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Performance Analysis of SIR-Based Triple Selection Diversity Over Correlated Weibull Fading Channels

Petar Spalević, Nikola Sekulović, Zachos Georgios, and Edis Mekić

Abstract: In this paper, closed-form expressions for the probability density function (PDF) and cumulative distribution function (CDF) of the signal-to-interference ratio (SIR) at the output of triple selection combining (SC) receiver over correlated Weibull fading channels are obtained. These expressions are used to study important system performance criteria such as the outage probability, average bit error probability (ABEP) and average output SIR. Numerical results are also graphically presented showing the effect of various systems parameters such as the average input SIR, fading severity and level of correlation on the systems performance.

Keywords: Cochannel interference, correlated Webull fading channels, selection diversity.

1 Introduction

DIVERSITY combining [1] is an efficient and widely employed technique in digital communication receivers for mitigating the multipath fading effects and upgrading transmission reliability at relatively low cost. Space diversity techniques [2, 3] combine input signals from multiple receive antennas in some way to ameliorate systems quality-of-service (QoS). The most popular linear diversity techniques are selection combining (SC), equal-gain combining (EGC) and maximal-ratio combining (MRC) [4]. MRC is optimal combining technique in the sense that it achieves the highest output signal-to-noise ratio (SNR) regardless of

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P. Spalević is with is with Faculty of Technical Sciences, Dept. of Electrical Engineering, K. Miloša 7, 38220 Kosovska Mitrovica, Serbia (e-mail: petarspalevic@yahoo.com). N. Sekulović, Z. Georgios, and E. Mekić are with Faculty of Electronic Engineering, A. Medvedeva 14, 18000 Niš, Sebia.

the fading statistics on the diversity branches. However, MRC requires the knowledge of the channel fading amplitudes and phases of each diversity branch which must be continuously estimated by the receiver. This estimations require separate receiver chain for each branch of the diversity system increasing its complexity. EGC provides performance comparable to MRC, but with simpler implementation complexity. EGC does not require the estimation of the channel fading amplitudes since it combines signals from all branches with the same weighting factor. Selection combining is the least complicated technique. It is reposed on processing only one of the diversity branches. SC combiner chooses the branch with the highest SNR, or equivalently, with the strongest signal assuming equal noise power among the branches. In wireless communication systems, the influence of the thermal noise may be negligible as compared to the influence of the cochannel interference (CCI). In that case, SC combiner processes the branch with the highest signal-to-interference ratio (SIR-based selection diversity) [5].

Despite the fact that Weibull fading model confirms experimentally attained fading channel measurements, for both indoor [6], as well as for outdoor environments [7], it has not yet received as much attention as Rayleigh, Rice and Nakagami-m fading models. Very recently the topic of digital communications over Weibull fading channels has begun to receive some interest. In paper [8], useful analytical expressions for the probability density function, cumulative density function, moment-generating function (MGF) and product moments for the bivariate and multivariate Weibull distribution have been derived and applied to analyze the performance of dual and multibranch SC, EGC and MRC receivers. The performance of a class of generalized-selection combining (GSC) receivers over independent and nonidentical Weibull fading channels has been studied in [9]. L-branch EGC and MRC receivers operating over nonidentical Weibull fading channels have been considered in [10]. Performance of dual selection combining receiver over correlated Weibull fading channels in the presence of Weibull distributed CCI has been presented in [11]. In [12], closed-form expression for the outage probability of SIR-based multibranch selection combining receiver over correlated Weibull fading channels has been derived.

In this paper, triple selection combining receiver operating over correlated Weibull fading channels in the presence of Weibull distributed CCI is considered. Expressions for the probability density function and cumulative density function of SIR at the output of selection combining receiver are presented and applied to analyze the performance of triple selection combining receiver. Various numerically evaluated results for the outage probability, average bit error probability (ABEP) and average output SIR are presented showing the effect of the average input SIR, fading severity and correlation coefficient on the systems performance.

