FACTA UNIVERSITATIS (NIŠ)
SER.: ELEC. ENERG. vol. 23, no. 3, Decmber 2010, 259-272

Magnetic Field Determination for Different Block Permanent Magnet Systems

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Abstract: The paper presents magnetic field calculation of three characteristic permanent magnet systems, which component parts are block magnets homogeneously magnetized in arbitrary direction. Method used in this publication is based on a system of equivalent magnetic dipoles. The results obtained using this analytical method are compared with results obtained using COMSOL Multiphysics software. Magnetic field and magnetic flux density distributions of permanent magnet systems are also shown in the paper.

Keywords: Magnetic field, permanent magnet, magnetic dipole.

1 Introduction

PERMANENT magnetism is one of the oldest continuously studied branches of the science. There are many properties of permanent magnets that are taken into consideration when designing a magnet for a certain device. Most often the demagnetization curve is the one that has the greatest impact on its usability. Curve shape contains information on how the magnet will behave under static and dynamic operating conditions, and in this sense the material characteristic will constrain what can be achieved in the device's design. The *B* (magnetic flux density) versus *H* (magnetic field) loop of any permanent magnet has some portions which are almost linear, and others that are highly non-linear [1].

Magnetic materials are vital components of most electromechanical machines. An understanding of magnetism and magnetic materials is therefore essential for the design of modern devices. The magnetic components are usually concealed

Manuscript received on June 22, 2010.

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in subassemblies and are not directly apparent to the end user. Permanent magnets have been used in electrical machinery for over one hundred years. Scientific breakthroughs in materials study and manufacturing methods from the early 1940's to the present have improved the properties of permanent magnets and made the use of magnetic devices common. This work is motivated by the need for different shaped permanent magnets in great number of electromagnetic devices. Knowledge of the magnetic scalar potential, magnetic flux density or magnetic field is required to control devices reliably [2].

Determination of the magnetic field components in vicinity of permanent magnets, starts with presumption that magnetization, $\boldsymbol{M}$, of permanent magnet is known. The following methods can be used in practical calculation:

(a) Method based on determining distribution of microscopic ampere's current;

(b) Method based on Poisson and Laplace equations, determining magnetic scalar potential; and

(c) Method based on a system of equivalent magnetic dipoles [3].

The third method that is mentioned in the paper for magnetic field calculation is based on superposition of elementary results obtained for elementary magnetic dipoles.

Elementary magnetic dipole (Fig. 1) has magnetic moment

$$\mathrm{d}\boldsymbol{m} = \boldsymbol{M}\mathrm{dV}' \tag{1}$$

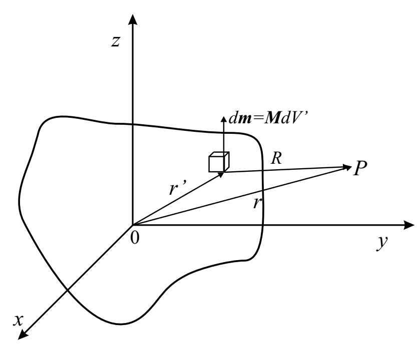

Fig. 1. Elementary magnetic dipole.

This magnetic moment produces, at field point P, elementary magnetic scalar potential

$$\mathrm{d}\,\varphi_{\mathrm{m}} = \frac{1}{4\pi}\frac{\boldsymbol{R}\mathrm{d}\boldsymbol{m}}{\mathrm{R}^3} = \frac{1}{4\pi}\frac{\boldsymbol{R}\boldsymbol{M}}{\mathrm{R}^3}\mathrm{dV}', \tag{2}$$

where $\boldsymbol{R} = |\boldsymbol{r} - \boldsymbol{r}'|$ is distance from the point where the magnetic field is being calculated to elementary source, and $\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}'$.

After integration magnetic scalar potential is obtained as

$$\varphi_m = \frac{1}{4\pi} \int_V \frac{\boldsymbol{RM}}{R^3} \mathrm{dV}'. \tag{3}$$

Magnetic field vector can be expressed as

$$\boldsymbol{H} = -\mathrm{grad}\varphi_m. \tag{4}$$

In publications [4]- [5] magnetic field components are determined for different shaped permanent magnets using methods presented above. This paper describes three typical examples of block permanent magnet systems.

