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ITERATIVE MULTIUSER DETECTION

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Abstract: In this paper, we describe iterative multiuser detection/decoding scheme, which was recently proposed in [8]. The multiuser detector and single–user turbo decoders are coupled such that after each turbo decoding iteration the extrinsic information of the interfering users is passed to the multiuser detector, and after each multiuser iteration, updated *a posteriori* probabilities are passed to the single–user turbo decoders as the soft input metrics. The impact of imperfect channel state information is taken into account. Simulation results indicate that amplitude estimation error of 1 dB results in performance degradation of 0.4 dB.

Key words: Multiuser detection, turbo codes, CDMA system.

1. Introduction

Recently, iterative multiuser techniques have attracted attention, following the discovery of powerful turbo codes [1]. Most of the work is related to convolutionally coded code–division multiple access (CDMA) systems. It was first pointed out by Hagenauer [2] that the CDMA channel can be viewed as a code. He suggested a suboptimal iterative decoding scheme, where the soft output multiuser receiver is based on interference cancellation. Similar iterative approach is considered in [5], while in [6] the soft output multiuser receiver is accomplished via interference cancellation and linear minimum mean squared error (MMSE) filtering. The optimal iterative multiuser detector based on the iterative techniques for cross–entropy minimization for synchronous multiuser systems with convolutional codes is derived in [3], and a suboptimum implementation with exponential complexity in the number of users is also presented. A similar work is reported

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in [4], where an iterative multiuser receiver is derived, and an approximate solution to avoid exponential complexity in the number of users is suggested. It was demonstrated in [3] that a combination of convolutional codes and random interleavers in a multiuser channel can achieve results close to the single–user performance, and in the case of high signal–to–noise ratios closely approach the multiuser capacity limit. The main drawback of that scheme is a relatively poor single–user performance of a convolutional code at low signal–to–noise ratio region. Namely, although the iterative multiuser detector in combination with convolutional decoding approaches the single–user convolutional code performance, the results may still be far from the multiuser capacity limit. It is reasonable to believe that this limitation can be mitigated with more powerful turbo codes. So far, there has not been much work in iterative multiuser detection for turbo coded CDMA systems. An iterative multiuser receiver for a turbo coded multiuser system using parallel concatenated convolutional codes (PCCC) is considered only in [7]. The authors take the same approach as in [4]. Multiuser detector (MUD) generates soft metrics and feed them into the bank of single–user turbo decoders. Turbo decoding of PCCC is done in a serial fashion [1], whereby maximum *a posteriori* probability (MAP) decoders [9] are activated one after the other. After several iterations, single–user turbo decoding is stopped, and *a posteriori* probabilities are used as *a priori* ones in the next multiuser iteration. The procedure is repeated iteratively. Another, iterative multiuser detector/decoder is proposed in [8]. Multiuser detection and single–user turbo decoding are tightly coupled to maximize the overall gain. Multiuser iterations are interlaced with single–user turbo decoding iterations and extrinsic instead of *a posteriori* probabilities are used as *a priori* ones in the next multiuser iteration.

In this paper, we describe iterative multiuser detection/decoding schemes proposed in [8] and investigate the impact of imperfect channel state information on the performance. In Section 2, the system model is presented. In Section 3 we describe the iterative receiver proposed in [8]. Section 4 shows numerical results while Section 5 contains conclusions.