2 Triple-branch SC output statistics

Since Weibull fading model exhibits an excellent fit to experimental fading channel measurements, especially in urban and nonhomogeneous environments, our efforts are concentrated on case in which both desired and interference signal envelopes follow the correlated Weibull distribution with joint PDFs [8, eq. (23) for $L=3$]:

$$\begin{aligned} p_{R_1R_2R_3}(R_1,R_2,R_3) =& \frac{\beta_1\beta_2\beta_3}{\Omega_{d1}\Omega_{d2}\Omega_{d3}(1-\rho)^2} \\ &\times \exp\left\{-\frac{1}{1-\rho}\left[\frac{R_1^{\beta_1}}{\Omega_{d1}}+\frac{R_3^{\beta_3}}{\Omega_{d3}}+\frac{(1+\rho)R_2^{\beta_2}}{\Omega_{d2}}\right]\right\} \\ &\times \sum_{k_1,k_2=0}^{\infty}\left[\frac{\sqrt{\rho}}{\sqrt[3]{\Omega_{d1}\Omega_{d2}\Omega_{d3}}(1-\rho)}\right]^{2(k_1+k_2)} \\ &\times \frac{R_1^{(k_1+1)\beta_1-1}R_3^{(k_2+1)\beta_3-1}R_2^{(k_1+k_2+1)\beta_2-1}}{(k_1!k_2!)^2} \end{aligned} \quad (1)$$

and

$$\begin{aligned} p_{r_1r_2r_3}(r_1,r_2,r_3) =& \frac{\beta_1\beta_2\beta_3}{\Omega_{c1}\Omega_{c2}\Omega_{c3}(1-\rho)^2} \\ &\times \exp\left\{-\frac{1}{1-\rho}\left[\frac{r_1^{\beta_1}}{\Omega_{c1}}+\frac{r_3^{\beta_3}}{\Omega_{c3}}+\frac{(1+\rho)r_2^{\beta_2}}{\Omega_{c2}}\right]\right\} \\ &\times \sum_{l_1,l_2=0}^{\infty}\left[\frac{\sqrt{\rho}}{\sqrt[3]{\Omega_{c1}\Omega_{c2}\Omega_{c3}}(1-\rho)}\right]^{2(l_1+l_2)} \\ &\times \frac{r_1^{(l_1+1)\beta_1-1}r_3^{(l_2+1)\beta_3-1}r_2^{(l_1+l_2+1)\beta_2-1}}{(l_1!l_2!)^2} \end{aligned} \quad (2)$$

respectively. Parameter β is Weibull fading parameter ($\beta > 0$) and represents fading intensity measure, ρ is correlation coefficient, $\Omega_{di} = \overline{R_i{}^2}$ and $\Omega_{ci} = \overline{r_i{}^2}$ are the average powers of desired and interference signal at ith branch, respectively. When value of parameter β increases, fading intensity decreases, while for β=2 Weibull distribution becomes Rayleigh distribution. Furthermore, it must be mentioned that the effect of only the strongest interference signal is considered, like in [5, 11–14].

Instantaneous value of SIR on the ith diversity branch of SC receiver can be defined as $\mu_i = R_i/r_i$, $(i = 1,2,3)$. Let μ_{sc} denotes the instantaneous SIR at the output of SC receiver, i.e. $\mu_{sc} = \max\{\mu_1,\mu_2,\mu_3\}$. The CDF of μ_{sc} is [12, eq. (10) for L=3]

$$
\begin{aligned}
F_{\mu_{sc}}(\mu) = {} & (1-\rho)^2 \sum_{k_1,k_2=0}^{\infty} \sum_{l_1,l_2=0}^{\infty} \frac{\rho^{k_1+l_1+k_2+l_2}}{(1+\rho)^{k_1+l_1+k_2+l_2+2}} \\
& \times \frac{\Omega_{c1}^{k_1+l_1+1-\frac{2}{3}(l_1+l_2)} \Omega_{c3}^{k_2+l_2+1-\frac{2}{3}(l_1+l_2)} \Omega_{c2}^{k_1+k_2+l_1+l_2+1-\frac{2}{3}(l_1+l_2)}}{(\Omega_{d1}\Omega_{d2}\Omega_{d3})^{1+\frac{2}{3}(k_1+k_2)} (k_1! l_1! k_2! l_2!)^2} \\
& \times \Gamma(k_1+l_1+2)\Gamma(k_2+l_2+2)\Gamma(k_1+k_2+l_1+l_2+2) \\
& \times \frac{\mu^{(k_1+1)\beta_1+(k_1+k_2+1)\beta_2+(k_2+1)\beta_3}}{(k_1+1)(k_1+k_2+1)(k_2+1)} \\
& \times {}_2F_1(k_1+l_1+2, k_1+1; k_1+2; -\frac{\Omega_{c_1}}{\Omega_{d1}}\mu^{\beta_1}) \\
& \times {}_2F_1(k_1+k_2+l_1+l_2+2, k_1+k_2+1; k_1+k_2+2; -\frac{\Omega_{c_2}}{\Omega_{d2}}\mu^{\beta_2}) \\
& \times {}_2F_1(k_2+l_2+2, k_2+1; k_2+2; -\frac{\Omega_{c_3}}{\Omega_{d3}}\mu^{\beta_3})
\end{aligned} \tag{3}
$$

where $\Gamma(\cdot)$ is the Gamma function and ${}_2F_1(a,b;c;d)$ is the Gaussian hypergeometric function.

