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SYSTEMS FOR GENERATING HOMOGENEOUS ELECTRIC FIELD

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Abstract: The systems of thin coaxial toroidal electrodes and hollow coaxial biconical electrodes are modeled for generating homogeneous electric field. The electrodes are of absolutely equal and opposite potential values, so that the potential of the symmetrical plane is zero. The electrodes dimensions and positions can be chosen so that practically homogeneous electric field can be obtained in central region of the structure. Along the system axis the potential function is expressed as polynomial series and the dimensions of system are determined so that only the linear term in these series is dominant. Thus, the primary cell of N-th kind which has N different degrees of degeneration is determined.

Better results are obtained with primary cell of higher kind. So the primary cell of the second kind provides the larger region with homogeneous electric field than the primary cell of the first kind. Also, better results are obtained with biconical electrode system then with system having two thin toroidal electrodes.

Obtained results show that in the case of basic biconical electrode system (primary cell), the electric field homogeneity is practically independent of the electrode length if length of electrode is great enough. The obtained results satisfy the requirements of high accuracy.

The same electrodes can be used for making systems serving for space protection against the overflow electric fields strength.

Key words: Homogeneous electric field, toroidal electrodes, biconical electrodes system.

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1. Introduction

The problems of electric and magnetic fields synthesis are of great importance in many technical applications where it is necessary to generate the field with a high accuracy and of required features. Existing fields for some applications need to be practically reduced or eliminated by specially generated electric and magnetic fields.

Perfect homogeneous electric field can be generated in closed area like excentrical spherical hole inside the sphere having uniform charge per unit volume, or in excentrical cylindrical hole inside the cylinder having uniform charge per unit volume. These areas are difficult to reach so various open systems for generation of homogeneous electric fields are developed.

In order to realize the homogeneous electrostatic field two coaxial thin toroidal electrodes can be used [2]-[4]. Then the homogeneous electric field exists in the central region of the system. More homogeneous electric field can be realized using the system of several thin coaxial toroidal electrodes [4]. Then several difficulties in the electrodes system design exist, like necessary precision of the electrode positions and of its dimensions. These difficulties are avoided by adopting a simple system of two thin hollow coaxial biconical electrodes. Present authors suggested biconical electrostatic systems for getting the homogeneous electric field, [5], [6]. The electrodes systems of that shape are used for electrostatic space protection system examination [7]-[9].

General procedure for modeling systems for generating electrostatic field of high order of homogeneity using toroidal and biconical electrodes is presented in this paper. The results for the electrical field strength on the system axis, the equipotential and equienergetic curves are presented. The deviation of the axial and radial electric field components near the origin from the field in the central point for the primary cell of the first kind for toroidal and biconical electrodes system is presented too.

The proposed systems can be used for eliminating external fields by specially generated electric and magnetic fields. Thus, protected space without electric and magnetic fields can be obtained on the Earth. It can be important because special physical conditions that reached only in satellites, that difference a lot from the Earth's conditions, can be realized. That is very useful because the investigation of the ordinary life and many of the technological processes in satellite conditions can be done on Earth.

New Equivalent electrodes method [1] will be used for all calculations. This method has been developed in our Department for nonrotational field solving. It has many advantages with respect to other convenient methods, as well as good efficiency, convergence and satisfactory accuracy.

2. Generation of Homogeneous Electric Field

As a basic element for generating homogeneous electric field, the so-called primary cell is used. It is consisted from two thin coaxial toroidal electrodes, Fig. 1(a), or from two hollow thin coaxial biconical electrodes, Fig. 1(b). The electrodes are of absolutely equal and opposite potential values, so that the potential of the symmetrical plane is zero, $\varphi = 0$, for $z = 0$. r and z are cylindrical co-ordinates, see in Figs. 1(a) and 1(b).

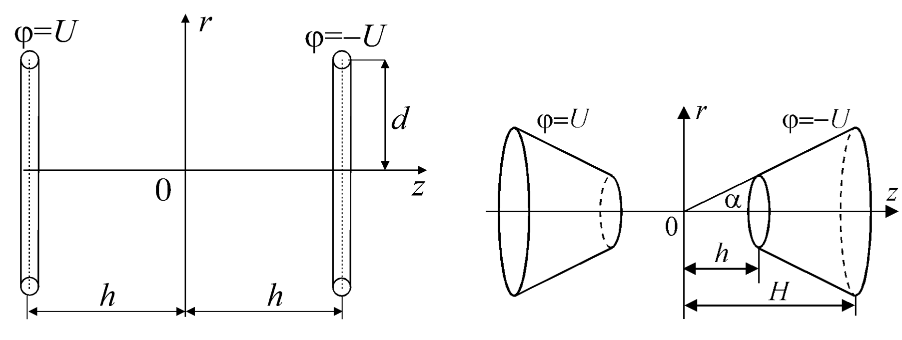


Fig. 1. The primary cell for generating homogeneous electric field.
(a) With toroidal electrodes.
(b) With biconical electrodes.

