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A METHOD OF DETERMINATION THE STATISTICAL CHARACTERISTICS OF ELECTRO-MAGNETIC SHIELDING EFFECTIVENESS

Zoran M. Vranić

Abstract. Elementary theory of the quantiles of B-distribution statistical method and its application on the measurement results of electromagnetic shielding effectiveness (SE) is given. The six modular shielded en closures are tested in accordance with the SE 1851 (ASTM) standard. Results are presented in the form of the binary functional matrices. Result gives lower and upper confidence limits of the probability of success as a function of assigned confidence level, the sample size and number of defective enclosures in the sample test.

Key words: Statistical method, B-distribution, shielding effectivness.

1. Introduction

The main goal of the experimental SE verification of the shielded enclosures is to give a probabilistic estimation of whole series compliance with the required level of SE. There are two goals for such estimation. The first, based on the obtained estimation it is possible to make corrections in construction, or even redesign the shielded volume, if the number of defective items is unacceptable large. The second, high price of testing, can be reduced because it is not necessary to measure all items. It is very important if an enclosure in a series is properly evaluated, and using a test procedure is ensured that it is adequate for its intended purpose to give an estimation in advance about other ones. Probabilistic approach in this problem is unavoidable, because the numerical simulation methods have a rather limited range of applications [1], which primary originates from rapidly increasing difficulties solution comlexiti.

This paper contains five sections. After the introduction, in the Sec. 2 elementary theory of the quantiles of the Beta distribution (B-distribution) is given.

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The author is with the College for Postal Services and Telecommunications 11000 Belgrade, Zdravka Čelara 16 (e-mail: zokivran@ptt.yu).

A table with confidence limits as applicable result of the developed softwareis presented. Sec. 3 present the method of determination SE base on the procedure defined in SE 1851 ASTM (American Society for Testing and Materials) Standard. Explanations of some modifications of the above mentioned test method are given in the same section. Sec. 4. present the results of six modular shielded enclosures (MSENC) with respect to SE. The experimental data and test results are presented in the form of binary functional matrices (BFM), which contain the test frequencies and normalized attenuation at each test point. Finally, the last Ses. 5 contains some concluding remarks, especially related to incapability of using numerical computer-based models, to solve the problem of SE determination [2].

2. Theory of the Quantiles of Betha-Distribution

Experimental verification of attained SE of the Shielded Enclosure (success) is a random event A, and it has almost always a predetermined probability level of realization $p = P(A)$ which has to be

$$p = P(A) > p_0, \tag{1}$$

where p_0 is a minimal acceptable level of probability $p = P(A)$. The probability p is usually unknown, so it is only possible to give its estimated value $\hat{p}$ base on the experimental results

$$\hat{p} = f = \frac{m}{n}, \tag{2}$$

where $\hat{p}$ is estimated value of unknown probability $p = P(A)$, f is relative frequency of defective item appearance, n is sample size and m is number of the event A realization.

If the numbers m and n are known from the experimental results and if the $X_n = m$ is a discrete random variable defined on the set of possible values

$$X_n \in \{0, 1, 2, \ldots, n\}, \tag{3}$$

then the probability of realization of one element from discrete set of X_n is

$$P(X_n = m) = \binom{n}{m} p^m (1-p)^{n-m}; \quad 0 < p \leq 1. \tag{4}$$

It is evident from (4) that probability $P(X_n = m)$ only depends on random variable $p = P(A)$ and may be represented as follows

$$P_{n,m}(p) = \binom{n}{m} p^m (1-p)^{n-m}. \tag{5}$$

Using equation

$$\sum_{m=1}^{n} P_{n,m}(p) = 1, \tag{6}$$

the probability of the event A (success) realization for zero-time, one-time, ... , m-time becomes

$$S_{n,m}(p) = S(p) = \sum_{k=0}^{m} P_{n,k}(p). \tag{7}$$

Replacing (5) in the previous equation, it is obtained

$$S(p) = \sum_{k=0}^{m} \binom{n}{k} p^k (1-p)^{n-k}. \tag{8}$$

From equation (8) it is evident that $S(p)$ is continuous function of $p = P(A)$, and its derivation is given by

$$\frac{dS}{dp} = (\sum_{k=1}^{m} \frac{n!}{(n-k)!(k-1)!} p^{k-1} q^{n-k} - \frac{n!}{(n-k-1)!k!} p^k q^{n-k-1}) - nq^{n-1}. \tag{9}$$