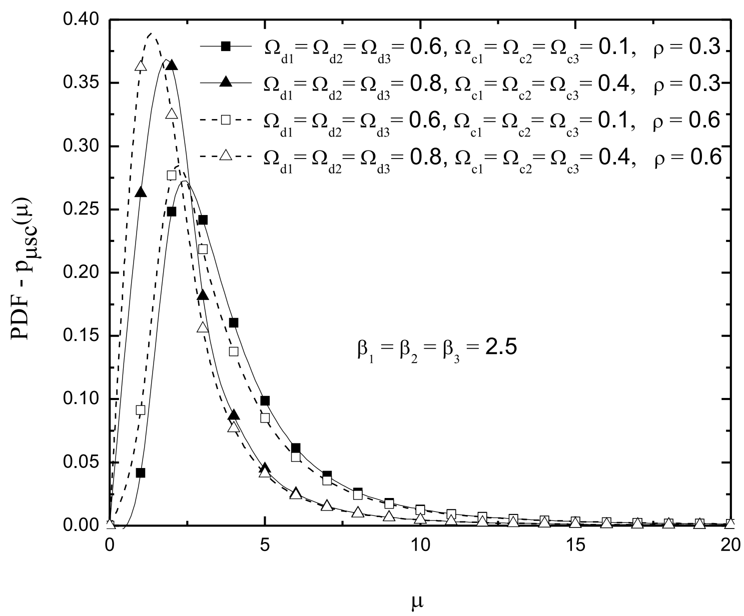

Fig. 1. Probability density function of instantaneous SIR at the output of triple SC receiver.

The PDF of the instantaneous SIR at the output of SC receiver is [12, eq. (12) for L=3]

$$
\begin{aligned}
p_{\mu_{sc}}(\mu) = {} & (1-\rho)^2 \sum_{k_1,k_2=0}^{\infty} \sum_{l_1,l_2=0}^{\infty} \frac{\rho^{k_1+l_1+k_2+l_2}}{(1+\rho)^{k_1+l_1+k_2+l_2+2}} \\
& \times \frac{\Omega_{c1}^{k_1+l_1+1-\frac{2}{3}(l_1+l_2)} \Omega_{c2}^{k_1+k_2+l_1+l_2+1-\frac{2}{3}(l_1+l_2)} \Omega_{c3}^{k_2+l_2+1-\frac{2}{3}(l_1+l_2)}}{(\Omega_{d1}\Omega_{d2}\Omega_{d3})^{1+\frac{2}{3}(k_1+k_2)} (k_1!k_2!l_1!l_2!)^2} \\
& \times \Gamma(k_1+l_1+2)\Gamma(k_2+l_2+2)\Gamma(k_1+k_2+l_1+l_2+2) \\
& \times \mu^{(k_1+1)\beta_1+(k_1+k_2+1)\beta_2+(k_2+1)\beta_3-1} \\
& \times \Bigg\{ \frac{\beta_1}{(k_2+1)(k_1+k_2+1)(1+\frac{\Omega_{c1}}{\Omega_{d1}}\mu^{\beta_1})^{k_1+l_1+2}} \\
& \times {}_2F_1(k_2+l_2+2, k_2+1, k_2+2, -\frac{\Omega_{c3}}{\Omega_{d3}}\mu^{\beta_3}) \\
& \times {}_2F_1(k_1+k_2+l_1+l_2+2, k_1+k_2+1, k_1+k_2+2, -\frac{\Omega_{c2}}{\Omega_{d2}}\mu^{\beta_2}) \\
& + \frac{\beta_2}{(k_1+1)(k_2+1)(1+\frac{\Omega_{c2}}{\Omega_{d2}}\mu^{\beta_2})^{k_1+k_2+l_1+l_2+2}} \\
& \times {}_2F_1(k_1+l_1+2, k_1+1, k_1+2, -\frac{\Omega_{c1}}{\Omega_{d1}}\mu^{\beta_1}) \\
& \times {}_2F_1(k_2+l_2+2, k_2+1, k_2+2, -\frac{\Omega_{c3}}{\Omega_{d3}}\mu^{\beta_3}) \\
& + \frac{\beta_3}{(k_1+1)(k_1+k_2+1)(1+\frac{\Omega_{c3}}{\Omega_{d3}}\mu^{\beta_3})^{k_2+l_2+2}} \\
& \times {}_2F_1(k_1+l_1+2, k_1+1, k_1+2, -\frac{\Omega_{c1}}{\Omega_{d1}}\mu^{\beta_1}) \\
& \times {}_2F_1(k_1+k_2+l_1+l_2+2, k_1+k_2+1, k_1+k_2+2, -\frac{\Omega_{c2}}{\Omega_{d2}}\mu^{\beta_2}) \Bigg\}
\end{aligned} \tag{4}
$$

Fig. 1 shows the PDF of SIR at the output of triple SC receiver for various values of correlation coefficient and average powers of desired and interference signal.

