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A NEW APPROACH FOR THE SS7 LEVEL-2 STEADY-STATE PERFORMANCE ANALYSIS

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Abstract In this paper, a new approach for arrival process characterization in the SS7 level-2 transmission protocol is presented. The SS7 level-2 protocol messages flow enables a very good aproximation by analytical model based on the continuous batch Markovian arrival process (BMAP). This analytical model generalize the Poisson assumptions. On this way, accurate predictions of message flows in the SS7 level-2 transmission protocol are achieved. Further, we analyse the SS7 level-2 protocol by way of the BMAP/G/1 queueing system. Using matrix analytic methods we determine actions for steady-state occupancy level computation of the SS7 level-2 buffers. The stationary queue length statistic calculation in SS7 level-2 transmission protocol at service completion times and at an arbitrary time is outlined in this paper

Key words: Transmission protocol, SS7 level-2 protocol, Markovian arrival process, Poisson assumtion.

1. Introduction

The Common Channel Signaling (CCS) network represents the latest step in the evolution of signaling systems for the Public Switched Telephone Network (PSTN). The signaling system uses the Signaling System No.7 (SS7), [1], protocol over its signaling links for transfer of signaling messages between exchanges, or other nodes in the telecommunications network, served by the system. SS7 defines three types of messages: MSUs (message signal units), LSSUs (link status signal units) and FISUs (fill-in signal units). MSUs carry application-specific signaling information and LSSUs carry link status information. FISUs are transmitted when there are no MSUs and LSSUs to transmit. Except in some circumstances, the traffic impact of LSSUs is negligible, and we shall ignore them.

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The Signaling System No.7 uses a layered protocol similar to the OSI protocol. The link-level (or level-2) basic error correction method is the well-known "go-back-N" protocol. Two buffers for keeping of signal units (except FISUs) need to be sent, is defined in the SS7 level-2 protocol. Basically, the protocol operates as follows: Whenever a message is sent out a communication link (from transmit buffer, TB), it is saved in a retransmit buffer, RB, (Fig. 1). The message is held in this buffer until a positive acknowledgment (+ve ack) is received indicating that the message was correctly received on the other end. If instead a negative acknowledgment (nack) is received then that message and all others that were transmitted after it are retransmitted. The retransmit buffer is of limited size ($K = 128$ messages as specified in [1]) and no new transmissions can be started if the retransmit buffer is full.

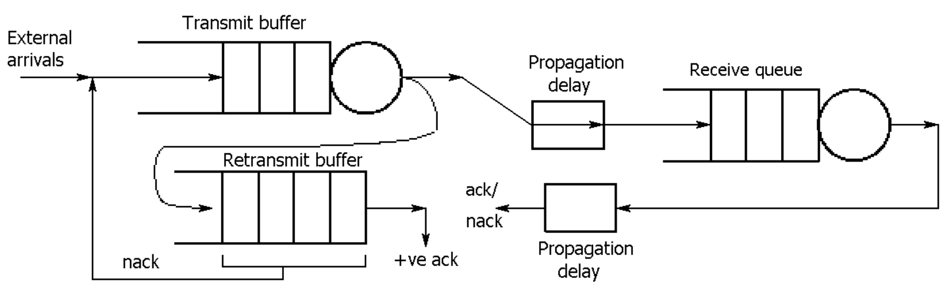

Fig. 1. An illustration of the SS7 level-2 protocol.

Accurate predictions of messages arrivals into SS7 level-2 transmit buffer are essential to efficient evaluate the SS7 level-2 protocol performance measures (for instance, waiting time or queue length distributions). The Poisson input process (interarrival time is exponential and orderliness of arrivals) are assumed by existing queueing models for performance evaluation of the SS7 level-2 protocol [1], [2]. However, can be see, exactly non-exponential interarrival time and batch (or bulk) arrivals (upon negative ack receive) are a natural features of the SS7 level-2 messages flow.