2 Problem definition

The aim of this paper is magnetic field components determination of block permanent magnet systems. To determine the magnetic field of a system the permanent magnet presented in the Fig. 2 will be considered first, [6]. The method described above is used for determining the magnetic field components of the block permanent magnet magnetized in arbitrary direction (Fig. 2).

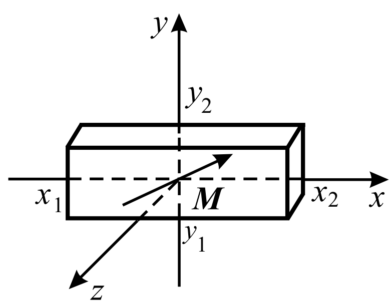

Fig. 2. Block permanent magnet magnetized in arbitrary direction.

The intensity of magnetic field depends on magnetization pattern and the magnet shape. In this case it is assumed that the magnitude of the magnetization M has the same value everywhere, it is oriented in xy plane and its direction can vary giving

$$\boldsymbol{M} = M(\cos(\alpha)\hat{x} + \sin(\alpha)\hat{y}). \tag{5}$$

Outside the permanent magnet, magnetic scalar potential, at the field point $P(x,y,z)$, could be presented using the expression (3).

As magnetization has two components, x and y, dot product $\boldsymbol{R}\cdot\boldsymbol{M}$ is formed as

$$\boldsymbol{R}\cdot\boldsymbol{M} = [(x-x')\hat{x}+(y-y')\hat{y}+(z-z')\hat{z}]M(\cos(\alpha)]\hat{x}+\sin(\alpha)\hat{y}), \tag{6}$$

therefore,

$$\boldsymbol{R}\cdot\boldsymbol{M} = M((x-x')\cos(\alpha)+(y-y')\sin(\alpha)). \tag{7}$$

Distance from the point where the magnetic field is being calculated to elementary source is

$$R = \sqrt{(x-x')^2+(y-y')^2+(z-z')^2}. \tag{8}$$

Finally, substituting the expressions Eq.7 and Eq. 8 in Eq. 3, magnetic scalar potential produced by a block magnet is formed as

$$\varphi_m = \frac{M}{4\pi}\int_{x_1}^{x_2}\int_{y_1}^{y_2}\int_{z_1}^{z_2}\frac{(x-x')\cos(\alpha)+(y-y')\sin(\alpha)}{[(x-x')^2+(y-y')^2+(z-z')^2]^{\frac{3}{2}}}\mathrm{d}x'\mathrm{d}y'\mathrm{d}z'. \tag{9}$$

The solution of this integral is

$$\begin{aligned}\varphi_m(x,y,z) =&\frac{M}{4\pi}((V[x-x_2,y-y_1,y-y_2,z-z_1,z-z_2]\\&-V[x-x_1,y-y_1,y-y_2,z-z_1,z-z_2])\cos\alpha\\&+(V[y-y_2,x-x_1,x-x_2,z-z_1,z-z_2]\\&-V[y-y_1,x-x_1,x-x_2,z-z_1,z-z_2]))\sin\alpha,\end{aligned} \tag{10}$$

where function V has the following form:

$$\begin{aligned}V(a,x_1,x_2,z_1,z_2) =&x_2\ln\frac{C_2}{C_3}+x_1\ln\frac{C_1}{C_4}+z_1\ln\frac{C_5}{C_8}+z_2\ln\frac{C_6}{C_7}\\&-2|a|\arctan\frac{C_5C_8+a^2+{z_1}^2+z_1(C_5+C_8)}{|a|(C_8-C_5)}\\&+2|a|\arctan\frac{C_7C_6+a^2+{z_2}^2+z_2(C_6+C_7)}{|a|(C_6-C_7)},\end{aligned} \tag{11}$$

and

$$\begin{aligned}&C_1 = z_1+\sqrt{a^2+{x_1}^2+{z_1}^2}; \quad C_2 = z_2+\sqrt{a^2+{x_2}^2+{z_2}^2};\\&C_3 = z_1+\sqrt{a^2+{x_2}^2+{z_1}^2}; \quad C_4 = z_2+\sqrt{a^2+{x_1}^2+{z_2}^2};\\&C_5 = x_1+\sqrt{a^2+{x_1}^2+{z_1}^2}; \quad C_6 = x_2+\sqrt{a^2+{x_2}^2+{z_2}^2};\\&C_7 = x_1+\sqrt{a^2+{x_1}^2+{z_2}^2}. \quad C_8 = x_2+\sqrt{a^2+{x_2}^2+{z_1}^2}.\end{aligned}$$