Introducing the following notation

$$a_k = \frac{n!}{(n-k)!(k-1)!} p^{k-1} q^{n-k}, \tag{10}$$

$$b_k = \frac{n!}{(n-k-1)!k!} p^k q^{n-k-1}, \tag{11}$$

(9) may be written as follows

$$\frac{dS(p)}{dp} = \sum_{k=1}^{m} a_k - \sum_{k=1}^{m} b_k - nq^{n-1}, \tag{12}$$

$$\frac{dS(p)}{dp} = (a_1 + a_2 + \cdots + a_m) - (b_1 + b_2 + \cdots + b_m) - nq^{n-1}. \tag{13}$$

From equations (10) and (11) it is obtained

$$a_{k+1} = \frac{n!}{(n-k-1)!k!} p^k q^{n-k-1} = b_k; \quad k \in [1, m] \tag{14}$$

and introducing (14) into (13) it is yielded

$$\frac{dS(p)}{dp} = a_1 - b_m - nq^{n-1}. \tag{15}$$

For $k = 1$ and $k = m$ from (10) and (11), it may be written

$$a_1 = nq^{n-1}, \tag{16}$$
$$b_m = \frac{n!}{(n-m-1)!m!} p^m q^{n-m-1}. \tag{17}$$

Returning to Eqs. (16) and (17), introducing them into (15) it is obtained

$$\frac{dS(p)}{dp} = -\frac{n!}{(n-m-1)!m!} p^m q^{n-m-1}. \tag{18}$$

Since function $S(p)$ is continuous function its value can be determined [3], from (18) as follows

$$S(p) = \int_0^p \frac{-n!}{(n-m-1)!m!} p^m (1-p)^{n-m-1} dp. \tag{19}$$

After substitutions $p \to 1-p \quad \Longrightarrow \quad dp = -dp$, the equation for $S(p)$ becomes

$$S(p) = \int_0^{1-p} \frac{n!}{(n-m-1)!m!} p^{n-(m+1)} (1-p)^m dp. \tag{20}$$

Equation (20) can be simplified by introducing Γ-function

$$\begin{aligned} \Gamma((n-m)+(m+1)) = \Gamma(n+1) &= n!, \\ \Gamma(m+1) &= m!, \\ \Gamma(n-m) &= (n-m-1)!, \end{aligned} \tag{21}$$

In order to use equations (21), the integral (20) may be rewritten as indicated below

$$S(p) = \int_0^{1-p} \frac{\Gamma(n-m+m+1)}{\Gamma(n-m)\Gamma(m+1)} p^{n-(m+1)} (1-p)^m dp. \tag{22}$$

Thanks to the relation between Euler's functions of the first and the second type (B- and Γ-functions)

$$\frac{1}{B(x,y)} = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}, \tag{23}$$

the integral (22) becomes

$$S(p) = \int_0^{1-p} \frac{1}{B(n-m;m+1)} p^{n-(m+1)} (1-p)^m dp. \tag{24}$$

Introducing new parameters (a, b) and the variables (q, x)