When the electrodes system is axially symmetric and the system axis is in coincidence with z-axis of cylindrical co-ordinate system, a potential, φ, is the solution of Laplace's equation

$$\Delta\varphi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) + \frac{\partial^2\varphi}{\partial z^2} = 0. \tag{1}$$

The solution is obtined using method of separaton variables,

$$\varphi = \sum_{n=0}^{\infty} f_{2n}(z) r^{2n}, \tag{2}$$

where

$$\begin{aligned} f_{2n}(z) &= (-1)^n \frac{f_0^{(2n)}(z)}{[(2n)!!]^2} \\ &= (-1)^n \frac{\varphi^{(2n)}(r=0,z)}{[(2n)!!]^2}. \end{aligned} \tag{3}$$

So that, it is possible to determine the potential in the point (r, z) if it is known potential value on the system axis, so-called axial potential $\varphi(r = 0, z)$.

Along the system axis, axial potential can be expanded as

$$\varphi(r = 0, z) = \sum_{n=0}^{\infty} \varphi_{2n+1} z^{2n+1}. \tag{4}$$

Because the series (4) are decreasing, it is possible to choose primary cell dimensions so that some series terms vanish and there is the domination of only one term. This dominant term provides corresponding distribution of the electric field. For example, in order to provide homogenous electric field in central system area, potential has to have the linear function versus the distance z from central system point. Equating the coefficient of z^{2N+1} with zero gives the relation

$$\varphi_{2N+1} = 0, \tag{5}$$

where N defines kind of primary cell. As this equation has N real positive roots, the primary cell of N-th kind is obtained, having n degrees of degeneration, where $n = 1, \ldots, N$. Each of these solutions defines only one degenerative primary cell.

In order to eliminate the influence of other series terms from z^3 to z^{2N-1}, the following equations system has to be satisfied

$$\varphi_{2n+1} = 0, \quad \text{for} \quad n = 1, 2, \ldots, N-1. \tag{6}$$

The rest of the higher order terms can be neglected with respect to linear term. Using this procedure, systems for generating electrostatic field of high order of homogeneity in the central region are obtained. The field is more homogeneous if the primary cell is of higher kind.

3. Toroidal Electrodes Systems

For primary cell with toroidal electrodes, Fig. 1(a), the potential is

$$\varphi(r, z) = \frac{Q}{2\pi^2 \varepsilon} \left[\frac{K(\frac{\pi}{2}, k)}{\sqrt{(d+r)^2 + (z+h)^2}} - \frac{K(\frac{\pi}{2}, p)}{\sqrt{(d+r)^2 + (z-h)^2}} \right], \tag{7}$$

where Q is total charge on the toroidal electrode, r and z are cylindrical coordinates, and $K(\pi/2, k)$ is complete elliptic integral of the first kind having modules

$$k^2 = \frac{4dr}{(d+r)^2 + (z+h)^2} \quad \text{and} \qquad p^2 = \frac{4dr}{(d+r)^2 + (z-h)^2}. \tag{8}$$

The electric field has radial and axial components,

$$E_r = \frac{Q}{4\pi^2 \varepsilon r}\left[\frac{K(\frac{\pi}{2},k) - \frac{(z+h)^2 + d^2 - r^2}{(d-r)^2 + (z+h)^2}E(\frac{\pi}{2},k)}{\sqrt{(d+r)^2 + (z+h)^2}} - \frac{K(\frac{\pi}{2},p) - \frac{(z-h)^2 + d^2 - r^2}{(d-r)^2 + (z-h)^2}E(\frac{\pi}{2},p)}{\sqrt{(d+r)^2 + (z-h)^2}}\right], \tag{9}$$

$$E_z = \frac{Q}{2\pi^2 \varepsilon}\left[\frac{\frac{z+h}{(d-r)^2 + (z+h)^2}}{\sqrt{(d+r)^2 + (z+h)^2}}E(\frac{\pi}{2},k) - \frac{\frac{z-h}{(d-r)^2 + (z-h)^2}}{\sqrt{(d+r)^2 + (z-h)^2}}E(\frac{\pi}{2},p)\right] \tag{10}$$

and

$$E_\theta = 0, \tag{11}$$

$E(\pi/2, k)$ is complete elliptic integral of the second kind. Elliptic integrals can be numerically calculated very correctly and simply using Landen's transformation [10].

Electric field strength is

$$E = E(r,z) = \sqrt{E_r^2 + E_z^2}. \tag{12}$$

In the system axis, $r = 0$, the potential can be put in closed form,

$$\varphi(0,z) = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{\sqrt{d^2 + (z+h)^2}} - \frac{1}{\sqrt{d^2 + (z-h)^2}}\right]. \tag{13}$$

Function (13) can be expressed as the following series

$$\varphi(0,z) = \sum_{n=0}^{\infty} \varphi_{2n+1} z^{2n+1}, \tag{14}$$

where are

$$\varphi_{2n+1} = -\frac{QP_{2n+1}(\cos\alpha)}{2\pi\varepsilon D^{2n+2}}, \tag{15}$$

$D = \sqrt{d^2 + h^2}$ and $P_n(x)$ is Legendre polynomial of the first kind.