3 Magnetic field determination

3.1 Example I

The first system which is considered is presented in the Fig. 3. The magnetic scalar potential solution obtained for the permanent magnet presented above can be used for each block of the system if it is taken that $\alpha = 0$ or $\alpha = \pi$ [7]. Therefore, each of these block permanent magnets is homogeneously magnetized in longitudinal direction,

$$\boldsymbol{M} = M(\pm\hat{x}). \tag{12}$$

The certain modification of this system might find its application in modern hard drives.

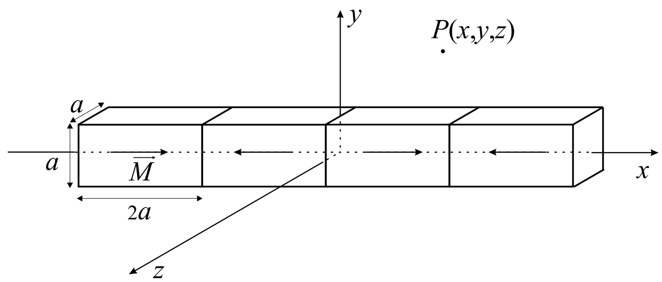

Fig. 3. First block permanent magnet system.

Magnetic scalar potentials of the block magnets may be presented using the function V, Eq. 11, as

$$\begin{aligned}\varphi_{m1} = &\frac{M}{4\pi}(V[x+4a, y+\frac{a}{2}, y-\frac{a}{2}, z+\frac{a}{2}, z-\frac{a}{2}] \\ &-V[x+2a, y+\frac{a}{2}, y-\frac{a}{2}, z+\frac{a}{2}, z-\frac{a}{2}]),\end{aligned} \tag{13}$$

$$\begin{aligned}\varphi_{m2} = &\frac{M}{4\pi}(V[x, y+\frac{a}{2}, y-\frac{a}{2}, z+\frac{a}{2}, z-\frac{a}{2}] \\ &-V[x+2a, y+\frac{a}{2}, y-\frac{a}{2}, z+\frac{a}{2}, z-\frac{a}{2}]),\end{aligned} \tag{14}$$

$$\begin{aligned}\varphi_{m3} = &\frac{M}{4\pi}(V[x-2a, y+\frac{a}{2}, y-\frac{a}{2}, z+\frac{a}{2}, z-\frac{a}{2}] \\ &-V[x, y+\frac{a}{2}, y-\frac{a}{2}, z+\frac{a}{2}, z-\frac{a}{2}]),\end{aligned} \tag{15}$$

$$\begin{aligned}\varphi_{m4} = &\frac{M}{4\pi}(V[x-2a, y+\frac{a}{2}, y-\frac{a}{2}, z+\frac{a}{2}, z-\frac{a}{2}] \\ &-V[x-4a, y+\frac{a}{2}, y-\frac{a}{2}, z+\frac{a}{2}, z-\frac{a}{2}]).\end{aligned} \tag{16}$$

Magnetic scalar potential of the whole system is the sum of magnetic scalar potentials obtained for each magnetized block.

3.2 Example II

The second example that is considered is calculation of the external magnetic field generated by domains of periodic parallel structure with domains width a, whose magnetization $\boldsymbol{M}$ is the same in magnitude but reverses orientation from domain to domain (Fig. 4) [8],

$$\boldsymbol{M} = M(\pm\hat{y}). \tag{17}$$

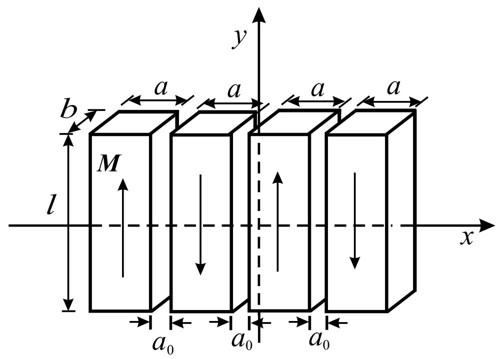

Fig. 4. Second block permanent magnet system.