From this reasons, I propose a new approach for the arrival process characterization in SS7 level-2 transmission protocol, which is based on the continuous batch Markovian arrival process (BMAP). Based on such defined arrival process, the steady-state analysis of the SS7 level-2 protocol is accomplished. Further, a computation method for obtaining of steady-state queue length statistics in the SS7 level-2 protocol (i.e. steady-state TB and

RB occupancy level statistics) at MSU emission completion time and at an arbitrary time is presented. This is done by the stationary probability matrices (Π) and (Ψ) calculation. From these matrices is simple calculated the mean values and probability distribution functions for the steady-state TB and RB occupancy level.

2. The arrival Model Definition

2.1 Mathematical model

Let $N(t)$ be the number of messages in transmit buffer up to time t and $J(t)$ be the number of messages in retransmit buffer at time t. Obviously, $N(t)$ is the counting variable and $J(t)$ is the phase variable, on the state space $\{(i,j) : i \geq 0, 0 \leq j \leq K\}$. Then the batch Markovian arrival process (BMAP) $\{N(t), J(t) : t \geq 0\}$ is a 2-dimensional Markov process whose generator matrix is given by

$$Q = \begin{bmatrix} D_0 & D_1 & D_2 & D_3 & \dots & D_K & \dots \\ & D_0 & D_1 & D_2 & \dots & D_{K-1} & \dots \\ & & D_0 & D_1 & \dots & D_{K-2} & \dots \\ & & & D_0 & \dots & D_{K-3} & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \end{bmatrix}$$

(empty entries shall represent the zero matrix).

D_k, $k \geq 0$, are $K \times K$ non-negative matrices, and D_0 has negative diagonal elements and non-negative off-diagonal elements. D_k basically represents the transitions among the phases when the batch of size k arrives. The generator of the underlying Markov process can be given by

$$D = \sum_{i=0}^{K} D_i \tag{1}$$

The matrix D_0 governs the transitions at the same level, between phases without any arrival in the level-2 transmit buffer, while the D_k governs the transitions between the phases (the number of messages in retransmit buffer) of level i and level $(i+k)$ due to the arrival of batch of size k. That BMAP has the following parametre matrices:

$$D_0 = \begin{bmatrix} -[\lambda+\delta(0)] & \delta(0) & 0 & \cdots & 0 \\ \gamma_p(1) & -[\gamma_a(1)+\delta(1)+\lambda] & \delta(1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & -[\gamma_a(k)+\delta(k)+\lambda] & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \gamma_p(K) & -[\gamma_a(K)+\lambda] \end{bmatrix}$$

and,

$$D_1 = \begin{bmatrix} \lambda & 0 & \cdots & 0 \\ \gamma_n(1) & \lambda & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda \end{bmatrix}, D_k = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_n(k) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \leftarrow k\text{th row}(2 \leq k \leq K)$$

where λ is external arrival rate of MSU messages into level-2 transmit buffer, $\gamma_a(i)$ is ack rate when i messages are in level-2 retransmit buffer, $\gamma_p(i)$ is positive ack rate when i messages are in level-2 retransmit buffer, $\gamma_n(i)$ is negative ack rate when i messages are in level-2 retransmit buffer and $\delta(i)$ is increment rate of the number of messages are in level-2 retransmit buffer.

2.2 Computation of entries in the parametre matrices

$[D_0]_{i,i-1} = \gamma_p(i)$ and $[D_k]_{i,0} = \gamma_n(i)$ $(1 \leq i, k \leq K)$. Given a ack rate when i messages are in level-2 retransmit buffer ($\gamma_a(i)$), the corresponding positive ack rate can be obtained as $\gamma_p(i) = (1-p_m)\cdot\gamma_a(i)$ where p_m denotes the message error ratio. Similary, we get $\gamma_n(i) = p_m \cdot \gamma_a(i)$. A simple way to compute $\gamma_a(i)$ is by using Norton's theorem. That is, we consider a closed queueing network of two stations. The first station representing the round-trip propagation delay (t_L). $t_L = 2\cdot T_p$, where T_p is data channel propagation time, [1].

