Non-uniform Threshold as an Alternative to Uniform Threshold in Denoising in Wavelet Domain

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Abstract: In this paper we present the advantage of non-uniform over uniform threshold wavelet shrinkage denoising method, applied on noisy signals with signal dependent noise. We illustrate our results by comparing the noise energy after using the both filtration methods on the same set of artificially noise contaminated images. The experiments are made with NPR-QMF filter banks instead with the filter banks that are commonly used in wavelet applications.

Keywords: Denoising, filter bank, signal-dependent noise, threshold, wavelet domain filtering.

1 Introduction

L ATELY, there are many developed methods for image noise filtration in a transformation domain [1–8]. In the last decade the stress on researches in this field is put on the signal processing in the wavelet domain.

The reason of using the wavelet transform for denoising purposes is that adequately chosen wavelet basis groups the coefficients in two groups - one with a few coefficients with high SNR, and other with a lot of coefficients with low SNR. In case of white Gaussian noise, the noise level is same through whole signal and for all the wavelet coefficients, independently on the signal. So, choosing a global threshold shrinks all the coefficients for an equal portion. But, in some signals,

like nuclear medicine (NM) images, the noise level is proportional to the local signal intensity. Obviously, denoising them with a global threshold is not the best solution.

In this paper we present results obtained by using our non-uniform threshold shrinkage method for removal of signal-dependent noise. We illustrate that noise energy in the filtered signal is bigger when any global threshold is used compared to the case when the proposed non-uniform threshold is used. We disclose some results of denoising of standard test images when our method and known methods are used. The paper is organized as follows. The method uses standard wavelet filtering outlined in Section II. In Section III we discuss how to estimate the varying threshold. In Section IV we verify the validity of our approach on deterministic signals contaminated with signal dependent noise. At the end, Section V concludes the paper.

2   Wavelet Shrinkage Method

The most popular form of conventional wavelet-based signal filtering [9], can be expressed by:

$$
\begin{aligned}
\{\mathbf{A}^{(k)},\mathbf{D}^{(1)},\mathbf{D}^{(2)},\dots,\mathbf{D}^{(k)}\} &= \mathrm{DWT}(\mathbf{s}+\mathbf{n}) \\
\mathbf{s}^{*} &= \mathrm{IDWT}(f(\mathbf{A}^{(k)},\mathbf{h}^{(1)}.\times\mathbf{D}^{(1)},\mathbf{h}^{(2)}.\times\mathbf{D}^{(2)},\dots,\mathbf{h}^{(k)}.\times\mathbf{D}^{(k)})),
\end{aligned}
\tag{1}
$$

where $\mathbf{s}$ is noise-free signal, $\mathbf{n}$ is noise, $\mathbf{s}^{*}$ is filtered signal, $\mathbf{A}^{(k)}$ and $\mathbf{D}^{(k)}$ are approximation and detail coefficients at level k, respectively, f is a function of the modified detail and approximation coefficients, $.\times$ is element-by-element multiplying and

$$
\mathbf{h}^{(k)} = [h_1^{(k)},h_2^{(k)},\dots,h_j^{(k)}]^T
$$

are weighting coefficients of the corresponding detail coefficients at level k.

In case of conventional hard threshold filtering the weighting coefficients are

$$
h_j^{(k)}(hard) = \begin{cases} 1, & \text{if } |D_j^{(k)}| > \tau^{(k)} \\ 0, & \text{otherwise} \end{cases},
\tag{2}
$$

while for the soft threshold filtering they are

$$
h_j^{(k)}(soft) = \begin{cases} 1 - \dfrac{\tau^{(k)}\mathrm{sgn}|D_j^{(k)}|}{D_j^{(k)}}, & \text{if } |D_j^{(k)}| > \tau^{(k)} \\ 0, & \text{otherwise} \end{cases},
\tag{3}
$$

where $\tau^{(k)}$ is user specified threshold for the k-th level details.

Having in mind that the noise is proportional to the local signal intensity, instead of using a global threshold $\tau^{(k)}$ in (3), we propose using a user specified varying threshold, $\tau_j^{(k)}$, that depends on the details position j.