2 System Model

The system model is shown in Fig. 1. We consider a PCCC coded synchronous baseband multiuser system with K users, employing unit energy time limited spreading waveforms $s_k(t) \in \mathcal{R}(0,T)$, $k = 1, 2, \cdots, K$. A block of message bits, $\boldsymbol{u}_k$, is encoded by a constituent encoder 1 to create a parity sequence, $\boldsymbol{x}_{k,1}$, and interleaved and encoded by a constituent encoder 2 to

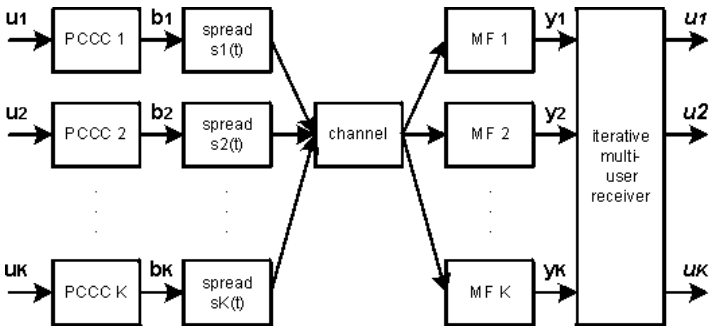

Fig. 1. A PCCC coded CDMA system with iterative multiuser receiver.

create the second parity sequence, $\boldsymbol{x}_{k,2}$. The systematic bits, $\boldsymbol{u}_k$, and the parity bits, $\boldsymbol{x}_{k,1}$ and $\boldsymbol{x}_{k,2}$, are then multiplexed to create a coded output sequence $\boldsymbol{b}_k = [\boldsymbol{u}_k, \boldsymbol{x}_{k,1}, \boldsymbol{x}_{k,2}]^T$, resulting in the rate 1/3 code. The coded bits at the output of the PCCC encoder are modulated by the spreading waveform $s_k(t)$ and then transmitted over a multiuser channel. The received signal can be written as

$$r(t) = \sum_{k=1}^{K} \sum_{i=0}^{N-1} A_k b_k(i) s_k(t - iT) + n(t), \tag{1}$$

where A_k is the amplitude of the k–th user, $b_k(i) \in \{+1, -1\}$ is the value of the k–th user's i–th coded bit, T is the coded bit duration, $n(t)$ is additive white Gaussian noise with two–sided power spectral density $N_0/2$ and N is the coded packet length. The sufficient statistic for demodulation of K coded bits in the i–th interval is given by the vector $\boldsymbol{y}(i) = [y_1(i), \cdots, y_K(i)]$, whose k–th component is the output of a filter matched to $s_k(t)$

$$y_k(i) = \int_{iT}^{(i+1)T} s_k(t - iT) r(t) dt, \; k = 1, 2, \cdots, K. \tag{2}$$

The vector of sufficient statistic can be written as

$$\boldsymbol{y}(i) = \boldsymbol{R}\boldsymbol{A}\boldsymbol{b}(i) + \boldsymbol{n}(i), \tag{3}$$

where $\boldsymbol{R}$ denotes the normalized cross–correlation matrix of spreading waveforms, $\boldsymbol{R}_{k,l} = \rho_{kl} \triangleq \int_0^T s_k(t) s_l(t) dt$, $\boldsymbol{A} \triangleq \mathrm{diag}(A_1, \cdots, A_K)$, $\boldsymbol{b}(i) = [b_1(i), \cdots, b_K(i)]^T$ and $\boldsymbol{n}(i) \sim \mathcal{N}(0, \frac{N_0}{2}\boldsymbol{R})$ is a Gaussian noise vector.

3. Iterative Receiver

The principal block diagram of the iterative receiver proposed in [8], on a two user example, is presented in Fig. 2. The iterative procedure can be summarized as follows

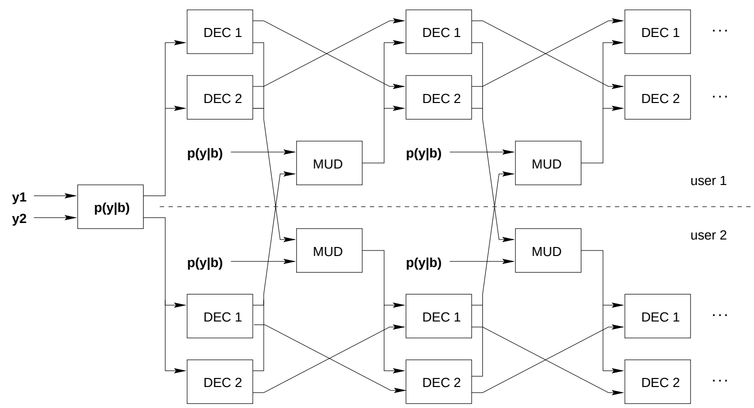

Fig. 2. Principal block diagram of the iterative multiuser for two user case, as proposed in [8]; $\boldsymbol{y}_k$ *is the matched filter output of user* k.