It is considered the case of magnetization vector $\boldsymbol{M}$ orientation perpendicular to a sample surface as shown in the figure. It is also assumed that the thickness of the block is b length l and the distance between two neighboring blocks is a_0 [7].

The magnetic scalar potential generated by the block permanent magnet is magnetized in positive direction of y axis is obtained from the expression (10) when $\alpha = \pi/2$, and magnetic scalar potential of the block magnetized in opposite direction in the case when $\alpha = -\pi/2$. Using the derivation presented above the magnetic scalar potential obtained for blocks are given by following expressions:

$$\begin{aligned}\varphi_{m1} =& \frac{M}{4\pi}(V[y-\frac{l}{2},x+2a+\frac{3a_0}{2},x+a+\frac{3a_0}{2},z+\frac{b}{2},z-\frac{b}{2}] \\ &-V[y+\frac{l}{2},x+2a+\frac{3a_0}{2},x+a+\frac{3a_0}{2},z+\frac{b}{2},z-\frac{b}{2}]),\end{aligned} \tag{18}$$

$$\begin{aligned}\varphi_{m2} =& \frac{M}{4\pi}(V[y+\frac{l}{2},x+a+\frac{a_0}{2},x+\frac{a_0}{2},z+\frac{b}{2},z-\frac{b}{2}] \\ &-V[y-\frac{l}{2},x+a+\frac{a_0}{2},x+\frac{a_0}{2},z+\frac{b}{2},z-\frac{b}{2}]),\end{aligned} \tag{19}$$

$$\begin{aligned}\varphi_{m3} =& \frac{M}{4\pi}(V[y-\frac{l}{2},x-\frac{a_0}{2},x-a-\frac{a_0}{2},z+\frac{b}{2},z-\frac{b}{2}]\\ &-V[y+\frac{l}{2},x-\frac{a_0}{2},x-a-\frac{a_0}{2},z+\frac{b}{2},z-\frac{b}{2}]),\end{aligned} \tag{20}$$

$$\begin{aligned}\varphi_{m4} =& \frac{M}{4\pi}(V[y+\frac{l}{2},x-a-\frac{3a_0}{2},x-2a-\frac{3a_0}{2},z+\frac{b}{2},z-\frac{b}{2}]\\ &-V[y-\frac{l}{2},x-a-\frac{3a_0}{2},x-2a-\frac{3a_0}{2},z+\frac{b}{2},z-\frac{b}{2}]).\end{aligned} \tag{21}$$

In cases where system has N block permanent magnets, the magnetic scalar potential of the whole system is equal to sum of magnetic scalar potentials formed by all of its magnetized parts:

$$\varphi_m = \sum_{i=1}^{N} \varphi_{m_i}. \tag{22}$$

3.3 Example III

The method that is described above is also used for determining the magnetic field components of the block permanent magnet system presented in the Fig.5 [9].

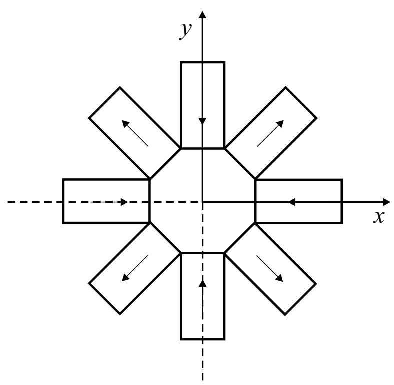

Fig. 5. Third block permanent magnet system.

With multiple translations and rotations of coordinate system, the block shown in the Fig. 2 may be positioned like it's shown on Fig. 5 to obtain suitable system.