3 Non-uniform Threshold determination

The approximation coefficients contain the low frequency part of the signal. Therefore one could say that the information part or "identity" of the signal is contained in the approximation. So, if we assume that the noise is proportional to the local signal intensity at certain level k, then the non-uniform threshold vector should be expressed as

$$\tau^{(k)} = \alpha |\mathbf{A}^{(k)}|, \tag{4}$$

where $\mathbf{A}^{(k)}$ is matrix with the approximation coefficients at level k, and α is a constant which should be determined by equalizing of the energy of the modified vectors of the approximation and the detail coefficients, $\mathbf{A_n}^{(k)}$ and $\mathbf{D_n}^{(k)}$, respectively, by using the following formulae:

$$\sum_i (D_{n,i}^{(k)})^2 = \sum_i (\alpha A_{n,i}^{(k)})^2. \tag{5}$$

The modification of the detail coefficients $D_n^{(k)}$ is obtained from $D^{(k)}$ by using the following reasoning. Since $D^{(k)}$ contain the high part of the spectrum of the original signal, its coefficients frequently change their polarity with the position (time). Whenever the change of the polarity of two consecutive local extremes (peeks) occur, one can discard all the detail coefficients on the positions between those peaks (due to their negligible magnitudes) in order to obtain $D_n^{(k)}$ from $D^{(k)}$.

Hence, $\mathbf{D_n}^{(k)}$ is a vector identical to $\mathbf{D}^{(k)}$ on all positions j, except on the positions between the consecutive peeks with opposite polarities in $\mathbf{D}^{(k)}$ where $\mathbf{D_n}^{(k)}$ is zero, while vector $\mathbf{A_n}^{(k)}$ is constructed by using

$$\mathbf{A_n}^{(k)} = \mathbf{A}^{(k)} . \times \text{sign}(\mathbf{D_n}^{(k)}). \tag{6}$$

Since the coefficients $\mathbf{D_n}$ and $\alpha \mathbf{A_n}$ have equal energy, it holds that for the positions j, for which $|D_{n,j}| > \alpha |A_{n,j}|$ the signal is less noise contaminated, and where $|D_{n,j}| < \alpha |A_{n,j}|$ the signal is more noise contaminated.

In general, we can assume that for noise stands polynomial dependence on the local signal intensity, hence, for the threshold τ at some level k the following can be written:

$$\tau_j^{(k)} = \alpha^{(k)} \cdot [1 A_j^{(k)} (A_j^{(k)})^2 \cdots (A_j^{(k)})^n]^T, \qquad j = 0, \cdots L-1, \qquad (7)$$

where $\alpha^{(\mathbf{k})} = [\alpha_0^{(k)} \alpha_1^{(k)} \ldots \alpha_n^{(k)}], L$ is the length of the vectors $\mathbf{A}$ and τ. The coefficients $\alpha_0, \alpha_1, \cdots$ can be obtained by minimizing:

$$E^{(k)} = \frac{1}{2} \sum_j (|D_{n,j}^{(k)}| - \tau_j^{(k)})^2 \qquad (8)$$

in the smallest squares sense.

4   Experimental Results

In this Section, we illustrate the effects of denoising the artificially contaminated signals (images) by applying the conventional shrinkage methods and our proposed non-uniform threshold approach. The noise energy in the filtered signal is higher when any global threshold is used compared to the case when the proposed non-uniform threshold is used.

The noise contaminated images are generated by superpositioning of shifted 2D random Gaussian functions (centered at position (i, j)) with energies proportional to the pixel intensities at position (i, j) in the noise-free images.