$$a = n - m, \tag{25}$$

$$b = m + 1, \tag{26}$$

$$q = 1 - p, \tag{27}$$

$$x = p, \tag{28}$$

(24) becomes

$$S(p) = \int_0^{1-p} \frac{1}{B(a;b)} x^{a-1} (1-x)^{b-1} dx. \tag{29}$$

And finally,

$$S(p) = \int_0^{1-p} f(x) dx, \tag{30}$$

where

$$f(x) = \frac{1}{B(a;b)} x^{a-1} (1-x)^{b-1}. \tag{31}$$

The function $f(x)$ is a two-parameter B-function with parameters a and b or as it is expressed by (25) and (26), with parameters m and n. In order to use the presented solution, the confidence limits (lower $\tilde{p}_1$ and upper $\tilde{p}_2$) of unknown probability $p = P(A)$ has to be determined using auxiliary probability density function $f(x)$, which random variable x has corresponding confidence limits x_1 and x_2. Using the quantiles of B-distribution function, the confidence limits x_1 and x_2 may be determined (see Fig. 1a).

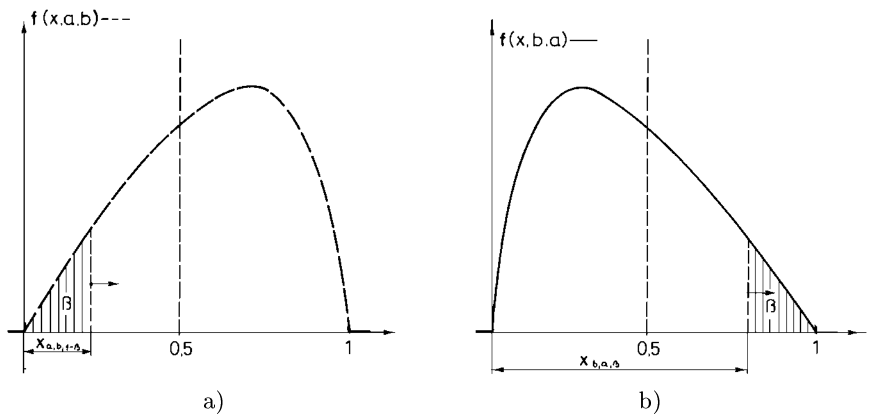

Fig. 1. a) Quantiles of B-distribution function$f(x; a, b)$.
b) Upper quintile of B-distribution function $f(x; b, a)$

As illustrated in Fig. 1, the lower and upper quntiles of B-distribution function $f(x;a,b)$ may be defined by the following relations:

$$\beta = \int_{x_{a,b,\beta}}^{1} f(x;a,b)dx = x_{a,b,\beta}, \tag{32}$$

$$1-\beta = \int_{x_{a,b,1-\beta}}^{1} f(x;a,b)dx = x_{a,b,1-\beta}. \tag{33}$$

In the tables of the inverse B-distribution function, only the upper quintile usually is given. To determine the lower quantile, it has to be used the symmetry of B-distribution functions $f(x;a,b)$ and $f(x;b,a)$ regard to straight line $x = 0.5$ (see Fig. 2a).

$$\beta = \int_{x_{b,a,\beta}}^{1} f(x;b,a)dx = x_{b,a,\beta}. \tag{34}$$

From (33) and (34), as a consequence of B-distribution function symmetry, the relation between lower quantile of the distribution $f(x;a,b)$ and upper quantile of the distribution $f(x;b,a)$ given by

$$x_{a,b,1-\beta} = 1 - x_{b,a,\beta}, \tag{35}$$

where $x_{a,b,1-\beta}$ is lower quantile of $f(x;a,b)$ distribution function and $x_{b,a,\beta}$ is upper quantile of $f(x;b,a)$ distribution function.

It will be able to obtain the lower x_1 and upper x_2 confidence limits of the random variable x for $\beta = \beta_1$ and $\beta = \beta_2$ as

$$\begin{aligned} x_1 &= 1 - x_{b,a,\beta_1}, \\ x_2 &= x_{a,b,\beta_2}. \end{aligned} \tag{36}$$

Confidence limits $\tilde{p}_1$ and $\tilde{p}_2$, of an unknown probability $p = P(A)$ will be obtained by the substitution $1-p \rightarrow x$ into the equation (30)

$$\begin{aligned} \tilde{p}_1 &= 1 - x_2 \qquad \text{for} \qquad \beta_2 = \alpha_1, \\ \tilde{p}_2 &= 1 - x_1 \qquad \text{for} \qquad \beta_1 = \alpha_2. \end{aligned} \tag{37}$$

where α_1 is assumed lower risk and α_2 is assumed upper risk.