At the origin electric field strength is

$$E(0,0) = E_z(r=0, z=0) = \frac{Q}{2\pi\varepsilon}\frac{h}{(d^2+h^2)^{3/2}}. \tag{16}$$

4. Degenerative Primary Cells with Toroidal Electrodes

If the coefficient of z^3 is equal to zero, then from Eq. (15)

$$P_3(\cos\alpha) = 0. \tag{17}$$

This equation has only one root, $\cos\alpha = \cos\alpha_1 = x_{3,1} \approx 0.7745966692$, and the primary cell of the first kind ($N = 1$ in Eq. (5)), with dimensions $h_1 = 0.7745966692D$ and $d_1 = 0.632455532D$ is obtained.

Equating the coefficient of z^5 with zero, then the primary cell of the second kind is obtained, which dimensions are: $d_1 = 0.422892524D$, $h_1 = 0.906179846D$, $d_2 = 0.8426544122D$ and $h_2 = 0.538469310D$.

In general case equating the coefficient of z^{2N+1} with zero in Eq. (4), then the primary cell of the N-th kind is obtained.

Table 1 presents the dimensions and positions of primary cells of the N-th kind and of the different degree for toroidal electrodes (from 1^{st} to 10^{th}). All degenerative primary cells are placed on sphere surface of radius $D = \sqrt{d_n^2 + h_n^2}$.

Table 1. Dimensions of primary cells of the N-th kind for toroidal systems.

	d_{10}/D	d_9/D	d_8/D	d_7/D	d_6/D	d_5/D	d_4/D	d_3/D	d_2/D	d_1/D	
N	0.990	0.958	0.906	0.834	0.745	0.640	0.521	0.392	0.254	0.112	10
1	0.775	0.263	0.263	0.263	0.263	0.263	0.263	0.263	0.263	0.263	9
2	0.906	0.538	0.294	0.294	0.294	0.294	0.294	0.294	0.294	0.294	8
3	0.949	0.742	0.406	0.334	0.334	0.334	0.334	0.334	0.334	0.334	7
4	0.968	0.836	0.613	0.324	0.385	0.385	0.385	0.385	0.385	0.385	6
5	0.978	0.887	0.730	0.519	0.270	0.963	0.855	0.683	0.462	0.208	5
6	0.984	0.918	0.802	0.642	0.448	0.230	0.946	0.790	0.549	0.250	4
7	0.988	0.937	0.848	0.724	0.571	0.394	0.201	0.914	0.671	0.315	3
8	0.991	0.951	0.880	0.782	0.658	0.513	0.351	0.178	0.843	0.423	2
9	0.992	0.960	0.903	0.823	0.721	0.601	0.465	0.317	0.160	0.632	1
10	0.994	0.967	0.920	0.853	0.768	0.667	0.552	0.424	0.288	0.146	N
	h_1/D	h_2/D	h_3/D	h_4/D	h_5/D	h_6/D	h_7/D	h_8/D	h_9/D	h_{10}/D	

5. Primary Cell of the First Kind with Toroidal Electrodes

All calculations in this paper are done for the primary cell of the first kind. In this case the potential distribution along system axis, z, is

$$\varphi(0,z) = -\frac{Q}{2\pi\varepsilon D^2} P_1(\cos\alpha_1) z - \frac{Q}{2\pi\varepsilon D^6} P_5(\cos\alpha_1) z^5 - \cdots, \qquad (18)$$

so that is approximate

$$\varphi(0,z) \approx -\frac{Q}{2\pi\varepsilon D}\sqrt{\frac{3}{5}}\left(\frac{z}{D} - \frac{27}{50}\frac{z^5}{D^5}\right), \quad \text{for} \quad z < D. \qquad (19)$$

The electric field is homogeneous and equal to electric field strength in the origin

$$E(0,z) \approx E(0,0) = \frac{Q}{2\pi\varepsilon D^2}\sqrt{\frac{3}{5}}, \quad \text{for} \quad z < D. \qquad (20)$$

It is very useful to consider deviation of the field from the field at the origin. The deviation of the axial electric field component is denoted by η and is defined by