By applying of the conventional and proposed method we obtain filtrated images $\mathbf{s_1}$ (normalized to the energy of the noise free images $\mathbf{s}$), and compare with the energy of the noise free images by using the following formula

$$E_n = \sum_{i,j} (s_{i,j} - s_{1\,i,j})^2. \qquad (9)$$

When the signal in Fig. 1a is filtered by using the proposed method we obtained 1586 for the noise energy E_n. The proposed algorithm uses NPR-QMF filters with length 12, stopband frequency 0.7π, and overall reconstruction error of the designed QMF bank 0.001 [10]. In addition, we filtered the signal by using standard technique of wavelet shrinkage [9] and used different wavelets and different values of the uniform threshold τ. The graph for dependence of E_n on τ for values of τ between 0 and maximal intensity in the detail coefficients is plotted in Fig. 2a. The threshold value $\tau = 0$ means that all the detail coefficients are kept, while the value $\tau = 1$ (which corresponds to a threshold equal to the maximal intensity in the detail coefficients) means that all the detail coefficients are discarded. From Fig. 2a it can be noticed that for any value of the uniform threshold, the energy of the remained noise is not smaller than 1586. This comes from the fact that using a uniform threshold for removing signal-dependent noise is not an adequate solution.

(a)

(b)

(c)

(d)
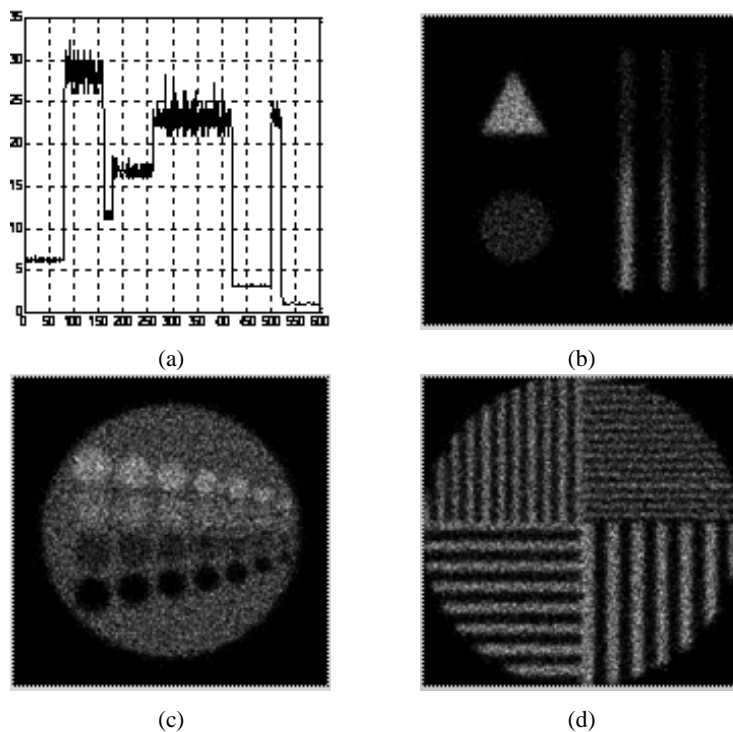
Fig. 1. Deterministic test noisy signals.

Similar results are presented in Fig. 2b. The graph shows dependence of E_n on τ after applying operation of variance normalization [1] on the images before they are filtered by using standard wavelet shrinkage.

(a)

(b)

Fig. 2. (a) Dependence of the noise energy E_n on the uniform threshold τ for the test signal in Fig. 1; (b) Dependence of the noise energy E_n on τ after the variance stabilizing operation is applied.

(a)

(b)

(c)

Fig. 3. Noisy images.