Finally, introducing (36) in (37) it is obtained

$$\tilde{p}_1 = 1 - x_{a,b,\alpha_1}, \tag{38.1}$$

$$\tilde{p}_2 = 1 - x_{b,a,\alpha_2}. \tag{38.2}$$

The determined lower confidence limit $\tilde{p}_1$ of an unknown probability $p = P(A)$ is given by expression (38.1), using upper quantile of the B-distribution function $f(x;a,b)$ with parameters $a = n - m$ and $b = m + 1$ for assumed lower risk α_1. The exactly determined upper confidence limit $\tilde{p}_2$ is given by expression (38.2).

Based on the analytical results given by expressions (38), the computer program is developed [4], for calculation the confidence limits $[\tilde{p}_1, \tilde{p}_2]$ as the function of test results (n-number of trials and m-number of success) and assumed lower and upper risk $\alpha_1 = \alpha_2 = \alpha$ as variable parameters. This statistical method provides insight into the significance of above mentioned parameters, and also benchmark against which practical realization of the MSENC can be checked.

Since in the experimental part of this paper six MSENC are tested, after imputing number of trials (n) and assumed high level of confidence $\wp = 0.9$ ($\alpha_1 = \alpha_2 = 0.05$) into computer program, the obtained results of confidence limits are presented in a tabulated form in Table 1.

Table 1. The confidence limits of $p = P(A)$

	$n = 6$; $\alpha_1 = \alpha_2 = 0.05$	
$n = \{0, 1, \dots, 5\}$	$\tilde{p}_1$	$\tilde{p}_2$
0	0.008512	0.393038
1	0.062850	0.581803
2	0.153161	0.728662
3	0.271338	0.846839
4	0.418197	0.931150
5	0.606962	0.991488

3. Experimental Determination of the Shielding Effectiveness

Shielding performance specifications especially refer to the shielding effectiveness of a shielded enclosure, that is, the amount of attenuation of unwanted energy required *versus* the type of field (electric, magnetic, plane wave or microwave) and frequency. In this work, SE is measured in accordance with standard [5], E 1851 ASTM. The primary objective of this standard was to provide uniform test techniques. Other objectives were: to develop a test method, to produce a high repeatability and excellent reliability, and to minimize uncertainty in the measuring of SE. The method is targeted for a room-size transportable and modular shielded enclosure, that does not have any equipment, or equipment racks in it. Five specified frequencies are used for testing, two in magnetic field and three in plane wave field, as given by (39)

$$F[MHz] = \{0.01; 0.15; 30; 400; 1000\} \tag{39}$$

The simplified view of plane wave SE measurement is given in Fig. 3. Transmitting antenna is placed outside the enclosure, and the receiving antenna is inside, so their propagation axes are collinear and perpendicular to the surface of the enclosure at a test point. Receiving antenna inside the enclosure has to move to scan the entire test point area to cover all seams with its antenna plane, parallel to the plane of transmitting antenna. At the point of highest signal, receiving antenna is moved about 0.5 wavelength to measure the local maximum signal. During the measurement, antenna is rotated to explore both the horizontal and vertical polarization.