$$\eta = \frac{E_z(r,z) - E(0,0)}{E(0,0)}. \qquad (21)$$

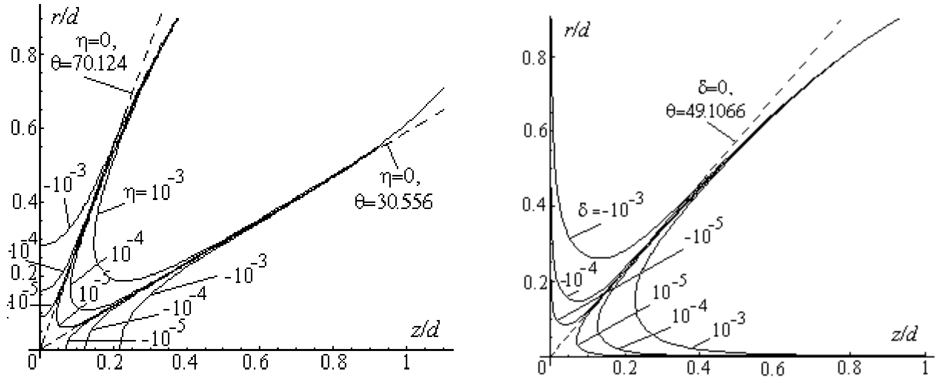
Fig. 2(a) demonstrates that for a region approximately $|z| \leq d/14$ from the origin the axial electric field is uniform to 1 part in 100 000 and it is uniform to 1 part in 10 000 over a region of roughly $|z| \leq d/8$. In special directions from the origin, the deviation of the axial field is zero. These directions are given by setting $\eta = 0$ in Eq. (21). These directions are with the polar angles equal to $\theta = 30.5556°$ and $\theta = 70.1243°$, [2], and are showed in Fig. 2(a).

If the deviation of the radial electric field is denoted by δ and is defined by

$$\delta = \frac{E_r(r,z)}{E(0,0)}. \qquad (22)$$

This equation shows that the radial electric field is zero along the axis and on the $z = 0$ plane, as it is required by symmetry considerations. The radial electric field near the origin is also zero along the directions when the polar angle is equal to $\theta = 49.1066°$, [2], Fig. 2(b).

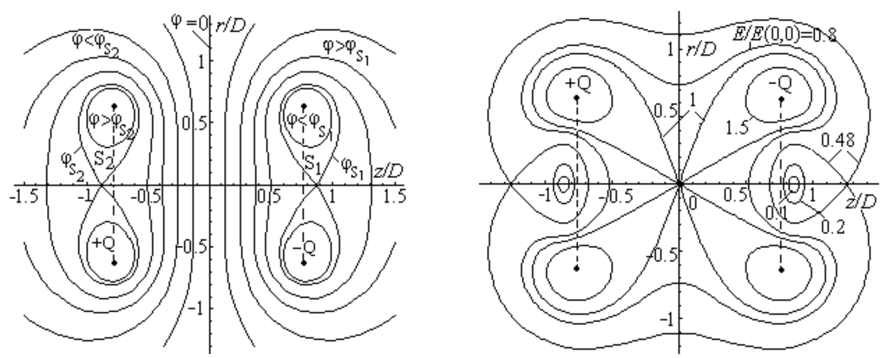
The equipotential curves for optimal system are presented in Fig. 3(a). The singular equipotential curves exist for $\varphi = \varphi_{S_1} \approx -0.498U$ and for $\varphi = \varphi_{S_2} \approx 0.498U$.



(a) (b)

Fig. 2. Deviations from the central field value.
(a) For the axial electric field.
(b) For the radial electric field.

The equienergetic curves, where $E = \sqrt{E_r^2 + E_z^2}$ is constant, are presented in Fig. 3(b).



(a) (b)

Fig. 3. Toroidal primary cell of the first kind.
(a) The equipotential curves.
(b) The equienergetic curves.

6. Biconical Electrodes Systems

Homogeneous electric field can be get using biconical electrodes. The mathematical model of the electrostatic system consisting of thin hollow coaxial electrodes is presented in Fig. 4(a). Equivalent electrodes method

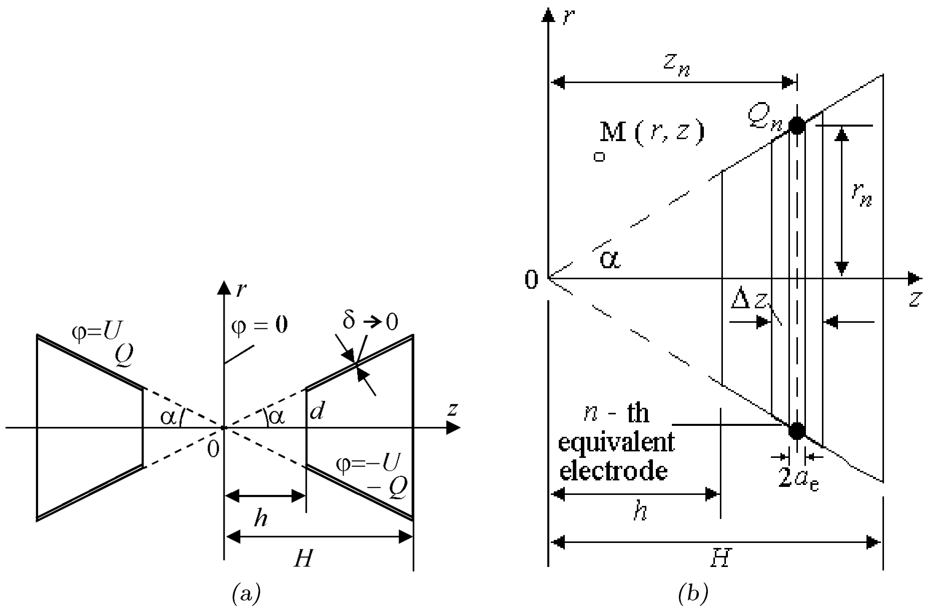


Fig. 4. Biconical electrode system.
(a) Mathematical model.
(b) Forming of equivalent electrodes.

is used for all calculations [1]. The idea of this method is that the biconical electrode divides on N parts that are so small that each of them can be replaced by toroidal electrodes, so-called equivalent electrodes.

