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QPSK PERFORMANCE WITH IMPERFECT CARRIER PHASE RECOVERY IN THE PRESENCE OF ATMOSPHERIC NOISE AND INTERFERENCE

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Abstract: The purpose of this paper is to provide the theoretical approach for determining a coherent quaternary phase-shift-keying performance by means of the system error probability at very low frequencies. An expression for the bit error probability is determined when the signal, thermal Gaussian noise, atmospheric noise, interference and imperfect carrier phase recovery are taken into consideration. Phase locked loop, as the constituent part of the receiver, is used in providing the synchronization reference signal extraction, which is assumed to be imperfect in this paper. The reference carrier is extracted by the non-linear first order PLL model with primary emphasis on the degradation in the system performance produced by imperfect carrier signal extraction.

Key words: QPSK, phase locked loop, atmospheric noise, interference

1. Introduction

The performance evaluation of binary and M–ary ($M > 2$) phase–shift–keying communication systems has been analyzed in a great variety of papers, which have appeared in the literature [1]-[7]. Quaternary phase-shift-keying (QPSK or 4-PSK) systems have the greatest practical interest of all non-binary (multi-position) systems of digital transmission of messages by phase modulated signals. Currently, QPSK is one of the prevalent modulations in use for digital communication systems (since bandwidth efficiency) [1], [2]. System with four–phase PM provides the possibility to double the transmission rate in the same frequencies band in comparison with the binary PM systems. However, even more important, QPSK can provide the

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same transmission rate in a twice narrower band without losses or with insignificant losses of noise immunity. The only significant penalty factor is an increased sensitivity to carrier phase synchronization error [3].

Any successful transmission of information through a digital phase-coherent communication system requires a receiver capable of determining or estimating the phase and the frequency of the received signal with as few errors as possible; any noise associated with carrier leads to degradation of the detection performance of the system. In practice, quite often the phase locked loop (PLL) is used in providing the desired reference signal [4]-[7]. Frequently, a PLL system must operate in such conditions where the external fluctuations due to the additive noise are so intense that classical linear PLL theory neither characterizes adequately the loop performance, nor explain the loop behavior [8]. The direct linearization cannot be used in loop performance explanation and characterization in the region of the operation in many practical situations. So, the analytical approach in developing an exact non–linear theory of PLLs, based on Fokker-Planck theory was investigated in [8], [9]. Numerical results for QPSK system are presented so that these results combined with the characteristic of the phase recovery circuit will enable the best practical design of a QPSK system.

The performance of QPSK and other systems at very low frequencies is strongly influenced by the non–Gaussian nature of atmospheric noise. Atmospheric noise model, used in this paper, agrees well with actual noise statistics. It should be noted that this noise is observed through the passband of some receiver filter. If the receiver is sufficiently narrow band, the noise at the receiver output can be reasonably assumed to be modeled well as a Gaussian process. This follows from the fact that the narrow band filtered noise is a sum of contributions from many independent lightening discharges, none of which is dominant at the filter output. The goal of this atmospheric noise model is the formulation of an analytical model that is reasonably descriptive of the received noise and suitable for application to the calculation of very low frequencies communication system performance.

Measured data on atmospheric noise indicate that atmospheric noise has a Gaussian behavior at low amplitudes and an approximately log–normally distributed envelope for large amplitudes. Namely, measured atmospheric noise usually consists of the effect of many lightening discharges around the world [10]. When no single discharge dominates at any instant of time, then, applying the central limit theorem, a Gaussian behavior should be expected. On the other hand, when a particular individual discharge dominates, the measured amplitude should have the statistical characteristics of the individual discharge, which is essentially log-normal in character. Since the larger amplitudes have the greater influence on the performance of any communication system, log–normal characteristic of atmospheric noise is concerned [11].