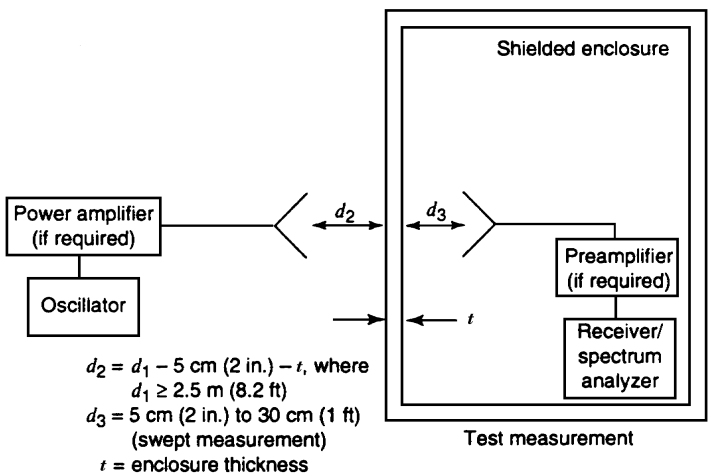

Fig. 3. Simplified test setup of plane wave SE test procedure.

The second objective in developing the test method was a clearly defining the test points on surfaces of the enclosure. Figure 4 shows test points assigned on a front wall and a part of side wall.

Such assignment of the test points permits recording the measuring results of SE in the form of binary functional matrix (BFM), which can be further processed using the quantiles of B-distribution statistical method. This is an adequate approach for six-sided clamp-up a MSENC system, which uses galvanized metal sheet (partial shield) bonded to both sides on a wood core and tied together by framework made by plated steel. With reference to Fig. 4. SE of the partial shield $E^{(i)}$ may be written in matrix form as follows

$$E^{(i)}(f_j) = \|e_{kl}^{(i)}(f_j)\|_{m,n}; \quad f_j \in F; \quad i = 1, 2, \ldots, w, \tag{40}$$

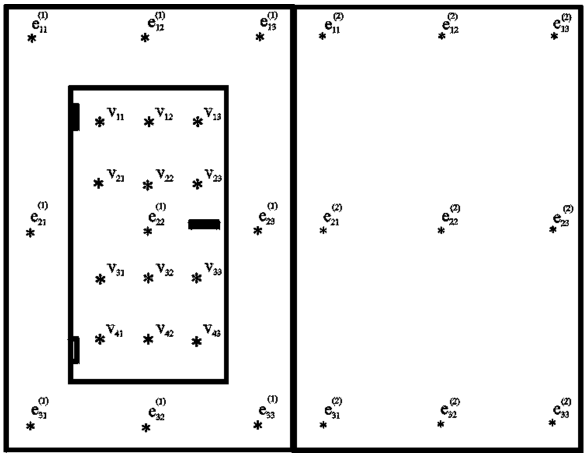

Fig. 4. Front and side wall test point assignments.

where $E^{(i)}(f_j)$ is the binary functional matrix, $e_{kl}^{(i)}(f_j)$ is the element of matrix $E^{(i)}(f_j)$, f_j is test frequency and F is set of test frequencies.

The matrix $E^{(i)}(f_j)$ is a functional one, because its elements are function of test frequencies on the set F.

$$F = \{f_1, f_2, \ldots, f_p\}. \tag{41}$$

The matrix $E^{(i)}(f_j)$ is a binary one, because its elements may have only two values "1" (one) or "0" (zero). If in the test point $e_{kl}^{(i)}(f_j)$ of partial shield $E^{(i)}$ at the frequency f_j the measurement result $S_{kl}^{(i)}(f_j)$ of SE is greater or equal to the required limit $S_0(f_j)$, then $e_{kl}^{(i)}(f_j) = 1$. If the measurement result of SE is less then $S_0(f_j)$, $e_{kl}^{(i)}(f_j) = 0$. This consideration may be written as follows

$$e_{kl}^{(i)}(f_j) = \begin{cases} 1; & S_{kl}^{(i)}(f_j) \geq S_0(f_j) \\ 0; & S_{kl}^{(i)}(f_j) < S_0(f_j). \end{cases} \tag{42}$$