3 System performance measures

Analytically extracted expressions for the PDF and CDF can be applied to analyze the performance of triple SC receiver operating in correlated Weibull fading environment. Several performance quality indicators are considered in this section.

3.1 Outage probability

Outage probability is standard system performance measure of diversity system operating over fading channels. In the interference-limited environment, the outage probability is defined as the probability that the output SIR falls below a specified threshold μ_{th}. It can be obtained by replacing μ with μ_{th}in the CDF expression given by (3).

$$P_{out} = P_R(\mu_{sc} < \mu_{th}) = \int_0^{\mu_{th}} p_{\mu_{sc}}(\mu)d\mu = F_{\mu_{sc}}(\mu_{th}) \tag{5}$$

Numerical results show that expression for the outage probability converges for all values of system parametres. As Table 1 indicates, the number of terms needed to be summed in (5) to achieve accuracy at the 4th significant digit strongly depends on the correlation coefficient. The number of terms increases as correlation coefficient increases.

Table 1. Terms need to be summed in expression for the outage probability to achieve accuracy at the 4th significant digit.

	Number of terms
$\rho = 0.2$	7
$\rho = 0.4$	14
$\rho = 0.6$	22

In Fig. 2, the outage probability of triple SC receiver is plotted as a function of the outage threshold for different system parameters. For lower values of the outage threshold, the outage probability decreases as Weibull fading parameter increases. When desired signal dominates (higher values of μ_{th}), increasing of Weibull fading parameter leads to deterioration of system performance. For a fixed values of Weibull fading parameters, system performance are better in the case of lower correlation coefficient.

3.2 Average bit error probability (ABEP)

Average bit error probability (ABEP) is another useful performance criterion characteristic of wireless communication systems. Conditional bit error probability (BEP) is a nonlinear function of the intstantaneous SIR and the nature of the nonlinearity is a function of the modulation/detection scheme employed by the system. The conditional BEP for a given SIR is

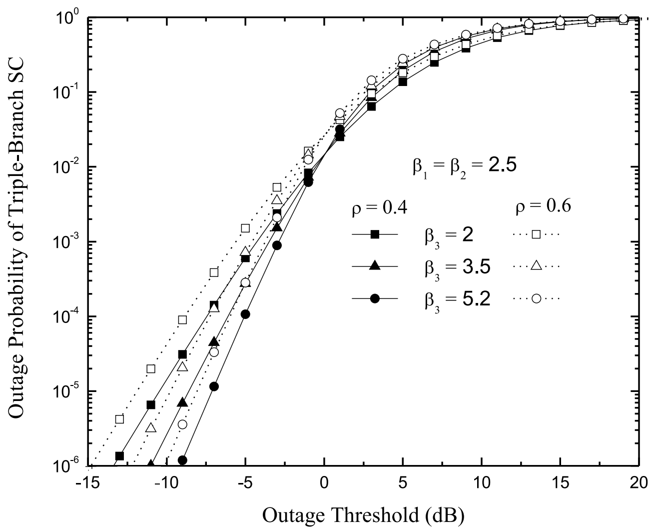

Fig. 2. Outage probability of triple SC as a function of the outage threshold.

$$P_e(\mu) = \frac{1}{2} e^{-g\mu^2} \tag{6}$$

where g denotes modulation constant, i.e. $g = 1$ for BDPSK and $g = 1/2$ for BFSK. ABEP at the SC output can be evaluated directly by averaging the conditional BEP over the PDF of μ_{sc}.

$$P_e = \int_0^\infty p_{\mu_{sc}}(\mu) P_e(\mu) d\mu \tag{7}$$

Based on (4), (6) and (7), the error performance of triple SC receiver can be obtained for several modulation schemes. ABEP of triple and dual [11] SC receiver with BFSK and BDPSK signaling is plotted in Fig. 3 as a function of the average SIRs at the input branches of the SC ($\Omega_{d1}/\Omega_{c1} = \Omega_{d2}/\Omega_{c2} = \Omega_{d3}/\Omega_{c3} = S$) for several values of ρ. Therefore, the case of balanced input SIRs is considered. The obtained performance evaluation results show that ABEP improves with decrease of ρ and increase of number of diversity branches. Moreover, system performance is better for BDPSK signaling. It is very interesting to observe that for lower values of S, ABEP performance of dual SC receiver with BDPSK signaling is better than ABEP performance of triple SC receiver with BFSK signaling.