The error probability, as a measure of systems quality, is an important issue and has received much attention in the literature. Noise influence and interference are often fundamental limiting factors in digital transmission systems. An expression for the bit error probability was calculated when the signal and Gaussian noise are applied at the input of the QPSK system [12]. The bit error probability of the QPSK system when the additive thermal Gaussian noise, atmospheric noise, interference and imperfect carrier phase recovery are considered as a source of degradation, are determined in this paper.

2. System Feature

Let the input at QPSK receiver consist of the signal, atmospheric noise, interference and noise

$$r(t) = A\cos(\omega_0 t) + a(t) + i(t) + m(t), \tag{1}$$

where A is a signal amplitude, ω_0 is a constant carrier frequency, $a(t)$ is an interference caused by atmospheric noise, $i(t)$ is the interference and $m(t)$ is a thermal Gaussian noise. Atmospheric noise model is represented as a narrow band process with a log–normal envelope with the form [11]

$$a(t) = A_1 e^{n(t)} \cos[\omega_0 t + \theta_1(t)], \tag{2}$$

where A_1 is a noise amplitude, $n(t)$ is a real stationary Gaussian process with zero mean, and $\theta_1(t)$ is an uniformly distributed phase with the probability density function

$$p(\theta_1) = \frac{1}{2\pi}, \qquad \{-\pi \leq \theta_1 \leq \pi\}. \tag{3}$$

Interference is represented as follows

$$i(t) = A_2 \cos[\omega_0 t + \theta_2(t)], \tag{4}$$

where A_2 is an interference amplitude, and $\theta_2(t)$ is, also, an uniformly distributed phase with the same probability density function as in the Eq.(3).

Input signal can be also written with the form

$$r(t) = AR\cos(\omega_0 t + \Psi) + m(t) \tag{5}$$

where

$$R = \sqrt{1 + e^{2n}\eta_1^2 + \eta_2^2 + 2e^n\eta_1\cos\theta_1 + 2\eta_2\cos\theta_2 + 2e^n\eta_1\eta_2\cos(\theta_1 - \theta_2)},$$
$$\Psi = \arctan\frac{\eta_1 e^n\sin\theta_1 + \eta_2\sin\theta_2}{1 + \eta_1 e^n\cos\theta_1 + \eta_2\cos\theta_2},$$
$$\eta_1 = \frac{A_1}{A},$$
$$\eta_2 = \frac{A_2}{A}$$

where η_1 and η_2 are noise to signal ratio and the interference to signal ratio respectively.

From now on, additive thermal Gaussian noise, atmospheric noise, interference and imperfect phase carrier recovery, are taken into account. All other functions are considered ideal. The block diagram of a QPSK receiver would be adopted is shown in Figure 1. The recovered carrier signal is assumed to be in the form of the sin wave. Also, it would be adopted that an original message is in binary form and that the primary goal is in determining the bit error rate.

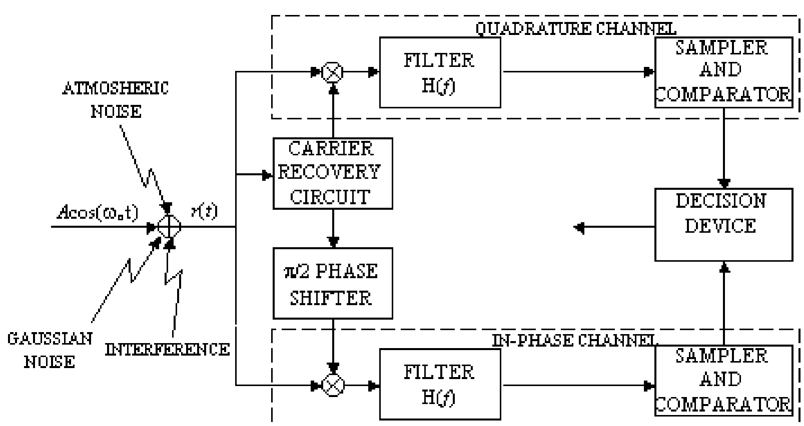

Fig. 1. Block diagram of a QPSK receiver.