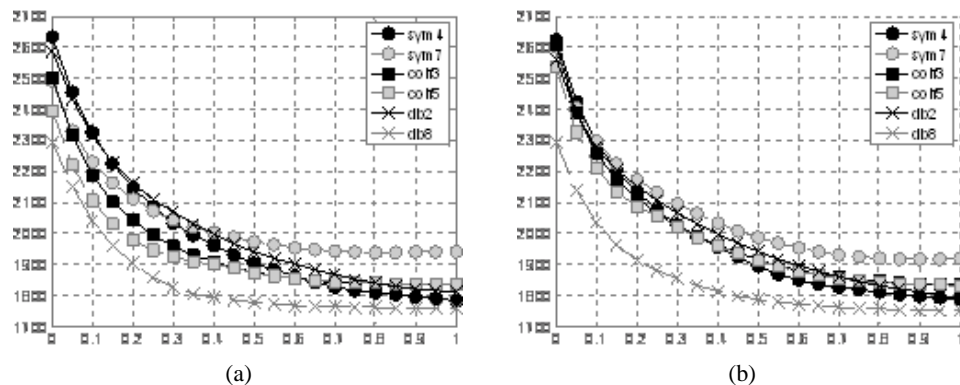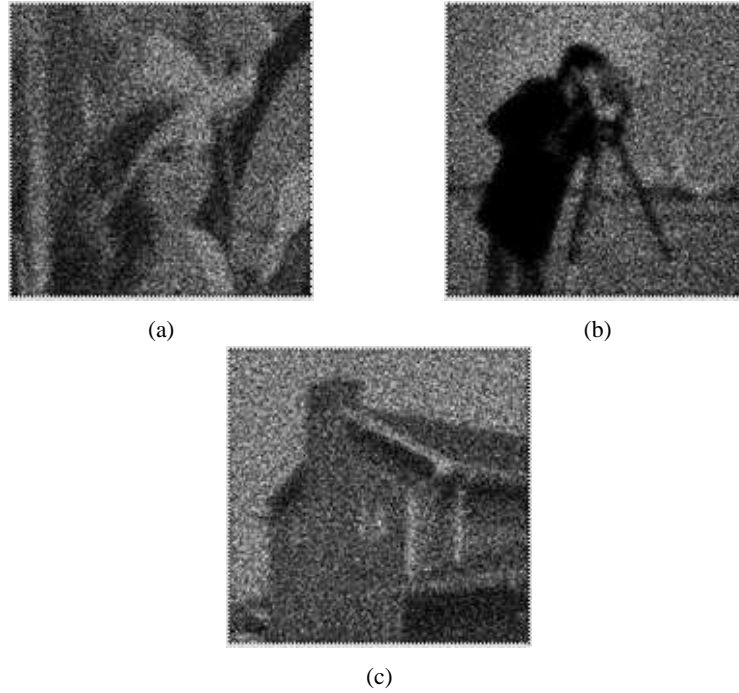Table 1. Comparison of the proposed with known methods in case of true signal estimating in the test images in Fig. 1.

Image	SNR_1	Wavelet	ΔSNR Known methods								ΔSNR Proposed algorithm	
			Visu Shrink [2] soft/hard	Sure Shrink [4]	Bayes Shrink [6]	Press [5]	Var. stab. [1]	Xu-Weaver [3]	Bi Shrink [7]	Prob Shrink [8]	Energy equalizing soft/hard	LS minimization soft/hard
Phantom	2.92dB	sym3	1.19/0.13	0.28	0.89	0.86	1.30	2.03	0.37	0.51	**2.47**/**1.71**	2.38/1.52
		sym5	1.19/0.14	0.28	0.88	0.86	1.29	1.79	0.33	0.50		
		db3	1.19/0.13	0.28	0.89	0.86	1.30	2.01	0.37	0.51		
		db5	1.19/0.10	0.28	0.73	0.86	1.18	1.98	0.37	0.44		
Circles	4.38dB	sym3	4.00/1.65	0.21	4.08	1.35	3.50	3.36	1.14	2.04	**4.46**/**2.67**	4.22/2.33
		sym5	3.97/1.55	0.21	4.06	1.34	3.49	2.98	1.00	1.98		
		db3	4.00/1.65	0.21	4.08	1.35	3.50	3.32	1.13	2.04		
		db5	3.81/1.19	0.21	4.07	1.34	3.38	3.22	1.08	1.75		
Bars	4.38dB	sym3	2.71/**2.02**	0.17	2.14	0.99	2.61	2.24	1.23	1.87	**2.85**/1.89	2.73/1.68
		sym5	2.71/**2.00**	0.17	2.20	1.00	2.62	2.03	1.07	1.85		
		sym7	2.74/**1.97**	0.17	2.22	1.00	2.64	2.20	1.18	1.84		
		db3	2.71/**2.02**	0.17	2.14	0.99	2.61	2.22	1.23	1.87		
		db6	2.74/**2.04**	0.17	2.21	0.99	2.64	2.22	1.24	1.88		
		coif3	2.74/**1.96**	0.17	2.25	1.00	2.64	2.19	1.19	1.84		
		coif5	2.74/**1.89**	0.17	2.30	1.00	2.66	2.19	1.17	1.80		
		bior9/7	2.70/**1.96**	0.17	2.19	1.00	2.60	2.14	1.18	1.85		

