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JENSEN'S INEQUALITY AS A CRITERION FOR COMPARISON OF BURSTY AND RANDOM ERRORS IMPACT

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Abstract. Use of Jensen's inequality as a criterion for comparison of impact of bursty and random errors on digital and signaling channels is presented in this paper. Some examples show that the availability of digital 64kb/s channels and CCS no 7 links is smaller under bursty errors. The CCS no 7 queueing delay time may be longer or shorter under bursty errors depending on MSU length and signaling load.

Key words: Digital communications, probability, availability.

1. Introduction

Jensen's inequality, [1], describes relationship between mean value of argument (or random variable) and mean value of a convex (or concave) function. In the analysis of the impact of errors on digital transmission the models with uniformly distributed or random errors are dominant. It is very interesting to calculate the impact of so called bursty errors or to compare models with random to models with bursty errors.

Model of channel with one kind of bursty errors, known as Gilbert-Elliott model, is described in [2]. As a result of measurement this model is described in [3], also, under the name "model with bimodal errors". In this short paper we use the Jensen's inequality to compare the impact of random and bursty errors on digital and signaling channel characteristics.

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2. Jensen's Inequality

Jensen's inequality states: if f is a convex function and X is a random variable then

$$E(f(X)) \geq f(E(X)) \tag{1}$$

where $E(V)$ is the mean value of variable V.

A function is convex (concave) if it always lies below (above) any chord.

For strictly convex function $E(f(X)) > f(E(X))$ holds and we always observe strictly convex (concave) functions in this paper. For concave (strictly concave) function, a reverse inequality holds.

3. Model of Digital Channel with Two Error Rates

The channel with two possible states is observed, like in [4]. It is the model similar to Gilbert-Elliott model, with the state of low (G) and the state of high (B) error rate, Fig. 23 in Ref. [4]. Errors are uniformly distributed in both states G and B, like in [4]. The time duration of being in state G(B) is random variable $T_g(T_b)$ with mean value $t_g(t_b)$. We assume that t_b is long enough that a stationary state may be established in digital or signaling channel.

The probability that the channel is in state G(B) is $P_g(P_b)$, $P_g+P_b=1$, $P_g/P_b=t_g/t_b$. The probability that the channel moves from state G to state B (from state B to state G) is $P_{gb}(P_{bg})$. It is clear that $P_g=P_{bg}/(P_{gb}+P_{bg})$ and $P_b=P_{gb}/(P_{gb}+P_{bg})$.

The bit error rate may be considered as a random variable which takes value $BERg(BERb)$ with probability $P_g(P_b)$. This model is called the model with bursty errors.

4. Comparison of Impact of Random and Bursty Errors

We observe the function $f(BER)$ that represents the dependence of some property or characteristic f of digital or signaling channel on BER (Bit Error Rate). Under random errors this property is designated with $f_r(BER)$ and under bursty errors with $f_{bur}(BER)$. This property may be: mean available and unavailable time of digital 64kb/s channel, mean queueing delay time of MSU sending on CCS (Common Channel Signaling) no 7 link, (un)availability of digital 64kb/s channel and CCS no 7 link etc. Let this function be strictly convex between points $A(BERg, f_r(BERg))$ and $B(BERb, f_r(BERb))$. Imagine the chord between points A and B. We can say the following:

The function $f_r(BER)$ between points A and B represents the set of values of property f for random BER and for each mean BER value, also.

$$f_r(BER) = f(E(BER)) = f(P_g BERg + P_b BERb), \quad P_g, P_b \in [0,1] \quad (2)$$

The chord between points A and B represents the set of mean values of properties f in the case of bursty errors i. e. f_{bur}

$$f_{bur}(BER) = E(f(BER)) = P_g f(BERg) + P_b f(BERb), \quad P_g, P_b \in [0,1] \quad (3)$$

According to Jensen's inequality it holds

$$\begin{aligned} f_{bur}(BER) =& P_g f(BERg) + P_b f(BERb) \\ & > f(P_g BERg + P_b BERb) = f_r(BER) \end{aligned}$$

We conclude that the property f, expressed by strictly convex function, has the greater values under bursty errors than under random errors.

For property f expressed by concave function f, the reverse conclusion is valid.

5. Examples

Example 1. We observe the functions of mean available, f_{ta}, and unavailable, f_{tu}, time of digital 64 kb/s channel on BER, Fig. 1. We see that both functions are strictly convex so the mean available and unavailable times are greater under bursty errors than under random errors. This is, especially, expressed if the values $BERg$ and $BERb$ are on the different sides of the knee of the function $f_{ta}(f_{tu})$. Similar conclusion may be drawn for mean changeover time and mean alignment time of a CCS no 7 link.