3. System Performance

Under the assumption of a constant phase in the symbol interval, the conditional error probability for the given phase error ϕ (the phase error ϕ is the difference between the receiver incoming signal phase and the voltage controlled oscillator output signal phase) can be written as (Appendix) [12]

$$P_{e/\phi}(\phi) = \frac{1}{4}\left\{\operatorname{erfc}\left[\sqrt{2R_b}\cos\left(\frac{\pi}{4}+\phi\right)\right] + \operatorname{erfc}\left[\sqrt{2R_b}\cos\left(\frac{\pi}{4}-\phi\right)\right]\right\}, \quad (6)$$

where the function $\operatorname{erfc}(x)$ is the well known complementary error function defined as

$$\operatorname{erfc}(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{x^2}{2}}\,dx. \quad (7)$$

The received signal to noise spectral density ratio in the data channel (demodulator), denoted by R_b, is given by

$$R_b = \frac{E}{N_0},$$

where E is the signal energy per bit duration T. N_0 represents the normalized noise power spectral density in W/Hz, referenced to the input stage of the demodulator, since the signal to noise ratio is established at that point. The signal detection in receiver is accomplished by cross–correlation and sampling operation. The effect of filtering due to $H(f)$ in Figure 1. is not considered here.

The conditional steady state probability density function, for the non–linear PLL model with a known signal and noise at the PLL input, of modulo 2π reduced phase error is given by the following approximation [8]

$$p(\phi/\theta_1,\theta_2,n) = \frac{e^{\beta\phi+\alpha\cos\phi}}{4\pi^2 e^{-\pi\beta}\mid I_{j\beta}(\alpha)\mid^2}\int_\phi^{\phi+2\pi} e^{-\beta x+\alpha\cos x}dx, \quad (8)$$

where $I_{j\beta}(\alpha)$ is the modified Bessel function of complex order $j\beta$ and real argument α. The range of definition for ϕ in the previous equation is any interval of width 2π centered about any lock point $2n\pi$, with n an arbitrary integer. The parameters α and β, that characterize Eq.(8), for the first order non–linear PLL model in this case are

$$\begin{aligned} \alpha &= \alpha_0 R, \\ \beta &= \beta_0 \Omega \end{aligned} \tag{9}$$

where α_0 and β_0 are constants [8], [13], and $R = R\{f(\theta_1, \theta_2, n)\}$. The parameter α is a measure of the loop signal to noise ratio in the sense that the larger the value of α, the smaller are the deleterious effects due to noise reference signal. The parameter β is a measure of the loop stress. Ω is the loop detuning, i.e. the frequency offset of the first term in Eq.(5) defined by

$$\begin{aligned} \Omega &= \frac{d}{dt}(\omega_0 t + \Psi) - \omega_0 \\ &= \frac{\eta(\eta + \cos\theta)}{R^2}\frac{d\theta}{dt}. \end{aligned} \tag{10}$$

Since $(d\theta/dt) = 0$, it follows $\Omega = 0$, i.e. $\beta = 0$. Therefore, the average steady–state probability density function of the phase error is

$$\begin{aligned} p(\phi) &= \int\limits_{\theta_1}\int\limits_{\theta_2}\int\limits_{n} p(\phi/\theta_1, \theta_2, n) p(\theta_1) p(\theta_2) p(n) d\theta_1 d\theta_2 dn \\ &= \frac{1}{8\pi^3}\int\limits_{-\pi}^{\pi}\int\limits_{-\pi}^{\pi}\int\limits_{-\infty}^{\infty} \frac{e^{\alpha_0 R\cos\phi}}{I_0(\alpha_0 R)}\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{n^2}{2\sigma^2}} d\theta_1 d\theta_2 dn. \end{aligned} \tag{11}$$

Substituting Eq.(5) into Eq.(11) yields the probability density function of the phase error that is shown in Figure 2. The values of parameters α_0 and η_1, η_2 are given in Figure. During the all numerical calculations, noise variance had a constant value ($\sigma = 1$).