The second station representing the receive time (t_R) plus the emission time at the receiving end of the CCS No7 signaling link. Here, the receive time is concerned with only the level-2 functionality (CRC checks and ack/nack generator) and will take less than $1ms$ per message, [2]. In SS7, the acks/nacks are embedded in regular messages and their processing causes negligible overhead. SS7 also allows for block acknowledgement of messages, but its effect is only to improve performance and, hence, can be ignored in our analysis. We shall assume the second station to be a single server with exponentially distributed service times. Then, $\gamma_a(i)$ is simply the throughput of this network.

Let is i messages in that closed network and $r(i)$ is expected "round-trip time" of a message between visits to the same station. $r(i) = w1(i) + w2(i)$, where are $w1(i)$ and $w2(i)$ mean waiting time at first and second stations respectively.

$$w1(i) = t_L^{-1} \qquad \text{and} \qquad w2(i) = d2(i) + (t_R + \bar{s}_R)^{-1} \tag{2}$$

where $d2(i)$ is denote the expected delay of an arrival at second station and $\bar{s}_R$ is the mean emission time at the receiving end of the signaling link. From Norton's theorem, [3], we also have $d2(i) = L2(i-1)/(t_R + \bar{s}_R)$ (where $L2(i)$ is the expected number of messages at second station). The throughput, respectively, ack rate when i messages are in level-2 retransmit buffer is

$$\gamma_a(i) = i/r(i) \tag{3}$$

and of course, $L2(i) = \gamma_a(i)w2(i)$. This relation enables to define the recursion procedure for $\gamma_a(i)$ calculation.

$[D_0]_{i,i+1} = \delta(i)$ is the transition rate from phase i (i messages are in level-2 retransmit baffer) to phase $i+1$ without any new arrival in level-2 transmit buffer (named, the retransmit buffer increment rate). This rate is equal $p(i) \cdot \mu$, where is μ the mean emission (service) rate (at the sending end) and $p(i)$ is probability of non-empty transmit buffer when i messages are pending in level-2 retransmit buffer. This probability is equal the efective link load, $p(i) = \rho_e(i) = \lambda_e(i)/\mu$, $(0 \le i \le K)$, when $\lambda_e(i) \le \mu$, or $p(i) = 1$ for $\lambda_e(i) > \mu$. Then,

$$\delta(i) = \begin{cases} \lambda, & \text{if } \ i = 0 \\ \min(\lambda_e(i), \mu), & \text{otherwise} \end{cases} \tag{4}$$

Effective arrival rate into level-2 transmit buffer we get from a simple flow balance:

$$\lambda_e(i) = \lambda + \gamma_n(i)(1 + r(i)\lambda_e(i)) \tag{5}$$

The number of messages put into the level-2 transmit buffer when a nack arrives (when i messages are pending in level-2 retransmit buffer) is the message in question plus the messages transmitted during its acknowledgemen delay ($r(i)$). This gives the $1 + r(i)\lambda_e(i)$ factor. With this, computation process of all non-null elements of the D_k ($k \geq 0$) matrices is completed.

Since, the matrix D in (1) is an irreducible generator matrix, it has got a stationary vector π, which is the unique solutions to the following set of equations

$$\pi D = 0, \qquad \pi e = 1$$

where e is a column vector of 1's. The vector d whose j-th component is the conditional arrival rate into level-2 transmit buffer, which starts with the arrival process in phase when j messages are in level-2 retransmit buffer, is given by

$$d = \sum_{k=1}^{K} k D_k e$$

Averaging over the phases, the effective (or, fundamental) arrival rate, λ_{eff} can be given by,

$$\lambda_{eff}^{-1} = \pi d \tag{6}$$

The results obtained using (6) for effective link load ($\rho_{eff} = \lambda_{eff}/\mu$), are almost the same as results obtained according to effective link load calculation method from [4].