If the original point is $P(x,y,z)$ it can be translated to point $P(x',y',z')$. New coordinates are

$$x' = x - x_1, \quad y' = y - y_1, \quad z' = z. \tag{23}$$

If the coordinate system is rotated clockwise around center point placed in the center of coordinate system new coordinates will be

$$x' = x\cos(\alpha) + y\sin(\alpha), \quad y' = -x\sin(\alpha) + y\cos(\alpha), \quad z' = z. \tag{24}$$

Therefore, using expression (10) and relations (23) and (24) magnetic scalar potential of each block of the treated system can be calculated [10]. As in previous two examples, the total magnetic scalar potential is equal to the sum of magnetic scalar potentials generated by all permanent magnet blocks (22). After determining magnetic scalar potential magnetic field component generated by permanent magnet systems may be calculated using the expression (4).

4 Numerical results

For the first considered permanent magnet system distribution of magnetic field, in $x0y$ plane, outside the system is illustrated in the Fig. 6. It is obtained using the analytical method for magnetic field determination. Magnetic field lines for the same system, obtained using the COMSOL Multiphysics software are presented in the Fig. 7. Distribution of magnetic flux density is shown in the same figure with arrows and its intensity is presented with gradient of gray. Magnetization of each block in the system is 750kA/m. Comparing these two figures it can be concluded that results of the analytical method are confirmed in satisfactory manner using COMSOL Multiphysics software.

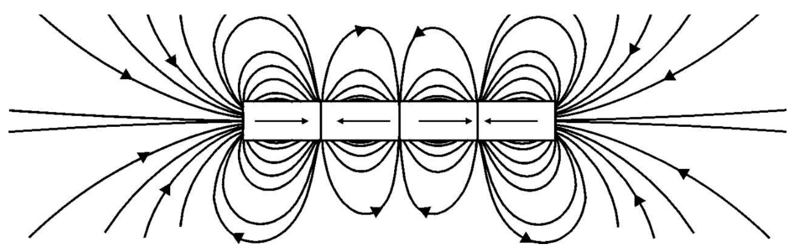

Fig. 6. Distribution of magnetic field obtained using the analytical method.

Fig. 8 presents magnetic flux density inside the system along the direction $y = 0$. It is obvious that magnetic flux density has the highest values inside of blocks and the lowest values in the border area between neighboring blocks.

The pictures from Fig. 9 to Fig. 12 present results obtained for the second example of permanent magnet system. The system consists of four blocks which dimensions are $a, b = a$ and $l = 3a$ and the distance between two neighboring blocks is $a_0 = a/5$. Magnetization of each block in the system is 150kA/m. Fig. 9 shows magnetic field distribution of the system as the result of the analytical method.

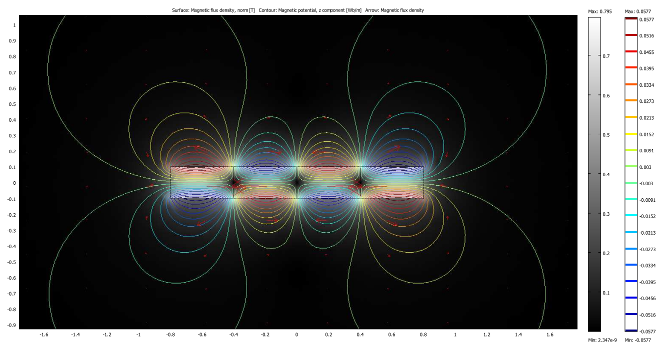

Fig. 7. Distribution of magnetic field for the first permanent magnet system (COMSOL Multiphysics software).

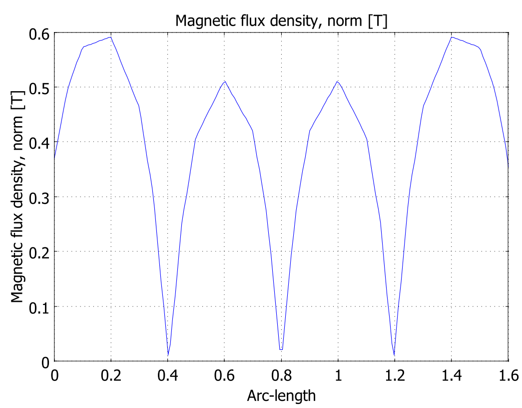

Fig. 8. Magnetic flux density inside the blocks of the system along the direction $y = 0$.