The total error probability is determined by averaging the conditional error probability over random variables, θ_1, θ_2, n and ϕ

$$P_e = \int\limits_{\theta_1}\int\limits_{\theta_2}\int\limits_{n}\int\limits_{\phi} P_{e/\phi} p(\phi/\theta_1, \theta_2, n) p(\theta_1) p(\theta_2) p(n) d\theta_1 d\theta_2 dn d\phi. \tag{12}$$

Substituting $R_b = R_1 R^2$ in Eq.(6), the average error probability becomes

$$\begin{aligned} P_e = &\frac{1}{32\pi^3}\int\limits_{-\pi}^{\pi}\int\limits_{-\pi}^{\pi}\int\limits_{-\infty}^{\infty}\int\limits_{-\pi}^{\pi}\Big\{\mathrm{erfc}\left[R\sqrt{R_1}(\cos\phi - \sin\phi)\right] \\ &+\mathrm{erfc}\left[R\sqrt{R_1}(\cos\phi + \sin\phi)\right]\Big\}\frac{e^{\alpha_0 R\cos\phi}}{I_0(\alpha_0 R}\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{n^2}{2\sigma^2}} d\theta_1 d\theta_2 dn d\phi \end{aligned} \tag{13}$$

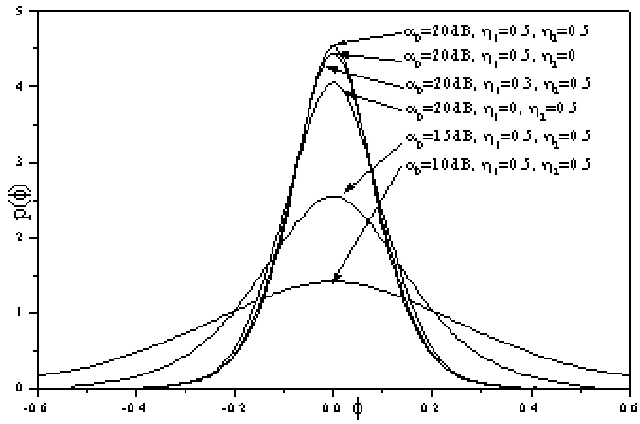

Fig. 2. Probability density function of the phase for the non–linear first order PLL model.

The total error probability is computed on the basis of the Eq.(13) and is plotted versus the signal to noise ratio at the demodulator input in Figures 3(a), 3(b) and 3(c) for various values of α_0, η_1 and η_2. The values of parameters are given in figures.

4. Numerical Results Analysis

The average error probability as a function of the signal to noise ratio in the demodulator input for constant values of parameter η_1 and η_2 and the various values of the parameter α_0 is plotted in Figure 3(a). The curve with $\alpha_0 = 20$ dB, $\eta_1 = 0.5$ and $\eta_2 = 0.5$ is split in two regions (denoted by AB and BC) for the explanation of signal to noise influence (α_0) on the error probability variations. In the AB region, the error probability decreases sharply with the parameter R_1 increasing. For example, if the parameter R_1 is changed from 10 dB to 25 dB, the error probability decreases 47.3 times. In the BC region the error probability variations with the R_1 increasing are fewer than in the previous case, i.e. if the parameter R_1 is changed from 29dB to 33dB, the error probability decreases only 1.385 times. In this region, the parameter R_1 is relatively large, and in comparison with the value of the parameter α_0, its influence on the error probability decreases. The error probability for great values of R_1 tends to constant value (BER floor). This BER floor can be reduced by increasing the parameter α_0 and decreasing the parameter η. Also, from the Figure 3(a) follows that the BER floor value for $\alpha_0 = 10$dB is greater 67.7 times than the BER floor value for $\alpha_0 = 20$ dB.