The thin toroidal equivalent electrodes are selected, because of the system axial symmetry and the fact that the biconical electrodes are hollow and very thin. If the biconical electrode is divided into N toroidal elements having width $\Delta z = (H - h)/N$, then equivalent electrode axis are on the circular curves as shown in Fig. 4(b), where

$$z_n = h + (2n-1)\frac{\Delta z}{2}, \quad r_n = z_n \tan\alpha, \quad \text{for} \quad n = 1, 2, \dots, N. \qquad (23)$$

The equivalent electrodes radius is $a_e = \Delta z/(4\cos\alpha)$.

If Q_n is the charge of n-th equivalent electrode, then the potential is

$$\varphi = \frac{1}{2\pi^2\varepsilon}\sum_{n=1}^{N}\left[\frac{Q_n K(\frac{\pi}{2}, k_n)}{\sqrt{(r_n + r)^2 + (z + z_n)^2}} - \frac{Q_n K(\frac{\pi}{2}, p_n)}{\sqrt{(r_n + r)^2 + (z - z_n)^2}}\right], \quad (24)$$

where r and z are cylindrical co-ordinates, $K(\pi/2, k)$ is complete elliptic integral of the first kind having modules

$$k_n^2 = \frac{4rr_n}{(r+r_n)^2 + (z+z_n)^2} \quad \text{and}$$
$$p_n^2 = \frac{4rr_n}{(r+r_n)^2 + (z-z_n)^2}. \tag{25}$$

Using boundary condition that the right biconical electrode potential is known and equal to U, the following linear equations system can be put,

$$U = \frac{1}{2\pi^2\varepsilon} \sum_{n=1}^{N} Q_n \left[\frac{K(\frac{\pi}{2}, k_{nm})}{\sqrt{(r_n+r_m)^2 + (z_m+z_n)^2 + a_e^2\delta_{nm}}} - \frac{K(\frac{\pi}{2}, p_{nm})}{\sqrt{(r_n+r_m)^2 + (z_m-z_n)^2}} \right], \quad m = 1, 2, \dots, N, \tag{26}$$

where

$$k_{nm}^2 = \frac{4r_nr_m}{(r_n+r_m)^2 + (z_n+z_m)^2 + a_e^2\delta_{nm}},$$
$$p_{nm}^2 = \frac{4r_nr_m}{(r_n+r_m)^2 + (z_n-z_m)^2} \tag{27}$$

and δ_{nm} is Kronecker delta symbol [10].

After solving this linear equation system and determining the unknown charges of the equivalent electrodes, the total charge of the right biconical electrode is

$$Q = \sum_{n=1}^{N} Q_n = 2CU, \tag{28}$$

where C is capacitance of biconical system.

The cylindrical components of the electric field are:

$$E_r = \frac{1}{4\pi^2\varepsilon r} \sum_{n=1}^{N} Q_n \left[\frac{K(\frac{\pi}{2}, k_n) - \frac{(z+z_n)^2 + r_n^2 - r^2}{(r-r_n)^2 + (z+z_n)^2} E(\frac{\pi}{2}, k_n)}{\sqrt{(r+r_n)^2 + (z+z_n)^2}} - \frac{K(\frac{\pi}{2}, p_n) - \frac{(z-z_n)^2 + r_n^2 - r^2}{(r-r_n)^2 + (z-z_n)^2} E(\frac{\pi}{2}, p_n)}{\sqrt{(r+r_n)^2 + (z-z_n)^2}} \right], \tag{29}$$

$$E_z = \frac{1}{2\pi^2\varepsilon}\sum_{n=1}^{N} Q_n \left[\frac{\dfrac{z+z_n}{(r-r_n)^2+(z+z_n)^2}}{\sqrt{(r+r_n)^2+(z+z_n)^2}} E(\frac{\pi}{2}, k_n) - \frac{\dfrac{z-z_n}{(r-r_n)^2+(z-z_n)^2}}{\sqrt{(r+r_n)^2+(z-z_n)^2}} E(\frac{\pi}{2}, p_n) \right] \tag{30}$$

and

$$E_\theta = 0, \tag{31}$$

where $E(\pi/2, k)$ is complete elliptic integral of the second kind.

