplus x _ { 0 } \oplus x _ { 6 3 } x _ { 6 0 } \oplus
\mp@subsup{x}{37}{}\mp@subsup{x}{33}{}\oplus\mp@subsup{x}{15}{}\mp@subsup{x}{9}{}\oplus\mp@subsup{x}{60}{}\mp@subsup{x}{52}{}\mp@subsup{x}{45}{}\oplus\mp@subsup{x}{28}{}(\mp@subsup{x}{33}{}\mp@subsup{x}{21}{}\oplus\mp@subsup{x}{63}{}\mp@subsup{x}{45}{}\mp@subsup{x}{9}{})\oplus\mp@subsup{x}{60}{}(\mp@subsup{x}{52}{}\mp@subsup{x}{37}{}\mp@subsup{x}{33}{}\oplus\mp@subsup{x}{63}{}(\mp@subsup{x}{21}{}\mp@subsup{x}{15}{}\oplus
```



So we reduced 7 nodes.

## 6 Conclusion

This presents a greedy local transformation-based method for optimizing ANDXOR expressions using literal count as objective. Our experimental results show that we achieve $23 \%$ reduction in the number of literals on average compared to the positive polarity Reed-Muller expression.

The main drawback of the discussed technique is that the optimization algorithm is greedy.

A greedy algorithm might get stuck quite fast in a local minimum instead of exploring all possibilities.

Further work remains to explore more sophisticated search approaches.
Another drawback of the presented algorithm is a simplistic optimization objective.

The use of literals as an objective function often results in poor circuit structures since the number of literals is only loosely correlated with the delay of the mapped circuit.

In our future work we will consider more complex objective functions, e.g. providing the choice of minimizing the number of literals or number of levels.

Table 2. Experimental results for NLFSRs.

|  |  | NLFSR in RM-form |  | NLFSR our Algorithm |  | difference Number of Cells | difference Area |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Cells | Area | Cells | Area |  |  |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 1 | VEST-1 | 31 | 194 | 19 | 110 | 38,71 | 43,30 |
| 2 | VEST-2 | 33 | 207 | 23 | 138 | 30,30 | 33,33 |
| 3 | VEST-3 | 30 | 188 | 22 | 128 | 26,67 | 31,91 |
| 4 | VEST-4 | 30 | 188 | 21 | 125 | 30 | 33,51 |
| 5 | VEST-5 | 32 | 207 | 23 | 135 | 28,13 | 34,78 |
| 6 | VEST-6 | 35 | 219 | 22 | 128 | 37,14 | 41,55 |
| 7 | VEST-7 | 32 | 200 | 24 | 144 | 25 | 28 |
| 8 | VEST-8 | 26 | 169 | 21 | 119 | 19,23 | 29,59 |
| 9 | VEST-9 | 27 | 169 | 20 | 113 | 25,93 | 33,14 |
| 10 | VEST-10 | 33 | 207 | 25 | 147 | 24,24 | 28,99 |
| 11 | VEST-11 | 31 | 194 | 23 | 135 | 25,81 | 30,41 |
| 12 | VEST-12 | 30 | 188 | 23 | 135 | 23,33 | 28,19 |
| 13 | VEST-13 | 35 | 219 | 26 | 150 | 25,71 | 31,51 |
| 14 | VEST-14 | 32 | 207 | 23 | 132 | 28,13 | 36,23 |
| 15 | VEST-15 | 32 | 200 | 25 | 147 | 21,875 | 26,5 |
| 16 | VEST-16 | 34 | 213 | 24 | 144 | 29,41 | 32,39 |
| 17 | VEST-17 | 32 | 200 | 22 | 132 | 31,25 | 34 |
| 18 | VEST-18 | 28 | 182 | 20 | 119 | 28,57 | 34,62 |
| 19 | VEST-19 | 27 | 169 | 20 | 116 | 25,93 | 31,36 |
| 20 | VEST-20 | 34 | 213 | 22 | 132 | 35,29 | 38,03 |
| 21 | VEST-21 | 33 | 207 | 23 | 138 | 30,30 | 33,33 |
| 22 | VEST-22 | 36 | 225 | 22 | 135 | 38,89 | 40 |
| 23 | VEST-23 | 32 | 200 | 22 | 128 | 31,25 | 36 |
| 24 | VEST-24 | 35 | 219 | 26 | 153 | 25,71 | 30,14 |
| 25 | VEST-25 | 32 | 207 | 22 | 132 | 31,25 | 36,23 |
| 26 | VEST-26 | 32 | 200 | 22 | 132 | 31,25 | 34 |
| 27 | VEST-27 | 34 | 213 | 26 | 153 | 23,53 | 28,17 |
| 28 | VEST-28 | 32 | 200 | 23 | 138 | 28,13 | 31 |
| 29 | VEST-29 | 32 | 200 | 23 | 132 | 28,13 | 34 |
| 30 | VEST-30 | 28 | 182 | 24 | 135 | 14,29 | 25,82 |
| 31 | VEST-31 | 25 | 157 | 20 | 122 | 20 | 22,29 |
| 32 | VEST-32 | 31 | 194 | 23 | 132 | 25,81 | 31,96 |
| 33 | Achterbahn-1 | 51 | 326 | 44 | 282 | 13,73 | 13,50 |
| 34 | Achterbahn-2 | 37 | 250 | 34 | 225 | 8,11 | 10 |
| 35 | Achterbahn-3 | 37 | 244 | 34 | 225 | 8,11 | 7,79 |
| 36 | Achterbahn-4 | 67 | 426 | 56 | 351 | 16,42 | 17,61 |
| 37 | Achterbahn-5 | 66 | 413 | 53 | 332 | 19,70 | 19,61 |
| 38 | Achterbahn-6 | 67 | 432 | 54 | 338 | 19,40 | 21,76 |
| 39 | grain 128 | 18 | 133 | 18 | 113 | 0 | 15,04 |
| 40 | grain80 | 39 | 263 | 38 | 244 | 2,56 | 7,22 |
|  | average |  |  |  |  | 24,43 | 28,92 |

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Table 3. Experimental results for NLFSRs after synthesis in RTLCompiler.