3. The BMAP/G/1 Analysis of the SS7 Level-2 Protocol

3.1 The embedded Markov process

If we observe $\{X(t), J(t) : t \geq 0\}$ at MSU emission completion times T_ν ($\nu \geq 0$), only, we obtain a discrete time-homogeneous Markov chain with transition probability matrix

$$\tilde{P}(x) = \begin{bmatrix} \tilde{B}_0(x) & \tilde{B}_1(x) & \tilde{B}_2(x) & \tilde{B}_3(x) & \dots & & \\ \tilde{A}_0(x) & \tilde{A}_1(x) & \tilde{A}_2(x) & \tilde{A}_3(x) & \dots & & \\ & \tilde{A}_0(x) & \tilde{A}_1(x) & \tilde{A}_2(x) & \dots & & \\ & & \tilde{A}_0(x) & \tilde{A}_1(x) & \dots & & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \end{bmatrix} \tag{7}$$

where $\tilde{A}_n(x)$ and $\tilde{B}_n(x)$ ($n \geq 0$) are the $K \times K$ matrices of mass functions defined by

$$[\tilde{A}_n(x)]_{ij} = P\{N(T_{\nu+1} - T_\nu) = n, J(T_{\nu+1}) = j, \\ T_{\nu+1} - T_\nu \leq x | X(T_\nu) > 0, J(T_\nu) = i\}$$

and

$$[\tilde{B}_n(x)]_{ij} = P\{N(T_{\nu+1} - T_\nu) = n+1, J(T_{\nu+1}) = j, \\ T_{\nu+1} - T_\nu \leq x | X(T_\nu) = 0, J(T_\nu) = i\}.$$

Futher, we define the stationary shape of these matrices $A_n = \tilde{A}_n(\infty)$, $B_n = \tilde{B}_n(\infty)$ $(n \geq 0)$ and $P = \tilde{P}(\infty)$. Beside of these matrices, nacessary is to determine one more matrix. Namely, the fundamental matrix G plays the key role in determining the steady-state distribution in the embedded Markov chain given by (7). In the context of the our queue problem, matrix G governs the number transmited MSUs during, and the duration of, the TB non-empty period.

3.2 Remarks on the matrices A_n, B_n and G computation

(a) The message service (emission) time distribution depends on the link capacity (C, which is a constant) and the message length distribution. In reality, we can find the service time distribution ($H(.)$) by observing and measuring the $H(.)$ as a set of $p(k)$ and $d(k)/C$, $(1 \leq k \geq N_{msu})$. $d(k)$ is the MSU length of type k MSU and $p(k)$ is the probability of observing a MSU of type k. N_{msu} is the number of MSU types.

(b) The matrix G computation is based on the key theorem for the fundamental period in the BMAP/G/1 queue, [5], which readily implies that

$$G = \int_0^\infty e^{D[G]x} dH(x) \tag{8}$$

in this relation $D[G] = \sum_{j=0}^\infty D_j G^j$ and may be computed using Horner's algorithm, [6]. Then from (8) and the remark (a), we see that G satisfies

$$G = \sum_{i=1}^{N_{msu}} p(i) e^{D[G]d(i)/C} \tag{9}$$

In this case, any routine which efficiently computes the matrix exponential can be used for the iteration implied by (9), starting with $G_0 = 0$.

(c) An efficient algorithm for computing the A_n's and B_n's may be found in [7]. This algorithm involves the numerical integration of the some scalar quantities. From the remark (a) numerical integrations may be avoided.