On the system axis potential is

$$\varphi(0,z) = \frac{1}{4\pi\varepsilon}\sum_{n=1}^{N} Q_n \left[\frac{1}{\sqrt{r_n^2+(z+z_n)^2}} - \frac{1}{\sqrt{r_n^2+(z-z_n)^2}} \right], \tag{32}$$

and it can be expressed as the following series,

$$\varphi(r=0,z) = \sum_{m=0}^{\infty} \varphi_{2m+1} z^{2m+1}, \tag{33}$$

where

$$\varphi_{2m+1} = -\frac{P_{2m+1}(\cos\alpha)}{2\pi\varepsilon}\sum_{n=1}^{N}\frac{Q_n}{R_n^{2m+2}} \tag{34}$$

and

$$R_n^2 = z_n^2 + r_n^2. \tag{35}$$

On the system axis field components are

$$E_z(r=0,z) = \frac{1}{4\pi\varepsilon}\sum_{n=1}^{N} Q_n \left\{ \frac{z+z_n}{[r_n^2+(z+z_n)^2]^{3/2}} - \frac{z-z_n}{[r_n^2+(z-z_n)^2]^{3/2}} \right\} \tag{36}$$

and

$$E_r(r=0,z) = 0. \tag{37}$$

In the central system point only axial electric field component exists

$$E(0,0) = E_z(r=0, z=0) = \frac{1}{2\pi\varepsilon}\sum_{n=1}^{N} Q_n \frac{z_n}{(r_n^2+z_n^2)^{3/2}}. \tag{38}$$

7. Degenerative Primary Cells with Biconical Electrodes

For the potential to be first order in z , it is necessary that the coefficient of z^3 have to be equal to zero in Eq. (33),

$$\varphi_3 = 0. \tag{39}$$

Since Q_n is not zero, Eq. (34) requires that $P_3(\cos\alpha) = 0$.

Therefore, the angular opening for the primary cell of the first kind is $\alpha = \arctan\sqrt{2/3} \approx 39.2315°$.

In general case equating the coefficient of z^{2N+1} with zero, Eq. (33), the primary cell of the N-th kind is obtained. Table 2 presents the angular openings of primary cells of the N-th kind and of the different degree of degeneration for biconical electrodes (from 1^{st} to 5^{th}).

Table 2. Angular openings of primary cells of the N-th kind.

N	1	2	3	4	5
α_1	39.2315	25.0173	18.3579	14.4971	74.3629
α_2		57.4205	42.1380	33.2766	58.7284
α_3			66.0559	52.1663	43.1009
α_4				71.0796	27.4936
α_5					11.9776

For each degenerative primary cell with biconical electrodes, two more parameters have to be determined: the position and the length of each electrode. The solutions of the Eq. (6) define these parameters. In that case, approximately linear distribution of the potential is provided.

8. Primary Cell of the First Kind with Biconical Electrodes

So-called "optimal system" of the primary cell of the first kind with angular opening $\alpha \approx 39.2315°$ and length of electrode $H = 2h$ is modelled using present procedure. The coefficients of the terms of higher order than z^3 are negligible in this case.

An expression for the corresponding potential along the system axis for the primary cell of the first kind, Fig. 1(b), near the origin approximately is

$$\varphi(0,z) \approx -\frac{1}{2\pi\varepsilon}\sqrt{\frac{3}{5}}\left(\sum_{n=1}^{N}\frac{Q_n}{R_n^2}z - \frac{27}{50}\sum_{n=1}^{N}\frac{Q_n}{R_n^6}z^5\right), \tag{40}$$

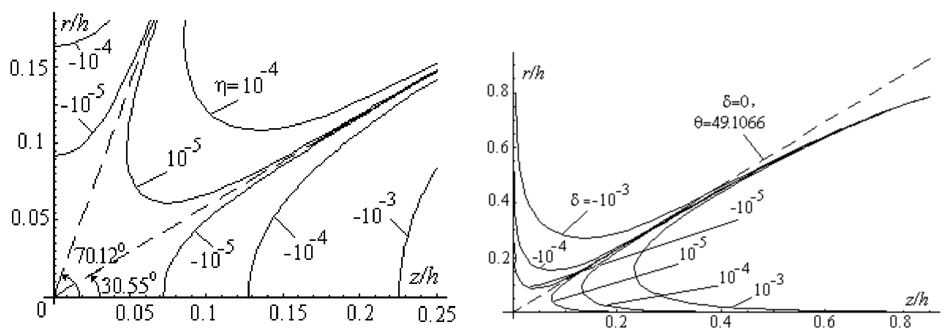
so that the electric field is approximately uniform and equal the field at the origin

$$E(0,z) \approx E(0,0) = \frac{1}{2\pi\varepsilon}\sqrt{\frac{3}{5}}\sum_{n=1}^{N}\frac{Q_n}{R_n^2}. \tag{41}$$

The deviation of the axial electric field η, is calculated by Eq. (21). Fig. 5(a) illustrates that for the region approximately $|z| \leq h/14$ the axial electric field is uniform and the deviation is less than 10^{-5} with respect to the electric field value in the origin and the deviation is less than 10^{-4} for the region approximately $|z| \leq h/8$. The deviation of the axial electric field is zero for biconical systems along the same directions as for toroidal systems. These directions have the polar angles θ, equal to $\theta = 30.5556°$ and $\theta = 70.1243°$ which is presented in Fig. 5(a).

The deviation of the radial electric field δ, defined by (22), is presented in Fig. 5(b). The radial electric field is zero along the axis, on the $z = 0$ plane, near the origin and along the direction when the polar angle is equal to $\theta = 49.1066°$, Fig. 5(b).