|  |  | NLFSR in RM-form |  | NLFSR our Algorithm |  | difference <br> Number of Cells | difference Area |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Cells | Area | Cells | Area |  |  |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 1 | VEST-1 | 15 | 60 | 12 | 44 | 20 | 26,67 |
| 2 | VEST-2 | 20 | 64 | 14 | 51 | 30 | 20,31 |
| 3 | VEST-3 | 11 | 51 | 11 | 43 | 0 | 15,69 |
| 4 | VEST-4 | 24 | 101 | 11 | 48 | 54,17 | 52,48 |
| 5 | VEST-5 | 27 | 113 | 17 | 67 | 37,03 | 40,71 |
| 6 | VEST-6 | 17 | 62 | 13 | 60 | 23,53 | 3,23 |
| 7 | VEST-7 | 21 | 75 | 15 | 48 | 28,57 | 36 |
| 8 | VEST-8 | 12 | 50 | 13 | 51 | -8,33 | -2 |
| 9 | VEST-9 | 14 | 53 | 10 | 40 | 28,57 | 24,53 |
| 10 | VEST-10 | 20 | 70 | 14 | 52 | 30 | 25,71 |
| 11 | VEST-11 | 21 | 63 | 13 | 49 | 38,10 | 22,22 |
| 12 | VEST-12 | 18 | 61 | 14 | 46 | 22,22 | 24,59 |
| 13 | VEST-13 | 27 | 86 | 17 | 65 | 37,04 | 24,42 |
| 14 | VEST-14 | 20 | 74 | 14 | 50 | 30 | 32,43 |
| 15 | VEST-15 | 16 | 55 | 19 | 69 | -18,75 | -25,45 |
| 16 | VEST-16 | 21 | 70 | 15 | 56 | 28,57 | 20 |
| 17 | VEST-17 | 16 | 66 | 15 | 60 | 6,25 | 9,09 |
| 18 | VEST-18 | 17 | 62 | 16 | 58 | 5,88 | 6,45 |
| 19 | VEST-19 | 16 | 56 | 12 | 44 | 25 | 21,43 |
| 20 | VEST-20 | 16 | 57 | 17 | 58 | -6,25 | -1,75 |
| 21 | VEST-21 | 19 | 81 | 17 | 57 | 10,53 | 29,63 |
| 22 | VEST-22 | 21 | 69 | 16 | 60 | 23,81 | 13,04 |
| 23 | VEST-23 | 22 | 93 | 18 | 65 | 18,18 | 30,12 |
| 24 | VEST-24 | 22 | 73 | 12 | 48 | 45,45 | 34,25 |
| 25 | VEST-25 | 22 | 77 | 16 | 65 | 27,27 | 15,58 |
| 26 | VEST-26 | 18 | 62 | 12 | 51 | 33,33 | 17,74 |
| 27 | VEST-27 | 14 | 48 | 14 | 48 | 0 | 0 |
| 28 | VEST-28 | 15 | 54 | 15 | 57 | 0 | -5,56 |
| 29 | VEST-29 | 20 | 77 | 15 | 58 | 25 | 24,68 |
| 30 | VEST-30 | 21 | 78 | 10 | 44 | 52,38 | 43,59 |
| 31 | VEST-31 | 15 | 60 | 10 | 44 | 33,33 | 26,67 |
| 32 | VEST-32 | 13 | 48 | 9 | 45 | 30,77 | 6,25 |
| 33 | Achterbahn-1 | 22 | 85 | 20 | 69 | 9,09 | 18,82 |
| 34 | Achterbahn-2 | 27 | 142 | 27 | 135 | 0 | 4,93 |
| 35 | Achterbahn-3 | 27 | 99 | 30 | 127 | -11,11 | -28,28 |
| 36 | Achterbahn-4 | 51 | 199 | 44 | 185 | 13,73 | 7,04 |
| 37 | Achterbahn-5 | 19 | 86 | 22 | 93 | -15,79 | -8,14 |
| 38 | Achterbahn-6 | 19 | 89 | 20 | 95 | -5,26 | -6,74 |
| 39 | grain128 | 15 | 80 | 15 | 80 | 0 | 0 |
| 40 | grain80 | 27 | 165 | 30 | 154 | -11,11 | 6,67 |
|  | average |  |  |  |  | 17,28 | 15,18 |

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