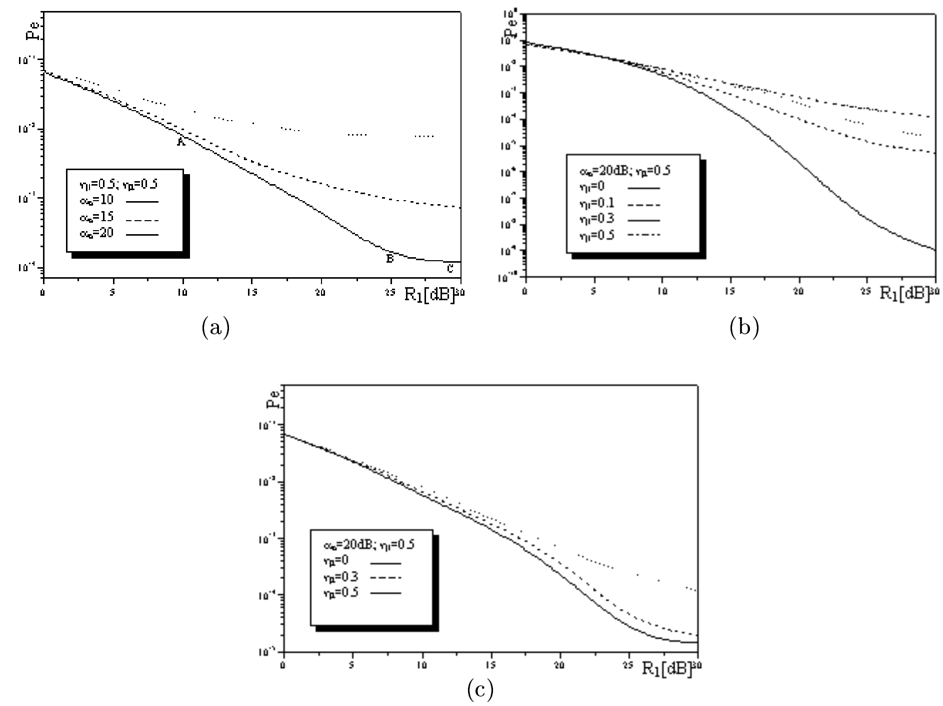

Fig. 3. Average error probability performance of a QPSK coherent detector with a noisy carrier synchronization reference signal when α_0 is a parameter, while η_1 and η_2 are constants (a), when η_1 is a parameter, while α and η_2 are constants (b), and when η_2 is a parameter, while α and η_1 are constants (c).

The atmospheric noise (η_1) influence on the error probability is evident from Figure 3(b). The following observation is significant. If the parameter η_1 is increased from 0.0 to 0.1 the BER floor value increases 5.25×10^3 times. But, if the parameter η_1 is increased from 0.1 to 0.5 the BER floor value increases only 20 times. It can be seen that the atmospheric noise has a significant influence on the system performance.

Observing the influence of atmospheric noise and interference, Figure 3(b) and 3(c), as the main point of this paper, it can be concluded that the atmospheric noise has greater influences than the interference.

5. Conclusion

The quaternary PSK system is analyzed by means of the system error probability at very low frequencies, in this paper. Bit error probability is determined when the signal, thermal noise, atmospheric noise, interference and imperfect carrier phase recovery are taken into consideration. The influence of the imperfect reference signal extraction is expressed by the probability density function of the PLL phase error.

The detailed analysis of the obtained numerical results is performed in this paper. The influence of the atmospheric noise, η_1, interference, η_2, as well as the influence of the parameter α_0, on the system error probability, are especially considered. One can conclude that the system error probability decreases with the increase of both, PLL signal to noise ratio α_0 and signal to noise ratio R_1, and with the decrease of both, the atmospheric noise and interference (η_1 and η_2.)