1. The MAP multiuser detector, based on the matched filter outputs and *a priori* information about the coded bits, provides the single–user turbo decoder with *a posteriori* probabilities of the coded bits.
2. Upon reception of the marginal coded bit probabilities and after one parallel single–user turbo decoding iteration, the interfering users supply the multiuser detector with the corresponding *a priori* information for the coded bits, which is equal to the extrinsic information obtained by the turbo decoding iteration.
3. Steps 1 and 2 are repeated desired number of times.

A more detailed description of the involved processing is provided below.

3.1 Multiuser detection – first iteration

In the first iteration, multiuser detection is activated disjointly of single–user turbo decoding. The purpose of multiuser detection is to provide the single–user turbo decoders with the soft input metrics. In the i–th interval, at the beginning of the iterative process, the MUD computes the likelihood vector $p(\boldsymbol{y}(i)|\boldsymbol{b}(i))$, which is a multivariate conditional Gaussian distribution. $p(\boldsymbol{y}(i)|\boldsymbol{b}(i))$ is computed only once, and it is used in subsequent multiuser iterations.

The marginal likelihood $p(\boldsymbol{y}(i)|b_k(i))$ is given by

$$p(\boldsymbol{y}(i)|b_k(i)) = \frac{\sum_{\substack{all\, b_j(i) \\ j\neq k}} p(\boldsymbol{y}(i)|\boldsymbol{b}(i)) \prod_{j=1}^{K} p(b_j(i))}{p(b_k(i))}, \tag{4}$$

where we assume statistical independence among the coded bits $b_j(i)$, $j = 1, \cdots, K$. The soft input metric for the single–user turbo decoder of user k is *a posteriori* distribution

$$p_{1,k}(b_k(i)) = \alpha p(\boldsymbol{y}(i)|b_k(i)), \tag{5}$$

where α is a constant that normalizes the probability mass. In the first iteration *a priori* probabilities for coded bits are all equal, *i.e.*, $p(b_j(i) = \pm 1) = 1/2$.

3.2 Single user soft output turbo decoder

The decoders of the constituent codes are as suggested in [10] activated simultaneously in order to avoid bias towards any of them. After parallel MAP decoding of both constituent codes (denoted as decoder 1 and 2) the outputs of both decoders contain the extrinsic probabilities for the systematic bits $\boldsymbol{u}_k$. *A posteriori* distribution of the l–th systematic bit after the n–th single–user turbo decoding iteration, $p^d_{n,k}(u_k(l))$ is given by [10]

$$p^d_{n,k}(u_k(l)) = \alpha p_{n,k}(u_k(l)) p^{e,1}_{n,k}(u_k(l)) p^{e,2}_{n,k}(u_k(l)), \tag{6}$$

where $p_{n,k}(u_k(l))$ is the input distribution for the systematic bit $u_k(l)$, generated by the multiuser detector in the n–th iteration, and $p^{e,1}_{n,k}(u_k(l))$ and $p^{e,2}_{n,k}(u_k(l))$ are the extrinsic distributions from decoders 1 and 2, respectively. For a rate $R = 1/\kappa$ turbo code, there are κ coded bits, $b_k(i)$, for every uncoded information bit, $u_k(l)$, so that $l = \lfloor i/3 \rfloor$, where $\lfloor x \rfloor$ denotes the largest integer not greater than x. In the following single–user turbo decoding iteration, *priori* distribution at the input of decoder 1 is set to