4. The Steady-State Occupancy Level of SS7 Level-2 Buffers

4.1 Probability statistics at message emission completion times

The distribution of TB and RB occupancy level at MSU emission completion times is the stationary distribution of the discrete Markov chain $\{X(T_\nu), J(T_\nu) : \nu \geq 0\}$. We are define $[\Pi]_{i,j}$ as the steady-state probability that number of MSUs in TB is equal i and number of MSUs in RB is equal j at MSU emission completion times, i.e.

$$[\Pi]_{i,j} = \lim_{\nu \to \infty} P\{X(T_\nu) = i, J(T_\nu) = j | X(T_0) = l, J(T_0) = k\}$$

for all $i, l \geq 0$ and $j, k = 0, 1, ..., K$. The entries of Π are calculate by following recursion

$$[\Pi]_{i,-} = \left[[\Pi]_{0,-} \bar{B}_i + \sum_{k=1}^{i-1} [\Pi]_{k,-} \bar{A}_{i+1-k} \right] (I - \bar{A}_1)^{-1}$$

where $[\Pi]_{i,-}$ $(i \geq 1)$ denote ith row of the Π matrix. Matrices $\bar{A}_v = \sum_{i=v}^{\infty} A_i G^{i-v}$ and $\bar{B}_v = \sum_{i=v}^{\infty} B_i G^{i-v}$ $(v \geq 0)$ can be implemented efficiently by choosing a large index k, and setting $\bar{A}_k$ and $\bar{B}_k = 0$. The other required matrices can be computed by following backward recursions: $\bar{A}_i = A_i + \bar{A}_{i+1} G$ and $\bar{B}_i = B_i + \bar{B}_{i+1} G$ for $i = k-1, k-2, ..., 0$.

So it only remains to compute the row $[\Pi]_{0,-}$. First we note that $[\Pi]_{0,j}$ is reciprocal of the mean recurrence time of the state $(0, j)$ in the Markov chain (7). Now we consider our Markov chain at its visit to level 0 only, i.e. we exclude the busy periods. So we obtain a discrete Markov chain, whose transition probability matrix will be denoted by S and a probability vector s such that $sS = s$. Now, the vector s^*, whose jth component gives the mean number of transitions between two consecutive visits to level 0, if the last state visited in level 0 was $(0, j)$. Therefore $s_j s_j^* / s_i$, is the mean number of transitions between successive visits to state i given that it visits state j in between. The steady-state probability of the state j can be given by $[\Pi]_{0,j} = s_j / ss^*$, therefore the row $[\Pi]_{0,-}$ can be given by

$$[\Pi]_{0,-} = \frac{s}{ss^*}$$

We only need to determine the matrix S and the vector s^* (for this see, e.g., [6]).

From the matrix Π by simple computations we get folloving steady-state statistics at message emission completion times:

(a) the mean value of the TB occupancy level (in numbers of MSUs), $E(X) = \sum_{i=0}^{\infty} i \sum_{j=0}^{K} [\Pi]_{i,j}$,

(b) the mean value of the RB occupancy level, $E(J) = \sum_{j=0}^{K} j \sum_{i=0}^{\infty} [\Pi]_{i,j}$,

(c) the mean value of the TB+RB occupancy level, $E(X+J) = \sum_{i=0}^{\infty} \sum_{j=0}^{K} (i+j)[\Pi]_{i,j}$.

On the same way, we are obtained corresponding steady-state probability distributions

$$F_X(x) = \sum_{i=0}^{x} \sum_{j=0}^{K} [\Pi]_{i,j}, \quad F_J(x) = \sum_{i=0}^{\infty} \sum_{j=0}^{x} [\Pi]_{i,j}, \quad F_{X+J}(x) = \sum_{i=0}^{x} \sum_{j=0}^{x-i} [\Pi]_{i,j}$$

4.2 Probability statistics at an arbitrary time

The probability distribution of TB and RB occupancy level at an arbitrary time is the stationary distribution of the Markov process $\{X(t), J(t) : t \geq 0\}$. We are define $[\Psi]_{i,j}$ as the steady-state probability that number of MSUs in TB is equal i and number of MSUs in RB is equal j at an arbitrary time, i.e.