The general numerical program is realised using presented theoretical analysis and numerous calculations of the electric field distribution are made.



(a) (b)

Fig. 5. Deviations from the central field value.
(a) For the axial electric field η, near the origin.
(b) For the radial electric field, δ.

The ratio of axial electric field strength $E(0,z) = E_z(r = 0, z)$ and electric field strength in the origin, $E(0,0)$, on the system axis, $r = 0$ for different angular opening electrode α and for length of electrode $H = 2h$, is presented in Fig. 6(a).

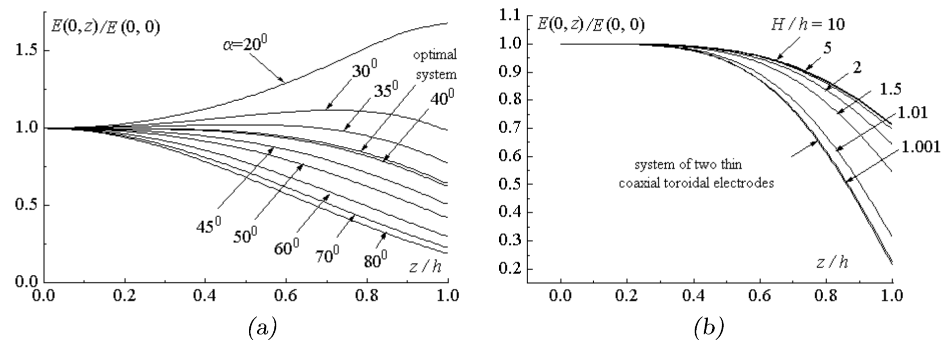


Fig. 6. The ratio $E(0,z)/E(0,0)$ on the system axis, $r=0$.
(a) For different angular opening electrode α, and for $H=2h$.
(b) For angular opening $\alpha = 39.2315°$ and different ratio H/h.

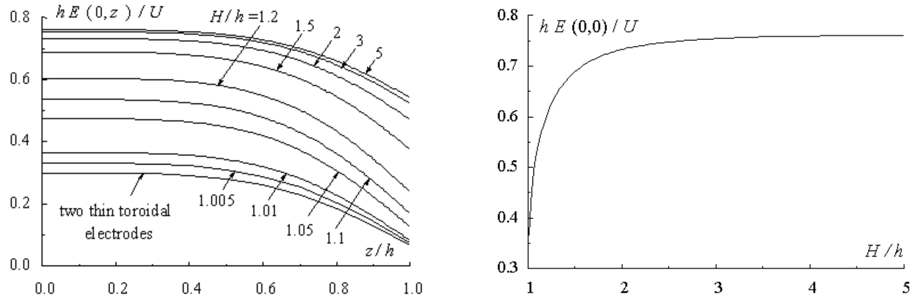
The ratio of axial electric field components and electric field strength in the central system point, $E(0,z)/E(0,0)$, on the system axis, for the primary cell of the first kind and for different ratio of the length and of the position of electrode, H/h, is presented in Fig. 6(b) and in Table 3. The axial electric field strength on the system axis, for the primary cell of the first kind for different ratio H/h, is presented in Fig. 7(a). Also, the comparisons between the results obtained for system having two thin toroidal electrodes and for the primary cell of the first kind for different ratio H/h are presented in Figs. 6(b) and 7(a). Better results are obtained with biconical electrodes.

Table 3. The ratio $E(0,z)/E(0,0)$ on the system axis $r=0$, for different H/h and z/h.

H/h z/h	2	3	4	5	6
0.00	1.000000	1.000000	1.000000	1.000000	1.000000
0.05	0.999997	0.999998	0.999998	0.999998	0.999999
0.10	0.999960	0.999965	0.999967	0.999968	0.999968
0.15	0.999796	0.999825	0.999832	0.999835	0.999837
0.20	0.999354	0.999444	0.999467	0.999477	0.999483
0.30	0.996701	0.997158	0.997277	0.997329	0.997361
0.40	0.989473	0.990930	0.991309	0.991474	0.991578
0.50	0.974106	0.977690	0.978621	0.979028	0.979281
0.60	0.946222	0.953675	0.955609	0.956450	0.956972
0.70	0.901256	0.914990	0.918542	0.920076	0.921020
0.80	0.835605	0.858623	0.864550	0.867077	0.868610

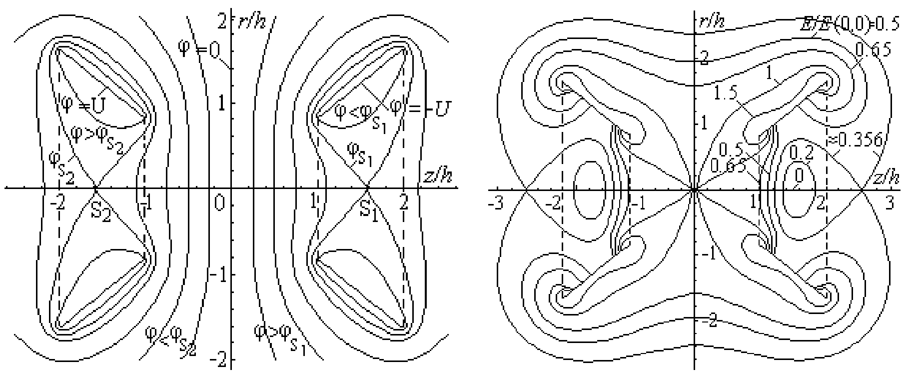
Fig. 7(b) shows the electric field strength in the central system point for the primary cell of the first kind for different ratio H/h.