If the partial shield $E^{(i)}$ does meet requirement at the frequency f_j, all elements of the matrix will be equal to one, but if not, one, a few or all-corresponding elements will be equal to zero. If the partial shield $E^{(i)}$ fulfills the required limit at frequency f_j, in accordance with preliminary statement, then the next equation will be satisfied

$$g^{(i)}(f_j) = \prod_{k=1}^{m} e_{kl}^{(i)}(f_j) = 1; \quad l = 1, 2, \ldots, n. \tag{43}$$

If the partial shield does meet requirement on the set of frequency F, then it is

$$G^{(i)} = \prod_{j=1}^{p} g^{(i)}(f_j) = 1. \tag{44}$$

Let the tested MSENC has w partial shields, then fulfilling of the requirement may be expressed as

$$G^{(1)} G^{(2)} \dots G^{(w)} = 1. \tag{45}$$

If the requirements are fulfilled in the all test points and on the all test frequencies which belong to set F, then the event A (success) is realized. Then, the following relation between the final SE result of MSENC testing and the realization of random event A, may be written as

$$G = \prod_{i=1}^{w} G^{(i)} = 1 \quad \Longleftrightarrow A. \tag{46}$$

4. Results

After testing of six MSENC, two of them did not meet the specified requirements on test frequency f_5. Test results of sample number 1 (MSENC-1) in BFM form are given by matrices (47) and (48)

$$V(f_5) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \tag{47}$$

$$E^{(10)}(f_5) = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}. \tag{48}$$

Test result of sample number 3. (MSENC-3) is given by matrix (49)

$$E^{(3)}(f_5) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}. \tag{49}$$

Obtained results are given in Table 2, where the success (occurrence of the random event A) is denoted by "1" and non-success (non-occurrence of the random event A) is denoted by "0".

According to the test results presented in Table 2, $n = 6$ and $m = 4$, it can be stated that the confidence limits $\tilde{p}_1 = 0.418$ and $\tilde{p}_2 = 0.931$ cover the unknown probability $p = P(A)$ of the event A with confidence $\wp = 0.9$ ($\alpha_1 = \alpha_2 = 0.05$).

Table 2. Occurrence of the random event A

f_i / G_{nj}	f_1	f_2	f_3	f_4	f_5	Event A
G_{1j}	1	1	1	1	0	0
G_{2j}	1	1	1	1	1	1
G_{3j}	1	1	1	1	0	0
G_{4j}	1	1	1	1	1	1
G_{5j}	1	1	1	1	1	1
G_{6j}	1	1	1	1	1	1

5. Conclusion

In this paper the contemporary approach to the probabilistic estimation of achieved SE of the MSENC, is presented. By application of the quantile of B-distribution statistical method on the experimental test results it is established that confidence limits $\tilde{p}_1 = 0.418$ and $\tilde{p}_2 = 0.931$ cover the unknown probability of the random event A realization with confidence $\wp = 0.9$ $(\alpha_1 = \alpha_2 = 0.05)$.

Sample size $(n = 6)$ from the statistical point of view is small, but the main question this paper answers is: which statistical method is the most appropriate in giving confidence limits for the whole serie with high confidence level, on the base of the test results of a small sample, if the tested series are always small and testing or corrections in the construction are too expensive. From the entire tables of the confidence limits for n= 1 to 30 and m=2 to 29 (the table 1. is only a part of them) it is evident that by increasing of the sample size, only lower confidence limit is increasing significantly. It is tending to the real value of the unknown probability P (A) while upper one is decreasing very slowly. This is inherent and the main advantage of the applied two parameter statistical method that it gives satisfactory results with small sample size $(n \leq 10)$.

By this and many other examples of realized SE testing over many years, it is established that there exists excellent correlation between the obtained test results and these predicted by theory. Additionally, this is a unique method yielding a possibility of SE value estimation, because none of the current numerical methods: MM (The Method of Moments), FE (The Finite Element Method), FD-TD (The Finite Difference Time-Domain Method) and TLM (The Transmission-line Modeling Method) can be applied to large electric structures, together with broad frequency coverage and the extremely large dynamic range of the electro-magnetic field.

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