$$[\Psi]_{i,j} = \lim_{t \to \infty} P\{X(t) = i, J(t) = j | X(0) = l, J(0) = k\}$$

for all $i, l \geq 0$ and $j, k = 0, 1, ..., K$. The components (rows) $[\Psi]_{i,-}$ $(i \geq 0)$ of Ψ can be obtained in terms of the sequence $\{[\Pi]_{i,-}\}$, by applying the key renewal theorem [5]. The component $[\Psi]_{0,-}$ is given by

$$[\Psi]_{0,-} = -\lambda_{eff}^{-1} [\Pi]_{0,-} D_0^{-1}$$

and $[\Psi]_{0,-} e = 1 - \rho$, as expected. For $i \geq 0$ the component $[\Psi]_{i,-}$ is given by

$$[\Psi]_{i+1,-} = \left[\sum_{j=0}^{i} [\Psi]_{j,-} D_{i+1-j} - \lambda_{eff}^{-1} ([\Pi]_{i,-} - [\Pi]_{i+1,-}) \right] (-D_0^{-1})$$

From the matrix Ψ, on the same way as above , can be obtained the steady-state statistics of the SS7 buffers occupancy level.

5. Numerical Example

Calculation of the stationary occupancy level of the SS7 level-2 buffers (TB and RB) is done for SS7 64kb/s link, with basic error correction method. Signaling traffic is characterized by two types of MSU (50% MSUs of 18 octets length and 50% MSUs of 54 octets length). External arrival rate of these MSUs into level-2 transmit buffer is $\lambda = 80 mess./s$. Signaling loop propagation time is $t_L = 30ms$. Fig. 2 shows the mean occupancy level (in number of MSUs) of the transmit buffer and the retransmit buffer against the signal unit error rate at MSU departure times. Fig. 3 shows the steady-state probability distribution functions of X and $X + J$ versus the signal unit error rate at MSU departure times. Comparison of obtained results (calculated from steady-state probabilities at MSUs departure times and an arbitrary times) is presented in Fig. 4. It can be seen that obtained results for entire stationary number of messages in the SS7 level-2 protocol, $(X + J)$, are very closely. However, exist certain a difference in distribution of these MSUs between SS7 buffers.

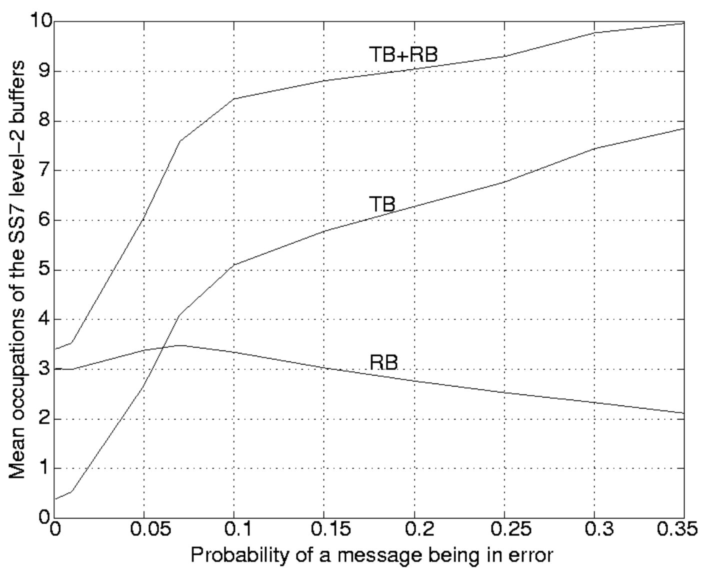

Fig. 2. Mean occupations of the level-2 buffers (in number of MSUs) against the signal unit error rate.