Further, we made experiments with the images in Fig. 1 and Fig. 3. They both contain signal-dependent noise with rather low SNR. The images in Fig. 1 are standard nuclear medicine test images, while the images in Fig. 3 are well known test images commonly used for comparing performances of different image processing

techniques. The maximal intensity in all three images in Fig. 1 is 22. The performances of the applied filtration methods obtained with various wavelet-based filtering methods, are presented in Table 1 in which SNR_1 is signal-to-noise ratio for the generated images while ΔSNR is the improved signal-to-noise ratio (after the filtering). When the proposed method is used with two differently generated thresholds (the last columns) it can be noticed that the filtering with non-uniform threshold determined through energy equalizing (Eq. 5) gives better results compared to the filtering with non-uniform threshold determined through LS minimization of the square measure (Eq. 8).

The results of filtering the images in Fig. 3 are shown in Table 2. They are similar to the results in Table 1. From both Table 1 and Table 2 the advantage of the non-uniform threshold shrinkage over the uniform threshold shrinkage is evident.

Table 2. Comparison of the proposed with known methods in case of true signal estimating in the images in Fig. 3

Image	SNR$_1$	Wavelet	Visu Shrink [2] soft/hard	Bayes Shrink [6]	Press [5]	Var. stab. [1]	Xu-Weaver [3]	Bi Shrink [7]	Prob Shrink [8]	Energy equalizing soft/hard	LS minimization soft/hard
			ΔSNR Known methods							ΔSNR Proposed algorithm	
Lena	5.27dB	sym3	5.00/**4.68**	2.81	1.49	4.38	3.72	2.13	3.83	**5.10**/3.04	4.81/2.63
		sym5	4.97/**4.69**	2.80	1.49	4.35	3.28	1.82	3.80		
		db3	5.00/**4.68**	2.81	1.49	4.38	3.67	2.13	3.83		
		coif5	4.98/**4.70**	2.80	1.49	4.33	3.47	1.88	3.80		
		bior9/7	4.90/**4.62**	2.79	1.47	4.27	3.61	1.96	3.75		
House	5.76dB	sym3	5.22/**4.95**	2.94	1.56	4.52	3.89	2.11	3.99	**5.29**/3.17	4.99/2.74
		sym5	5.14/**4.87**	2.89	1.56	4.47	3.39	1.80	3.96		
		db3	5.22/**4.95**	2.94	1.56	4.52	3.87	2.11	3.99		
		coif5	5.14/**4.84**	1.54	4.45	3.62	1.87	2.11	3.94		
		bior9/7	5.07/**4.72**	2.87	1.54	4.39	3.69	1.94	3.88		
Camera	5.32dB	sym3	4.58/**3.99**	3.19	1.42	4.00	3.48	1.69	3.27	**4.71**/2.86	4.46/2.50
		sym5	4.59/**4.07**	3.16	1.43	4.02	3.08	1.47	3.30		
		db3	4.58/**3.99**	3.19	1.42	4.00	3.46	1.68	3.27		
		coif5	4.58/**4.13**	3.11	1.43	4.02	3.23	1.49	3.49		
		bior9/7	4.54/**3.97**	3.15	1.42	3.96	3.33	1.58	3.26		

5 Conclusion

In this paper we compare non-uniform and uniform threshold filtering methods on denoising artificially noised deterministic test images. Experimental results show that for the signal-dependent noise filtering with non-uniform threshold outperforms uniform threshold filtering for any level of the threshold and all used wavelets we have experimented with.

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