$$p^{a,1}_{n+1,k}(u_k(l)) = p^{e,2}_{n,k}(u_k(l)), \tag{7}$$

while *a priori* distribution at the input of decoder 2 is set to

$$p^{a,2}_{n+1,k}(u_k(l)) = p^{e,1}_{n,k}(u_k(l)). \tag{8}$$

A priori probabilities are initialized as

$$p^{a,1}_{1,k}(u_k(l) = \pm 1) = p^{a,2}_{1,k}(u_k(l) = \pm 1) = \frac{1}{2}, \tag{9}$$

while at the end of turbo decoding process, the decision on $u_k(l)$ is based on

$$u_k(l) = \arg \max_{u_k(l)=\pm 1} (p^{d}_{n,k}(u_k(l))). \tag{10}$$

Denote with $p^{d,1}_{n,k}(u_k(l))$ *a posteriori* distribution at the output of decoder 1 and with $p^{d,2}_{n,k}(u_k(l))$ *a posteriori* distribution at the output of decoder 2. The extrinsic distributions $p^{e,1}_{n,k}(u_k(l))$ and $p^{e,2}_{n,k}(u_k(l))$ can be factored out from $p^{d,1}_{n,k}(u_k(l))$ and $p^{d,2}_{n,k}(u_k(l))$, respectively, since

$$p^{d,1}_{n,k}(u_k(l)) = \alpha p_{n,k}(u_k(l)) p^{a,1}_{n,k}(u_k(l)) p^{e,1}_{n,k}(u_k(l)), \tag{11}$$

$$p^{d,2}_{n,k}(u_k(l)) = \alpha p_{n,k}(u_k(l)) p^{a,2}_{n,k}(u_k(l)) p^{e,2}_{n,k}(u_k(l)). \tag{12}$$

The goal is to obtain the extrinsic information output which can be used as *a priori* information for $\boldsymbol{b}_k$ in the next multiuser iteration. In this case it is necessary to modify the MAP decoders to produce not only *a posteriori* probabilities of the systematic bits, but also of the parity bits. After this straightforward modification, in the n–th iteration, decoder 1 produces *a posteriori* probabilities for the systematic and parity bits of code 1, denoted as $p^{d,1}_{n,k}(\boldsymbol{u}_k)$ and $p^{d,1}_{n,k}(\boldsymbol{x}_{k,1})$, respectively. Similarly, decoder 2 produces *a posteriori* probabilities for the systematic and parity bits of code 2, denoted as $p^{d,2}_{n,k}(\boldsymbol{u}_k)$ and $p^{d,2}_{n,k}(\boldsymbol{x}_{k,2})$, respectively. With $p^{E}_{n,k}(\boldsymbol{b}_k) = [p^{E}_{n,k}(\boldsymbol{u}_k), p^{E}_{n,k}(\boldsymbol{x}_{k,1}), p^{E}_{n,k}(\boldsymbol{x}_{k,2})]^T$ we label the extrinsic distribution for the coded bits obtained by the n–th single–user turbo decoding iteration and therefore suitable to serve as *a priori* distribution in the $(n+1)$–th multiuser detection iteration. Extracting $p^{E}_{n,k}(\boldsymbol{b}_k)$ for the parity bits is straightforward, since the extrinsic distributions, $p^{E}_{n,k}(x_{k,1}(l))$ and $p^{E}_{n,k}(x_{k,2}(l))$, are simply given by

$$p^{E}_{n,k}(x_{k,1}(l)) = \alpha \frac{p^{d,1}_{n,k}(x_{k,1}(l))}{p_{n,k}(x_{k,1}(l))}, \tag{13}$$

$$p^{E}_{n,k}(x_{k,2}(l)) = \alpha \frac{p^{d,2}_{n,k}(x_{k,2}(l))}{p_{n,k}(x_{k,2}(l))}, \tag{14}$$