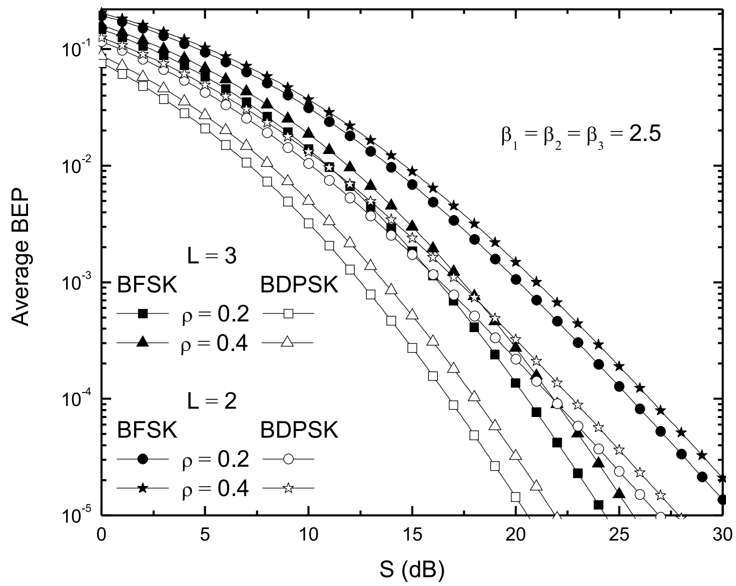

Fig. 3. ABEP of dual and triple SC with BFSK and BDPSK signaling as a function of the average input SIR.

3.3 Average output SIR

Average output SIR is very useful parameter for describing wireless communication systems in the presence of CCI and it can be evaluated as

$$\overline{\mu_{sc}} = \int_0^\infty \mu p_{\mu_{sc}}(\mu) d\mu \tag{8}$$

Based on (4) and (8), Fig. 4 demonstrates numerically obtained results for the average output SIR as a function of ρ for diffrent system parameters. It is obviously that diversity gain decreases as ρ and/or fading parameters increase. Average output SIR degrades rapidly for higher values of ρ. Increasing of S leads to amelioration of system performance. This amelioration is more significant for lower values of fading parameters.

4 Conclusion

The performance of triple SC receiver operating over Weibull fading channels in the presence of Weibull distributed CCI was studied. Closed-form expressions for the PDF and CDF of the instantaneous SIR at the output of SC combiner were

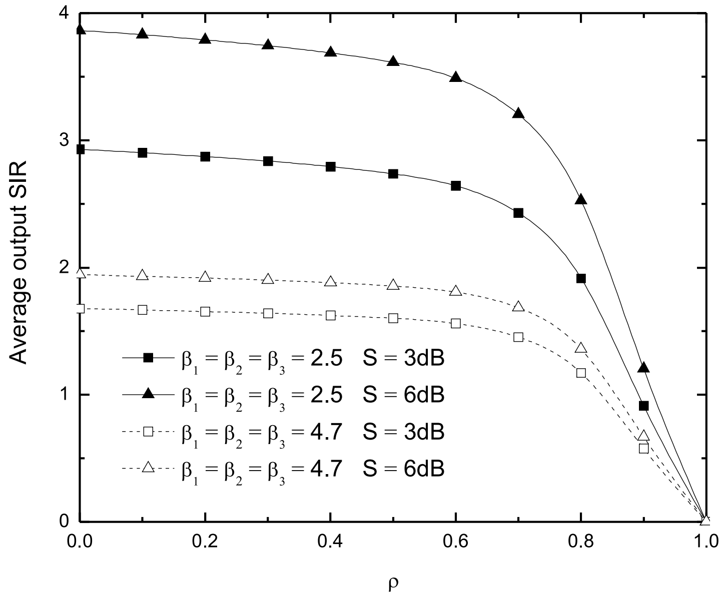

Fig. 4. Average output SIR as a function of the correlation coefficient.

presented and applied to analyze system performance measures, such as the outage probability, ABEP for several modulation schemes and average output SIR. Various numerical results of these performance measures are presented, describing their dependence on the average input SIR, correlation coefficient, fading severity and modulation format.

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