However, from all figures, the large signal to noise ratio system error tends to a constant value (BER floor). In the BER floor area, the signal to noise ratio is relatively large with respect to all parameters, α_0, η_1 and η_2, and has therefore a small influence on the system error probability. It is seen from figures that this BER floor can be reduced by increasing the parameter α_0 which depends on the applied PLL loop and by decreasing η_1 and η_2. On the basis of the shown analysis it is possible to determine the QPSK system parameter α_0 and useful signal power necessary to compensate the imperfect carrier extraction. This means that the presented conclusions can be useful in practice for the QPSK system design.

Appendix
Bit Error Probability for Gray Code

The bit error rate of a QPSK is related to the symbol error rate through the coding scheme relating to the quaternary symbols to the binary message. Gray's code is used in assigning pairs of bits in the original message to the transmitted phase levels. It must be noted that Gray mapping of the source onto the signal vectors (Figure A.) ensures that pairs of bits assigned to adjacent phases differ only in one of two positions. In order to derive the expression for the error probability it is assumed that the symbol 00 is transmitted. It is assumed that the first quadrant is the decision threshold area for symbol 00, the second quadrant is the decision threshold area for the symbol 01, and the third quadrant is the decision threshold area for 10. p_1 is the probability that the received phasor lies in the second quadrant, p_2 is the probability that lies in the third quadrant and p_3 is the probability that lies in the fourth quadrant.

The expected number of bit errors in a pair is $p_1 + 2p_2 + p_3$, or the bit error rate Pe of QPSK can be expressed as

$$\begin{aligned} P_e &= 0.5(p_1 + 2p_2 + p_3) \\ &= 0.5[(p_1 + p_2) + (p_2 + p_3)]. \end{aligned} \tag{A1}$$

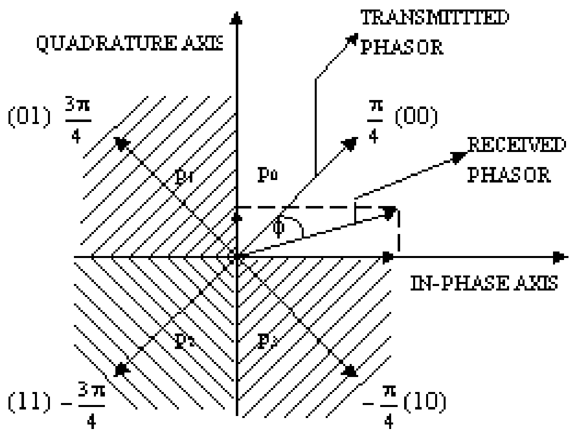

Fig. A. Detection of a quaternary PSK signal when the transmitted bit pair is 00.

Since $p_1 + p_2$ is the probability of detecting a transmitted bit pairs 00 as 01 or 11, from Figure A it is evident that

$$\begin{aligned} p_1 + p_2 &= \text{Probability [in-phase component of (signal+noise)} < 0] \\ &= \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{\left[x - \cos(\frac{\pi}{4} - \phi)\right]^2}{2\sigma^2} \right\} dx \\ &= 0.5\text{erfc}\left\{ \rho \cos\left[\frac{\pi}{4} - \phi\right] \right\}. \end{aligned} \tag{A2}$$

Similarly,

$$\begin{aligned} p_1 + p_2 &= \text{Probability [quadrature component of (signal+noise)} < 0] \\ &= \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{\left[x - \sin(\frac{\pi}{4} - \phi)\right]^2}{2\sigma^2} \right\} dx \\ &= 0.5\text{erfc}\left\{ \rho \cos\left[\frac{\pi}{4} + \phi\right] \right\}. \end{aligned} \tag{A3}$$

From the previous two equations it follows that the bit error probability for QPSK

$$P_e = 0.25\left\{\mathrm{erfc}[\rho\cos(\frac{\pi}{4}+\phi)] + \mathrm{erfc}[\rho\cos(\frac{\pi}{4}-\phi)]\right\}, \qquad (A4)$$

where $\rho = \sqrt{2R_b}$ is the signal to noise ratio per bit.

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