where $p_{n,k}(x_{k,1}(l))$ and $p_{n,k}(x_{k,2}(l))$ are the input distributions for $x_{k,1}(l)$ and $x_{k,2}(l)$, respectively. Computing the extrinsic distribution for the systematic bits is complicated by the fact that both decoders produce *a posteriori* distribution for $u_k(l)$. Therefore, we need to extract the extrinsic distribution from both decoders and then compute the combined extrinsic distribution. The extrinsic distribution obtained after decoding the code 1 can be found as

$$p_{n,k}^{E,1}(u_k(l)) = \alpha \frac{p_{n,k}^{d,1}(u_k(l))}{p_{n,k}(u_k(l))}, \tag{15}$$

while the extrinsic distribution obtained after decoding the code 2 is

$$p_{n,k}^{E,2}(u_k(l)) = \alpha \frac{p_{n,k}^{d,2}(u_k(l))}{p_{n,k}(u_k(l))}. \tag{16}$$

Finally, the combined extrinsic distribution, $p_{n,k}^{E}(u_k(l))$ is given by

$$p_{n,k}^{E}(u_k(l)) = \alpha p_{n,k}^{E,1}(u_k(l)) p_{n,k}^{E,2}(u_k(l)). \tag{17}$$

We want to point out the difference between $p_{n,k}^{E,1}(u_k(l))$ and $p_{n,k}^{e,1}(u_k(l))$ (or equivalently the difference between $p_{n,k}^{E,2}(u_k(l))$ and $p_{n,k}^{e,2}(u_k(l))$). $p_{n,k}^{E,1}(u_k(l))$ is the extrinsic distribution, obtained by decoding the code 1, relative to the input distribution $p_{n,k}(u_k(l))$, while $p_{n,k}^{e,1}(u_k(l))$ is the classical extrinsic distribution in turbo decoding of PCCC, computed relative to the input distribution $p_{n,k}(u_k(l))$ and *a priori* distribution $p_{n,k}^{a,1}(u_k(l))$.

3.3 Multiuser detection – after first iteration

The extrinsic distribution can be fed back as *a priori* distribution for next multiuser iteration. After the $(n+1)$–th multiuser iteration, updated marginal *a posteriori* distribution of the coded bits is equal to

$$p_{n+1,k}(b_k(i)) = \alpha \sum_{\substack{all\ b_j(i) \\ j \neq k}} p(\boldsymbol{y}(i)|\boldsymbol{b}(i)) \prod_{\substack{j=1 \\ j \neq k}}^{K} p_{n,j}^{E}(b_j(i)), \tag{18}$$

and it is fed as the soft input metric into the single–user turbo decoder of user k. The procedure is iteratively repeated until some predetermined stopping condition is met.

3.4 Complexity considerations

The complexity of this scheme is $O(K2^K + 2^{\nu+1})$ per bit per user per iteration, since $O(K2^K)$ computations are needed to compute eq. (18), and for the constituent convolutional codes of constraint length ν, the decoding complexity is $O(2^{\upsilon+1})$. It is evident from the complexity analysis that the main computational burden is imposed by the multiuser detector. At the expense of small degradation in performance, exponential complexity can be avoided by employing greedy multiuser detector [11].

4. Numerical Results

To illustrate the potential of the scheme proposed in [8], we consider a synchronous K–user symmetric channel that is characterized by the following cross–correlation matrix, $\boldsymbol{R}_{ii} = 1$ and $\boldsymbol{R}_{ij} = \rho$ when $i \neq j;\ i,j = 1,\cdots,K$. All users employ the same PCCC encoders of rate 1/3, obtained by using systematic, recursive, 4–state convolutional encoders of rate 1/2 with generator matrix $G = (1, 5_8/7_8)$ as constituent codes. Different users employ different pseudorandom interleavers that are changed for every new packet transmission. All users have the same power. The performance of the algorithm is compared to the multiuser capacity limit for Gaussian signalling. The limit for equal amplitudes can be computed from [12] as $C = 1/2\log\left(\det\left[\boldsymbol{I} + SNR\boldsymbol{R}\right]\right)$, where $\boldsymbol{I}$ is the identity matrix and SNR is the signal–to–noise ratio per coded bit.