This system is also analyzed using COMSOL Multiphysics software and the result is shown in the Fig. 10. The same picture presents the magnetic flux density, marked with arrows, while its intensity is presented with different colors. The light yellow color presents the highest magnetic flux density while the black color expresses the lowest magnetic flux density.

Fig. 11 and Fig. 12 represent magnetic flux density between two neighboring blocks, positioned in the center of the system, for $x = 0$, $z = 0$ and $-l/2 \leq y \leq l/2$.

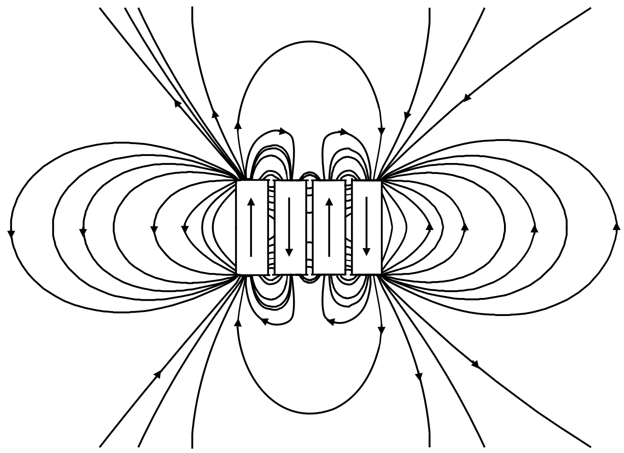

Fig. 9. Distribution of magnetic field for second permanent magnet system (analytical method).

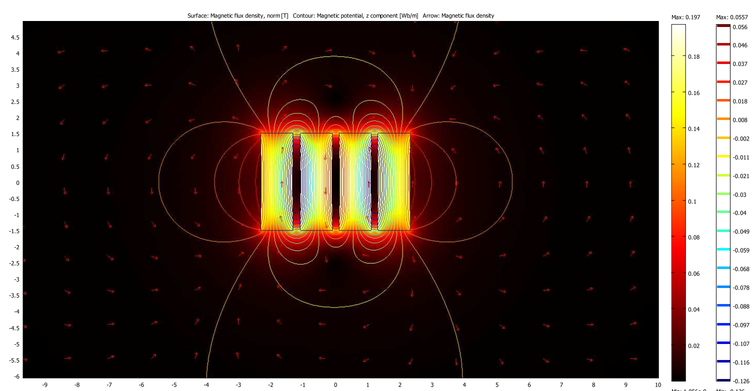

Fig. 10. Distribution of magnetic field for second permanent magnet system (COMSOL Multiphysics software) .

The first one is result of COMSOL Multiphysics software while, the second one is obtained using analytical method. Comparing these two diagrams it may be concluded that alignment exists between analytical and numerical methods of calculation. Certain deviation is present on block ends.

Table in the Figure 13 presents normalized magnetic field values for the second

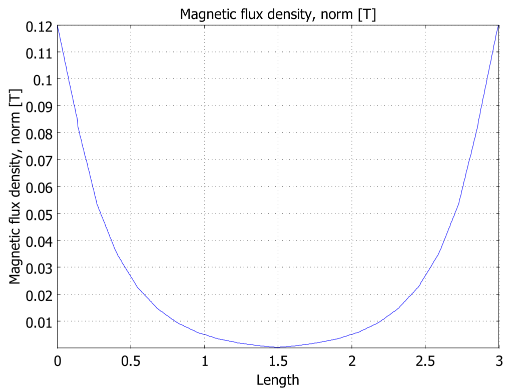

Fig. 11. Magnetic flux density between two neighboring blocks (COMSOL Multiphysics software).