It can be concluded that the ratio H/h does not influence the electric field homogeneity when $H/h \geq 2$.



(a) (b)

Fig. 7. Electric field strength for the primary cell of the first kind.
(a) On the system axis $r = 0$, for different ratio H/h and comparison with the system of two coaxial thin toroidal electrodes.
(b) In the origin for different ratio H/h.



(a) (b)

Fig. 8. Biconical primary cell of the first kind.
(a) The equipotential curves.
(b) The equienergetic curves.

The equipotential curves for optimal system are presented in Fig. 8(a). Two singular equipotential curves exist, for $\varphi = \varphi_{S_1} \simeq -0.802U$ and $\varphi = \varphi_{S_2} \simeq 0.802U$. The singular points S_1 and S_2 are on the system axis, for $|z| \simeq 1.6h$. As it is known, the electric field strength in the singular point is equal to zero.

The equienergetic curves, when electric field strength, Eq. (12), is constant, are presented in Fig. 8(b) for optimal system.

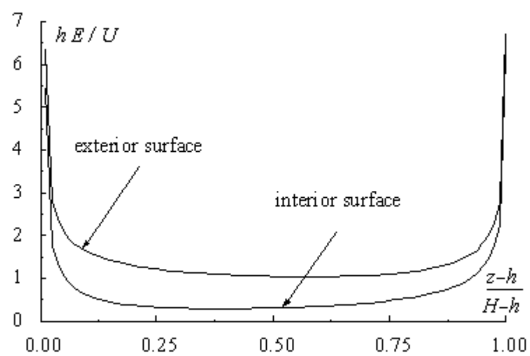


Fig. 9. Electric field strength on the electrode surface for optimal system for $\delta = 0$.

The electric field strength over the interior and exterior electrode surface, when the electrode tickness, $\delta = 0$, is neglected, is presented in Fig. 9.

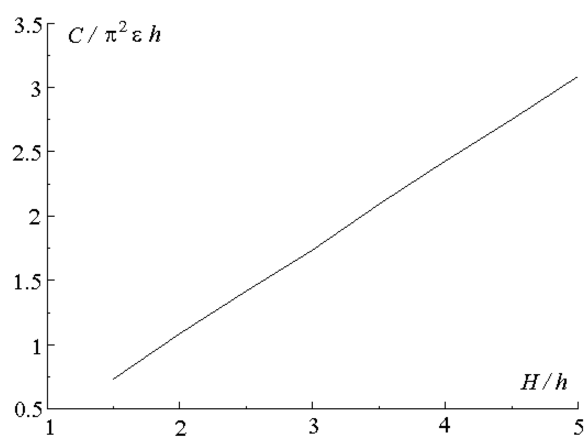


Fig. 10. Capacitance of optimal system for different ratio H/h.

The capacitance of optimal system for generating homogeneous electric field with biconical electrodes for different ratio H/h is presented in Fig. 10.

9. Conclusion

The authors propose one new very simple procedure for modelling electrostatic system for generating homogeneous electrostatic field. These systems are formed by thin coaxial toroidal electrodes and by hollow coaxial biconical electrodes, having opposite potentials.

Along the system axis, so-called axial potential can be expressed as series. Because this series is decreasing it is possible to choose primary cell dimension so that some series terms vanish and there is the domination of only linear term, which provides homogeneous electric field in central system area. Thus, the primary cell of N-th kind, which has N different degrees of degeneration, is determined. Better results are obtained with primary cell of higher kind; the primary cell of the second kind provides the larger region with homogeneous electric field than the primary cell of the first kind. Also, better results are obtained for primary cell of the first kind with biconical electrode system then with system having two thin toroidal electrodes.

Modelling systems with biconical electrodes is defined so-called "optimal system" for the primary cell of the first kind, Fig. 1(b), when electrode angular opening is $\alpha \simeq 39.2315°$ and with length of electrode is $H = 2h$. In this case practically homogeneous electric field exists in the central system region. The obtained results show that the electric field homogeneity is practically independent on the electrode length if it is $H \geq 2h$, Fig. 1(b).

The biconical electrode thickness, if they are thin enough practically do not make influence to the realised homogeneity. The electrode thickness influences to the electric field strength on the electrode surfaces is investigated in [6].

These electrodes systems are very useful in practice for getting systems for obtaining homogeneous electric field.

When the electrodes are grounded and placed in external electric field, then they are used for space protection against the overflow electric fields strength.

The potential and electric field strength is approximately determined using new numerical Equivalent electrode method.

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