The performance of the algorithm approaches multiuser capacity limit. The number of iterations is crucial for the good performance. Fig. 3 illustrates the performance behavior for 4 users, $\rho = 0.75$, packet size of 2000 bits and different numbers of iterations. The difference between 4 and 32 iterations is more than 2 dB. It appears that the performance can further be improved by increasing the number of iterations beyond 32, although the incremental gain diminishes.

Of interest is the performance of the proposed receiver in the presence of imperfect channel state information since in practical systems channel state information need to be estimated. In this paper we assume that cross-correlations between users are known, perfect timing is established and the received amplitude is estimated. This scenario is illustrated in Fig. 4 on a 4–user symmetric channel, with crosscorrelation $\rho = 0.75$, when estimated amplitude A_e is different from the received amplitude A. The results from the figure indicate that the iterative receiver is sensitive to the imperfect amplitude estimates. A 1 dB error in amplitude results in loss of 0.4 dB in

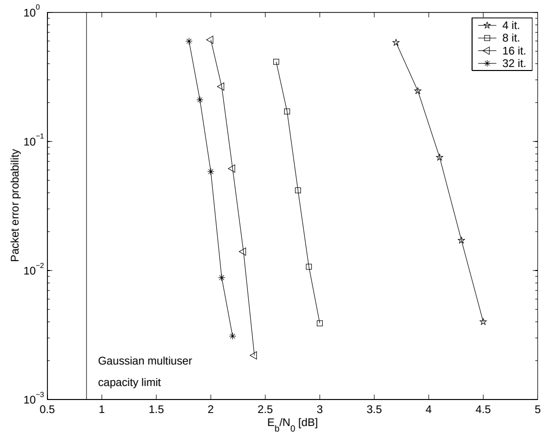

Fig. 3. Packet error probability for 4-user case, $\rho = 0.75$ and number of iterations as a prameter; interleaver size is 2000 bits.

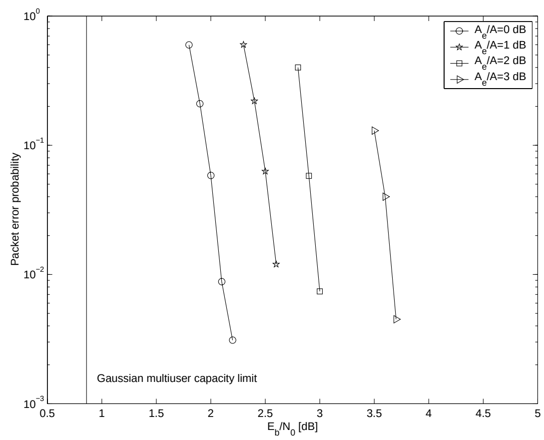

Fig. 4. Packet error probability for 4-user case, $\rho = 0.75$ imperfect amplitude estimation, 32 iterations and interleaver size is 2000 bits.

performance. Additional 1 dB error in amplitude results in performance loss of another 0.4 dB, while 3 dB amplitude error results in 1.6 dB degradation relative to perfect amplitude estimates.

5. Conclusions

The iterative approach for multiuser detection and single–user turbo decoding is described and the effect of imperfect channel information is studied. The simulation results for binary signalling and low signal–to–noise ratio show that the iterative approach proposed in [8] approaches multiuser capacity limit. Good channel state information is important for the good performance of the algorithm. We found in particular that 1 dB amplitude estimation error results in degradation of performance of 0.4 dB. Our current research is focused on iterative channel estimation and decoding.

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