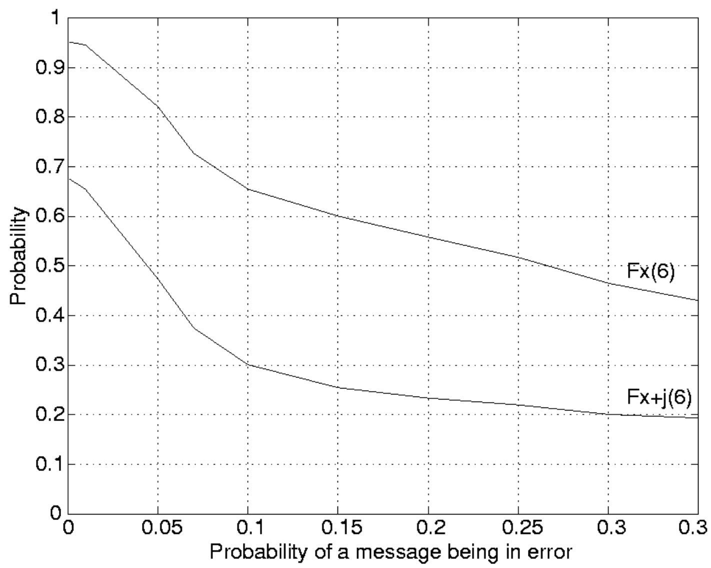

Fig. 3. Probability distribution functions of X and $X + J$ versus the signal unit error rate.

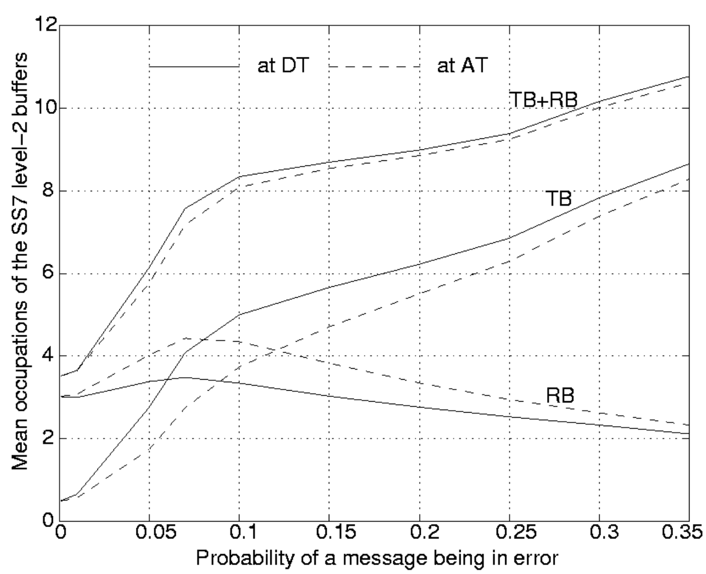

Fig. 4. The mean occupancy level of the SS7 level-2 buffers against the signal unit error rate, calculated at MSUs departure times (DT) and at an arrbitrary times (AT).

6. Conclusions

The arrival process defined as BMAP enables a very good aproximation of SS7 level-2 message flows (into/from) transmit and retransmit buffers. Note that, non-exponential interarrival time and batch (or bulk) arrivals (upon negative ack receive) are a natural features of SS7 level-2 function.

The arrival process defined on this way enables additionaly simplification of a procedure for SS7 level-2 protocol performance evaluation. When only the moments of the queue length are needed, then computation of the matrix G is sufficient. Note that this approach does not require the numerical evaluation and storage of the matrices A_n, [2], (M/G/1 approach). In this paper we developed analytic model that describe the steady-state queue length performance in the SS7 level-2 protocol. This model (BMAP/G/1) enables accurate predictions of the message flows on the SS7 level-2 protocol, and on that account the developed calculation method is very applicable in reality. Finally, the presented analytic model and obtained results may be used in analysis of the congestion control scheme in the signaling sistem No.7.

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