Example 2. The functions of availability, f_A, and unavailability, f_U, of the digital 64 kb/s channel, Fig. 2, are both concave. As a consequence the availability and unavailability of a digital 64 kb/s channel are greater under random errors than under bursty errors in the region of great f_A and f_U. Similar conclusion holds in the case of (un)availability of signaling CCS no 7 channel.

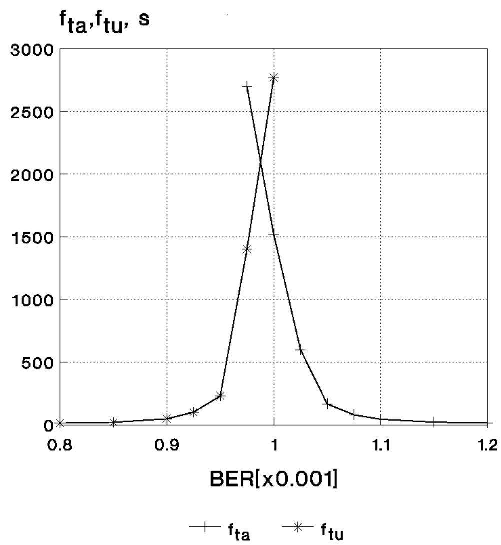

Fig. 1. Mean available time f_{ta} and mean unavailable time t_{tu} against BER (random errors) for digital 64 kb/s channel.

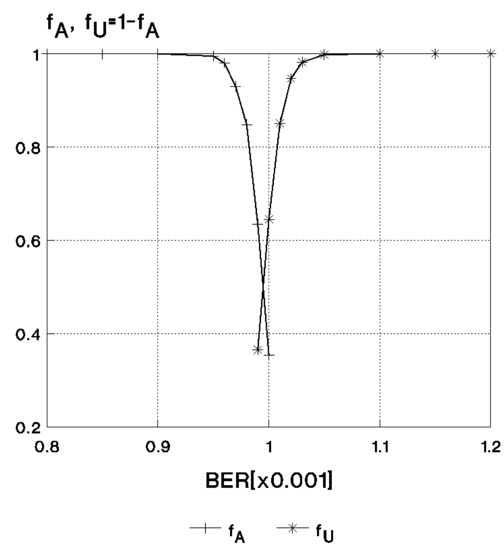

Fig. 2. Availability f_A and unavailability f_U of 64 kb/s channel against BER (random) calculated according to ITU-T Rec. G.821.

Example 3. Very interesting example is function of mean queueing delay time (*mqdt*) for MSUs (Message Signal Unit) on BER, f_Q, in signaling over a CCS no 7 link, [6]. This function may be strictly concave (a), strictly convex (c) and very close to straight line (b), Fig 3. The shape of function depends on MSU length. Increase of signaling load enlarges the convexity of function f_Q. Increase of MSU length causes function f_Q to become more concave. Comparison of bursty and random errors impact on *mqdt* must be calculated for each case. In the case shown in Fig. 3. the impact is equal for MSU length of cca 60 octets.

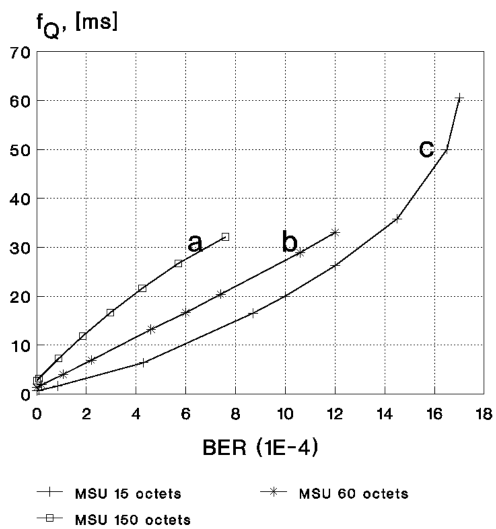

Fig. 3. Queueing delay time of CCSNo7 link f_Q against BER, $a = 0.2$, MSU lenght 15, 60 and 150 octets.

Example 4. We rearrange the formulae for calculation of MSU *mqdt*, f_Q, in the presence of disturbances on CCS no 7 link, ref. [6], table 2/Q.706, so that f_Q is dependent on BER instead on probability of errored MSU, P_u, as in [6]. Now, we can easily find the second derivative (by $MATHEMATICA$ for example), $f'' = \mathrm{d}f_Q/\mathrm{d}(BER)$, and determinate the convexity ($f'' > 0$) or concavity ($f'' < 0$).

6. Conclusion

Jensen's inequality may be used for comparison of digital and signaling channels properties under random and bursty errors. If the property of the digital channel is presented by convex function on BER the impact of bursty errors is greater and vice versa. If the property may be expressed as a mathematical function in explicit form then the impact of random or bursty errors may be estimated using the second derivative.

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