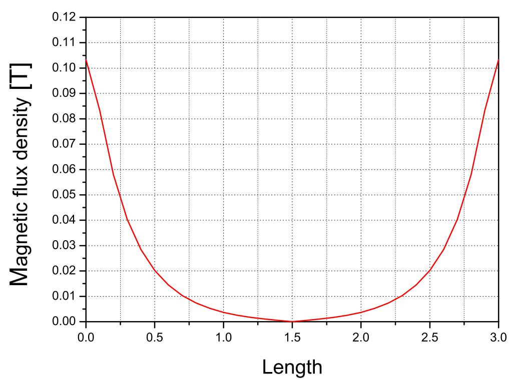

Fig. 12. Magnetic flux density between two neighboring blocks (analytical method).

block permanent magnet system with dimensions given above. It is obtained using the analytical method. The values along the direction $x = 0$, $z = 0$, in the gap between two neighboring blocks, are shown in the table.

Distribution of magnetic field generated by the third system, in $x0y$ plane, outside the permanent magnet is illustrated in the Fig. 14. It is obtained using the analytical method for magnetic field determination. Magnetic field distribution for the same system, obtained using the COMSOL Multiphysics software is presented in the Fig. 15. Distribution of magnetic flux density is shown in the same figure with arrows and its intensity is presented with gradient of gray. Magnetization of each block in the system is 750kA/m. Comparing these two figures it is obvious

y/a	H/M
0.1	0.002569
0.2	0.005440
0.3	0.008946
0.4	0.013481
0.5	0.019545
0.6	0.027796
0.7	0.039135
0.8	0.054822
0.9	0.076672
1.0	0.107373
1.1	0.151041
1.2	0.214233
1.3	0.307486
1.4	0.442088
1.5	0.548363

Fig. 13. Normalized values of magnetic field along the direction $x = 0$, $z = 0$.

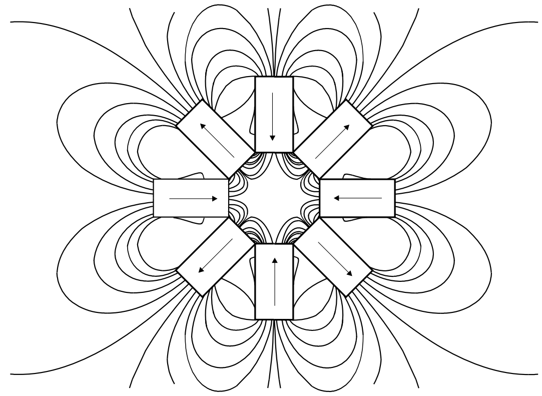

Fig. 14. Distribution of magnetic field for third permanent magnet system (analytical method).

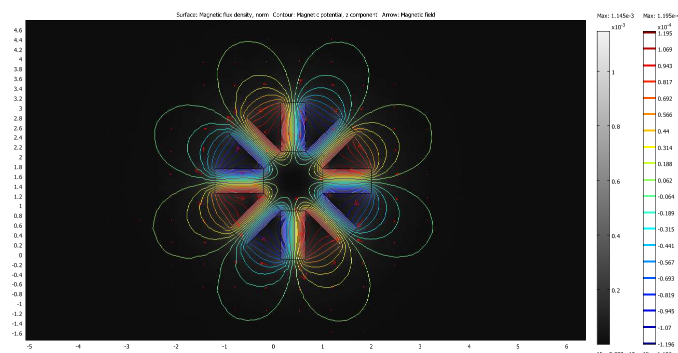

Fig. 15. Distribution of magnetic field for third permanent magnet system (COMSOL Multiphysics software).

that results of the analytical method are confirmed in satisfactory manner using COMSOL Multiphysics software.

This system may be applicable for approximation of the permanent magnets system found in rotation motors.

5 Conclusion

Three different permanent magnet systems which consist of block permanent magnets, homogeneously magnetized in known direction, are observed in the paper. Method that is used for magnetic field determination is based on superposition of results that are obtained for elementary magnetic dipoles. Magnetic field and magnetic flux density distributions of permanent magnet systems are also presented in the paper. Magnetic field lines have the same form and the same direction as magnetic flux density lines, outside the system. Results obtained by the analytical method are satisfactory confirmed using COMSOL Multiphysics software. This work is motivated by the need for different shaped permanent magnets in great number of electromagnetic devices [11].

Acknowledgments

The author would like to acknowledge the support of the Ministry of Science and Technological Development, Serbia.

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